PASSIVITY PRESERVING MODEL REDUCTION VIA INTERPOLATION OF SPECTRAL ZEROS

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Abstract

An algorithm is developed for passivity preserving model reduction of LTI systems. The derivation is justified analytically and implementation schemes are developed for both medium scale (dense) and large scale (sparse) applications. The algorithm is based upon interpolation of specified spectral zeros of the original transfer function to produce a reduced transfer function that has the specified roots as its spectral zeros. These interpolation conditions are satisfied through the computation of a basis for a selected invariant subspace of a certain blocked matrix which has the spectral zeros as its spectrum.

1 Introduction

This paper is concerned with linear time invariant (LTI) systems

$$\mathbf{\Sigma}: \quad \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t),$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}, \mathbf{B} \in \mathbb{R}^{n\times m}, \mathbf{C} \in \mathbb{R}^{p\times n}, \mathbf{D} \in \mathbb{R}^{p\times m}$. These systems arise frequently in many branches of engineering. Circuit simulation is an important example and in this application the system $\mathbf{\Sigma}$ is often *passive*. Model reduction with a passivity constraint is of great importance in circuit simulation and has been studied by many researchers, including Rohrer [9], Ober [8], Feldmann, Freund, and Bai [6], [7], [3], [4], [5] Gugercin and Antoulas [10] and others.

The goal is to produce a reduced model of much lower order that preserves important system properties and response characteristics. Projection methods construct matrices $\mathbf{V} \in \mathbb{R}^{n\times k}$ and $\mathbf{W} \in \mathbb{R}^{n\times k}$ such that $\mathbf{W}^T\mathbf{V} = \mathbf{I}_k$ that are used to obtain a reduced model

$$\hat{\mathbf{\Sigma}}: \quad \dot{\hat{\mathbf{x}}}(t) = \hat{\mathbf{A}}\hat{\mathbf{x}}(t) + \hat{\mathbf{B}}\mathbf{u}(t), \quad \hat{\mathbf{y}}(t) = \hat{\mathbf{C}}\hat{\mathbf{x}}(t) + \mathbf{D}\mathbf{u}(t), \quad (1)$$

where

$$\hat{\mathbf{A}} = \mathbf{W}^T\mathbf{A}\mathbf{V}, \ \hat{\mathbf{B}} = \mathbf{W}^T\mathbf{B}, \ \hat{\mathbf{C}} = \mathbf{C}\mathbf{V}. \quad (2)$$

A new projection method is developed here that preserves both stability and passivity. This method is quite novel because it obtains the projection matrices $\mathbf{V}, \mathbf{W}$ as a by-product of a certain eigenvalue problem. These matrices are constructed from a basis set for an invariant subspace associated with that problem. The mathematical foundation for the results follow from the recent work of Antoulas [1] characterizing passivity through interpolation conditions.

2 Passive Systems

Throughout the remainder of this discussion, it shall be assumed that $m = p$, i.e., that $\mathbf{B} \in \mathbb{R}^{n\times p}$, $\mathbf{C} \in \mathbb{R}^{p\times n}$. Moreover, the matrix $\mathbf{A}$ is assumed to be stable (the spectrum $\sigma(\mathbf{A})$ is contained in the open left half-plane), and that the system $\mathbf{\Sigma}$ is both observable and controllable. Finally, it is assumed that the system is passive and that $\mathcal{D} := \mathbf{D} + \mathbf{D}^T$ is positive definite so that $\mathcal{D} = \mathbf{W}_o^T\mathbf{W}_o$ with $\mathbf{W}_o$ nonsingular. The transfer function for $\mathbf{\Sigma}$ is denoted by $\mathbf{G}(s) = \mathbf{C}(s\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$. The results of this section are stated without proofs. For a complete discussion of these results including detailed proofs, see [11].

Passive Systems: Informally, a system $\mathbf{\Sigma}$ is passive if it cannot produce energy and strictly passive if it consumes energy. Formally, $\mathbf{\Sigma}$ is

Passive if $Re \int_{-\infty}^{t} \mathbf{u}(\tau)^T\mathbf{y}(\tau)\mathrm{d}\tau \geq 0$ for all $t \in \mathbb{R}$ and all $\mathbf{u} \in \mathcal{L}_2(\mathbb{R})$,

Strictly Passive if there is a $\delta > 0$ such that $Re \int_{-\infty}^{t} \mathbf{u}(\tau)^T\mathbf{y}(\tau)\mathrm{d}\tau \geq \delta \int_{-\infty}^{t} \mathbf{u}(\tau)^T\mathbf{u}(\tau)\mathrm{d}\tau$ for all $t \in \mathbb{R}$ and all $\mathbf{u} \in \mathcal{L}_2(\mathbb{R})$.

There is an equivalent condition for LTI systems that is more easily verified.

Positive Real: The system $\mathbf{\Sigma}$ is passive if and only if its transfer function $\mathbf{G}(s)$ is *positive real* , which means that:

(1) $\mathbf{G}(s)$ is analytic for $Re(s) > 0$),
(2) $\mathbf{G}(\bar{s}) = \overline{\mathbf{G}(s)}$ for all $s \in \mathbb{C}$,
(3) $\mathbf{G}(s) + (\mathbf{G}(s))^* \succeq \mathbf{0}$ for $Re(s) > 0$.

For real systems, (2) is always satisfied. Property (3) implies the existence of a stable rational matrix function $\mathbf{W}(s)$ (with stable inverse) such that $\mathbf{G}(s) + \mathbf{G}^T(-s) = \mathbf{W}(s)\mathbf{W}^T(-s)$. This is the *spectral factorization* of $\mathbf{G}$ and the quantity $\mathbf{W}$ is a *spectral factor* of $\mathbf{G}$. The numbers λ_i, $i = 1, \cdots, n$, such that $\det\mathbf{W}(\lambda_i) = 0$, are the *spectral zeros* of $\mathbf{G}$.

Let

$$\mathcal{A} := \begin{bmatrix} \mathbf{A} & & \mathbf{B} \\ & -\mathbf{A}^T & -\mathbf{C}^T \\ \mathbf{C} & \mathbf{B}^T & \mathbf{D}+\mathbf{D}^T \end{bmatrix} \text{ and } \mathcal{E} := \begin{bmatrix} \mathbf{I} & & \\ & \mathbf{I} & \\ & & \mathbf{0} \end{bmatrix}.$$

Then the spectral zeros of $\mathbf{G}$ are the set of all (finite) complex numbers λ such that

$$Rank(\mathcal{A} - \lambda\mathcal{E}) < 2n + p,$$

i.e. the finite generalized eigenvalues $\sigma(\mathcal{A}, \mathcal{E})$. The set of spectral zeros shall be denoted as $\mathcal{S}_{\mathbf{G}}$. It is easily seen that $\lambda \in \mathcal{S}_{\mathbf{G}} \Rightarrow -\bar{\lambda} \in \mathcal{S}_{\mathbf{G}}$ since $\mathcal{A}\mathbf{q} = \mathcal{E}\mathbf{q}\lambda \Rightarrow \tilde{\mathbf{q}}^T\mathcal{A} = (-\bar{\lambda})\tilde{\mathbf{q}}^T\mathcal{E}$, where $\mathbf{q}^* := [\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*]$, and $\tilde{\mathbf{q}}^* := [\mathbf{y}^*, -\mathbf{x}^*, \mathbf{z}^*]$.

Now, suppose a reduced model $\hat{\mathbf{\Sigma}}$ as defined in (1) has been obtained and let $\hat{\mathbf{G}}(s) := \hat{\mathbf{C}}(s\mathbf{I} - \hat{\mathbf{A}})^{-1}\hat{\mathbf{B}}$ be the reduced transfer function. It is desirable to place conditions on this reduced system that provide for the inheritance of passivity from the original system. The approach taken here is entirely motivated by the following theorem of Antoulas proved in [1] that is stated here in a form that is restricted to the problem at hand. This result indicates that a passive reduced model will be obtained if certain of the spectral zeros are preserved (interpolated) in the reduced model. For real systems, $\mathcal{S}_{\hat{\mathbf{G}}}$ must include conjugate pairs of spectral zeros as well as their reflections across the real axis.

Theorem 2.1 *(Antoulas) If $\mathcal{S}_{\hat{\mathbf{G}}} \subset \mathcal{S}_{\mathbf{G}}$ and $\hat{\mathbf{G}}(\lambda) = \mathbf{G}(\lambda)$ for all $\lambda \in \mathcal{S}_{\hat{\mathbf{G}}}$, then the reduced system $\hat{\mathbf{\Sigma}}$ is both stable and passive.*

The following discussion will establish that these interpolation conditions can be satisfied if a basis for an specified invariant subspace of $(\mathcal{A}, \mathcal{E})$ can be constructed. Suppose $\mathcal{A}\mathbf{Q} = \mathcal{E}\mathbf{Q}\mathbf{R}$ is a partial real Schur decomposition for the pair $(\mathcal{A}, \mathcal{E})$. Thus $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$ and $\mathbf{R}$ is real and quasi-upper triangular. Let $\mathbf{Q}^T = [\mathbf{X}^T, \mathbf{Y}^T, \mathbf{Z}^T]$ be partitioned in accordance with the block structure of $\mathcal{A}$. Then

$$\begin{bmatrix} \mathbf{A} & & \mathbf{B} \\ & -\mathbf{A}^T & -\mathbf{C}^T \\ \mathbf{C} & \mathbf{B}^T & \mathbf{D}+\mathbf{D}^T \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{bmatrix} = \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{0} \end{bmatrix} \mathbf{R}. \tag{3}$$

In the following discussion, it will be useful to have the following lemma.

Lemma 2.1 *Suppose that $\mathbf{R}$ in (3) satisfies $Re(\lambda) > 0$ for all $\lambda \in \sigma(\mathbf{R})$. Then $\mathbf{X}^T\mathbf{Y} = \mathbf{Y}^T\mathbf{X}$ is symmetric.*

The matrices $\mathbf{X}$ and $\mathbf{Y}$ will be used to construct the matrices $\mathbf{W}$ and $\mathbf{V}$ with $\mathbf{W}^T\mathbf{V} = I$. In this construction, it will be useful to know something about the ranks of $\mathbf{X}$ and $\mathbf{Y}$.

Lemma 2.2 *If $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{R}$ are as specified in equation (3), then $\mathbf{X}$ and $\mathbf{Y}$ are both full rank. Moreover,*

$$\mathbf{X}^T\mathbf{A}^T\mathbf{Y} + \mathbf{Y}^T\mathbf{A}\mathbf{X} = \mathbf{Z}^T\mathcal{D}\mathbf{Z} \tag{4}$$

$$\mathbf{C}\mathbf{X} + \mathbf{B}^T\mathbf{Y} = -\mathcal{D}\mathbf{Z}. \tag{5}$$

With these observations in hand, consider the following construction of $\mathbf{V}$ and $\mathbf{W}$. First, find a basis for an invariant subspace with all eigenvalues of $\mathbf{R}$ in the open right half-plane as in (3) above. Let $\mathbf{Q}_x\mathbf{S}^2\mathbf{Q}_y^T = \mathbf{X}^T\mathbf{Y}$ be the SVD of $\mathbf{X}^T\mathbf{Y}$ and note that $\mathbf{Q}_y = \mathbf{Q}_x\mathbf{J}$ where $\mathbf{J}$ is a signature matrix by virtue of the symmetry of $\mathbf{X}^T\mathbf{Y}$. For this discussion $\mathbf{S}$ is assumed to be nonsingular. The singular case may be handled by truncation (see [11] for details). With $\mathbf{S}$ nonsingular, put

$$\mathbf{V} = \mathbf{X}\mathbf{Q}_x\mathbf{S}^{-1}, \quad \mathbf{W} = \mathbf{Y}\mathbf{Q}_y\mathbf{S}^{-1}.$$

Then it is easily seen that $\mathbf{W}^T\mathbf{V} = \mathbf{I}$. Let $\hat{\mathbf{X}} := \mathbf{S}\mathbf{Q}_x^T$ and $\hat{\mathbf{Y}} := \mathbf{S}\mathbf{Q}_y^T$ and define

$$\mathcal{V} := \begin{bmatrix} \mathbf{V} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{W} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}, \text{ and } \mathcal{W} := \begin{bmatrix} \mathbf{W} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{V} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}.$$

Now, observe that $\mathcal{W}^T\mathcal{V} = \mathbf{I}$ and it is straightforward to see that

$$\hat{\mathcal{A}} := \mathcal{W}^T\mathcal{A}\mathcal{V} = \begin{bmatrix} \hat{\mathbf{A}} & & \hat{\mathbf{B}} \\ & -\hat{\mathbf{A}}^T & -\hat{\mathbf{C}}^T \\ \hat{\mathbf{C}} & \hat{\mathbf{B}}^T & \mathbf{D}+\mathbf{D}^T \end{bmatrix},$$

and

$$\begin{bmatrix} \hat{\mathbf{A}} & & \hat{\mathbf{B}} \\ & -\hat{\mathbf{A}}^T & -\hat{\mathbf{C}}^T \\ \hat{\mathbf{C}} & \hat{\mathbf{B}}^T & \mathbf{D}+\mathbf{D}^T \end{bmatrix} \begin{bmatrix} \hat{\mathbf{X}} \\ \hat{\mathbf{Y}} \\ \mathbf{Z} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{X}} \\ \hat{\mathbf{Y}} \\ \mathbf{0} \end{bmatrix} \mathbf{R}. \tag{6}$$

This shows that the spectral zeros $\mathcal{S}_{\hat{\mathbf{G}}}$ are a subset of the spectral zeros $\mathcal{S}_{\mathbf{G}}$ of the original system. Moreover, since $\mathcal{S}_{\hat{\mathbf{G}}} = \sigma(\mathbf{R}) \cup \sigma(-\mathbf{R}^T)$ and $\sigma(\mathbf{R})$ is in the open right half-plane, the reduced model has no spectral zeros on the imaginary axis.

It turns out that this construction gives a reduced model $\hat{\mathbf{\Sigma}}$ that is stable and passive. One could apply Antoulas' result in Theorem 2.1 to establish this. However, it is instructive to prove passivity and stability directly from the construction. This will be established with the following results.

First, it is useful to note that $\hat{\mathbf{Y}}\hat{\mathbf{X}}^{-1} = \mathbf{S}\mathbf{Q}_y^T\mathbf{Q}_x\mathbf{S}^{-1} = \mathbf{J}$, since $\mathbf{Q}_y = \mathbf{Q}_x\mathbf{J}$.

Lemma 2.3 *The reduced model $\hat{\mathbf{\Sigma}}$ satisfies*

$$\begin{aligned} \hat{\mathbf{A}}^T(-\mathbf{J}) + (-\mathbf{J})\hat{\mathbf{A}} &= -\mathbf{C}_o^T\mathbf{C}_o, \\ \hat{\mathbf{B}}^T(-\mathbf{J}) + \mathbf{W}_o^T\mathbf{C}_o &= \hat{\mathbf{C}}, \\ \mathbf{D}+\mathbf{D}^T &= \mathbf{W}_o^T\mathbf{W}_o, \end{aligned}$$

where $\mathbf{C}_o := -\mathbf{W}_o\mathbf{Z}\hat{\mathbf{X}}^{-1}$.

The results of Lemma 2.3 may be used to verify the necessary conditions for $\hat{\mathbf{\Sigma}}$ to be positive real required by the Positive Real Lemma [2] (Theorem (13.25) in [12]).

Lemma 2.4 *If $\mathbf{J} = -\mathbf{I}$ in Lemma (2.3), then $\mathbf{G}$ is positive real and the reduced order system $\hat{\mathbf{\Sigma}}$ is stable and passive.*

Passivity of the reduced system is established then by demonstrating that $\mathbf{J} = -\mathbf{I}$ follows from the passivity of the original system $\mathbf{\Sigma}$. To do this, it is sufficient to show that $\hat{\mathbf{X}}^T\hat{\mathbf{Y}}$ is negative definite.

Lemma 2.5 *The matrix $\hat{\mathbf{X}}^T\hat{\mathbf{Y}}$ in Lemma (2.3) is symmetric and negative definite.*

This result is obtained for strictly positive real systems by extending the partial Schur decomposition to one that includes all n of the spectral zeros of $\mathbf{\Sigma}$ in the open right half-plane. This gives extended matrices $\tilde{\mathbf{X}} := [\mathbf{X}, \mathbf{X}_2]$ in place of $\mathbf{X}$, $\tilde{\mathbf{Y}} = [\mathbf{Y}, \mathbf{Y}_2]$ in place of $\mathbf{Y}$, $\tilde{\mathbf{Z}} := [\mathbf{Z}, \mathbf{Z}_2]$ in place of $\mathbf{Z}$. Then Lemma 2.2 is applied to this system to conclude that $\tilde{\mathbf{X}}^T\tilde{\mathbf{Y}}$ is symmetric and negative semi-definite by showing it is similar to the solution $\mathcal{Q}$ of a certain Lyapunov equation of the form $\mathbf{A}^T\mathcal{Q} + \mathcal{Q}\mathbf{A} = \tilde{\mathbf{C}}_o^T\tilde{\mathbf{C}}_o$. Since $\mathbf{X}^T\mathbf{Y}$ is a leading principle submatrix of $\tilde{\mathbf{X}}^T\tilde{\mathbf{Y}}$, it must also be negative semi-definite. Moreover, since $\mathbf{Q}_x\mathbf{S}^2\mathbf{J}\mathbf{Q}_x^T = \hat{\mathbf{X}}^T\hat{\mathbf{Y}} = \hat{\mathbf{X}}^T\mathbf{V}^T\mathbf{W}\hat{\mathbf{Y}} = \mathbf{X}^T\mathbf{Y}$, it follows that the diagonal elements of $\mathbf{S}^2\mathbf{J}$ are non-positive. For every positive diagonal element of $\mathbf{S}$, the corresponding diagonal element of $\mathbf{J}$ must be negative, and since $\mathbf{S}$ was assumed to be nonsingular, it follows that $\mathbf{J} = -\mathbf{I}$. A continuity argument is used to establish the result for positive real systems.

3 Algorithms for Passivity Preserving Reduced Models

The results of the previous section establish the passivity of $\hat{\mathbf{\Sigma}}$ and, in addition, they establish that $\mathcal{S}_{\hat{\mathbf{G}}} \subset \mathcal{S}_{\mathbf{G}}$ corresponding to the spectral zeros appearing as eigenvalues of $\mathbf{R}$ in (3). This leads to the following algorithm.

function $[\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}]$ = Posreal$(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, k)$;

Compute a k-th order partial
real Schur decomposition
$\mathcal{A}\mathbf{Q} = \mathcal{E}\mathbf{Q}\mathbf{R}$;

$\mathbf{X} = \mathbf{Q}(1:n,:)$; $\mathbf{Y} = \mathbf{Q}(n+1:2n,:)$;
$[\mathbf{Q}_x, \mathbf{S}, \mathbf{Q}_y] = \mathrm{svd}(\mathbf{X}^T\mathbf{Y})$; $\mathbf{S} \leftarrow \mathbf{S}^{1/2}$;
$\mathbf{V} = \mathbf{X}\mathbf{Q}_x\mathbf{S}^{-1}$; $\mathbf{W} = \mathbf{Y}\mathbf{Q}_y\mathbf{S}^{-1}$;

$\hat{\mathbf{A}} = \mathbf{W}^T\mathbf{A}\mathbf{V}$; $\hat{\mathbf{B}} = \mathbf{W}^T\mathbf{B}$; $\hat{\mathbf{C}} = \mathbf{C}\mathbf{V}$;

Figure 1: Positive Real Model Reduction

In the algorithm shown in Figure 1, it is assumed that $\mathcal{A}$ and $\mathcal{E}$ represent the blocked matrices defined in Section 2. For small to medium scale dense problems, these matrices might actually be formed and then the desired partial Schur decompostion would be extracted from the full eigensystem. For large sparse problems, this would be impractical and inefficient. The algorithm as posed is appropriate for real matrices and all arithmetic stays real throughout. A real partial Schur decomposition is approproate since it will automatically keep complex congugate pairs of spectral zeros together. The parameter k that specifies the order of the reduced model will perhaps need to be adjusted by 1 to accomodate this. Modification of these algorithms to accomodate complex matrices is relatively straightforward.

For large scale problems, an implicitly restarted Arnoldi (IRA) method would be quite suitable. It naturally produces a partial real Schur form corresponding to a desired set of eigenvalues (spectral zeros here). One could use *eigs* in Matlab or ARPACK in Fortran to find such an invariant subspace. One choice for selecting the spectral zeros might be to compute the k eigenvalues of largest real part. However, there is another choice that seems quite natural and which works well with an IRA method.

A convenient spectral transformation is obtained with the Cayley transformation

$$\mathcal{C}_\mu := (\mu\mathcal{E} - \mathcal{A})^{-1}(\mu\mathcal{E} + \mathcal{A}),$$

where $\mu \geq 0$ is a real shift. With a proper choice of μ this will provide for rapid convergence to an invariant subspace corresponding to k transformed eigenvalues of largest magnitude:

$$(\mu\mathcal{E} - \mathcal{A})^{-1}(\mu\mathcal{E} + \mathcal{A})\mathbf{Q} = \mathbf{Q}\hat{\mathbf{R}}$$

so that

$$\mathcal{A}\mathbf{Q} = \mathcal{E}\mathbf{Q}\mathbf{R}, \text{ where } \mathbf{R} := \mu(\hat{\mathbf{R}} - \mathbf{I})(\hat{\mathbf{R}} + \mathbf{I})^{-1}.$$

This gives the partial Schur decomposition as required by the algorithm in Figure 1.

The implementation will require two sparse direct factorizations of

$$\mathbf{A} - \mu\mathbf{I} \text{ and } \mathbf{A} + \mu\mathbf{I}.$$

The Cayley transformation $\mathcal{C}_\mu$ may then be applied to an arbitrary vector using a blocked matrix-vector product followed by a blocked Gaussian elimination.

Interestingly enough, computing the k eigenvalues of largest magnitude for this Cayley transformation is related to computing k eigenvalues of largest real part for the original pair $(\mathcal{A}, \mathcal{E})$ in a very special way. A circle of radius $\rho > 1$ centered at the origin is the image of a circle $\mathcal{D}_\rho$ of radius $\frac{2\mu\rho}{\rho^2-1}$ centered at $\mu\frac{\rho^2+1}{\rho^2-1}$. If ρ is the radius of the circle centered at the origin drawn through the selected eigenvalue(s) of smallest magnitude, then the k selected eigenvalues are images of the spectral zeros interior to the circle $\mathcal{D}_\rho$ (shown in Figure 2) and this gives the interpolation points. As $\rho \to 1$ the interior circle $\mathcal{D}_\rho$ tends to include all of the right half-plane.

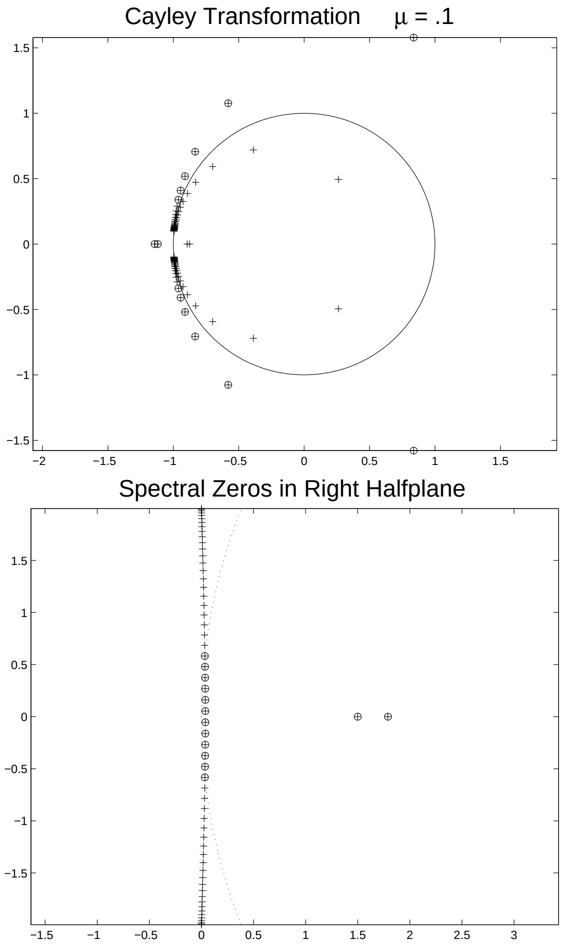

Figure 2: The Cayley transformed spectral zeros (+) and interpolation points selected (o) (reduced order spectral zeros).

Example. The following graphs show the result of applying this scheme to an RLC circuit of order 201. The circuit is an RLC ladder network. The state variables are as follows: x_1, the voltage across C_1; x_2, the current through L_1; x_3, the voltage across C_2; x_4, the current through L_2; and x_5, the voltage across C_3, etc. In general, n is odd and x_{2i-1} is the voltage across capacitor C_i for $i = 1, 2, \ldots \frac{n+1}{2}$, while x_{2i} is the current through inductor L_i for $i = 1, 2, \ldots \frac{n-1}{2}$. There are two resistors R_1, R_2 placed at either end of the "ladder" as shown in Figure 3 for an order $n = 5$ example.

The input is the voltage u and the output is the current y as shown in Figure 3. It is assumed that all the capacitors and inductors have unit value, while $R_1 = \frac{1}{2}$, $R_2 = \frac{1}{5}$. A minimal realization for the order $n = 5$ example is:

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -5 \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix},$$

$$\mathbf{C} = [0 \;\; 0 \;\; 0 \;\; 0 \;\; -2], \; \mathbf{D} = 1$$

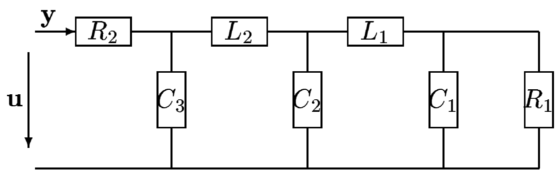

Figure 3: RLC ladder circuit of order 5

For general n The matrix $\mathbf{A}$ is simply extended with value 1 on the super diagonal, the value -1 on the subdiagonal and with the values -2, and -5 in the (1,1) and (n,n) positions. The vectors $\mathbf{B}, \mathbf{C}$ are extended in the obvious way by introducing zero entries in the first $n-1$ positions.

The method was applied to an RLC ladder system of order $n = 201$ using an IRA method with the Cayley transformation (shift $\mu = .1$). A reduced model of order 20 was constructed. The graphs in Figure 4 and Figure 5 illustrate the effectiveness of the procedure. The distribution of the spectral zeros are similar to those shown in Figure 2 for a smaller problem. That figure shows the effect of the transformation and the interpolation points.

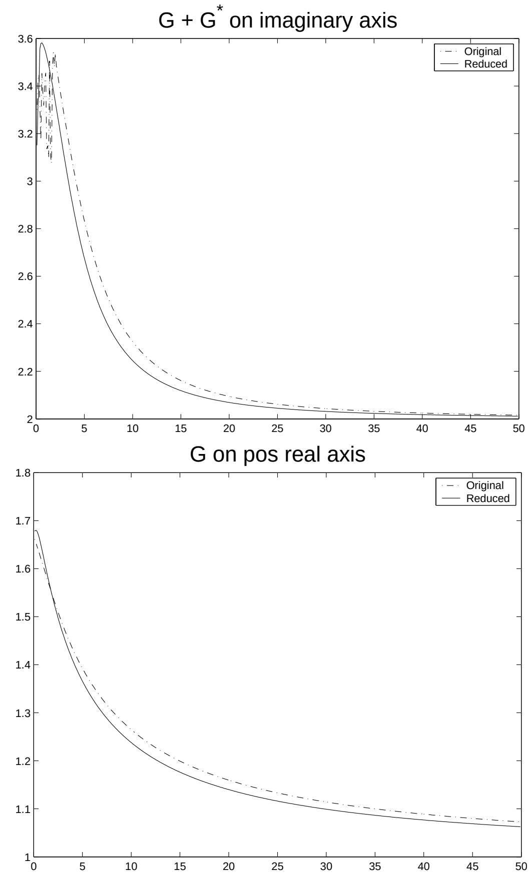

Figure 4: Plot of $\mathbf{G}^T(-i\omega) + \mathbf{G}(i\omega) \succeq 0$ for $\omega \in \mathbb{R}$ (left) and plot of $\mathbf{G}(s)$ real positive s (right) .

These results are encouraging, but limited. The particular ex-

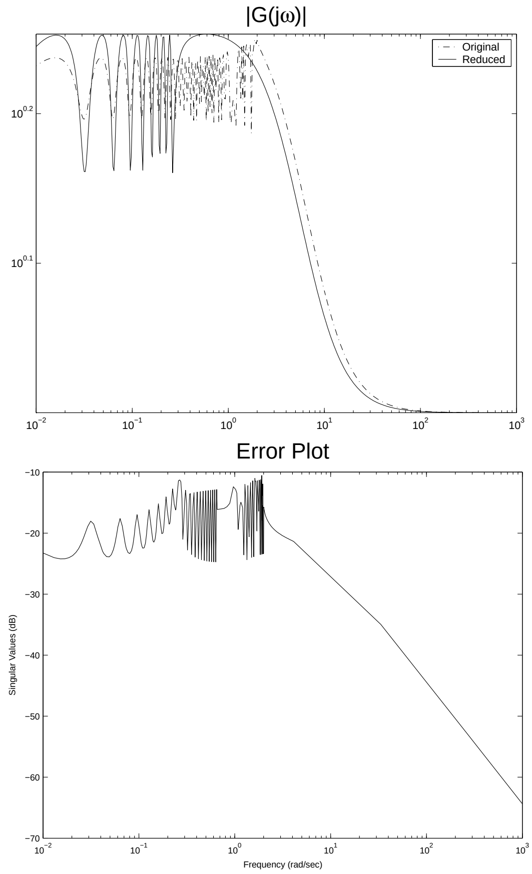

Figure 5: Comparison of absolute value of transfer functions on $j\omega$ axis (left) and sigma plot of the error (right).

ample given here is quite difficult to approximate with a low order reduced model. Additional research and experimentation is needed to better understand this approach. The following questions remain: 1) What is the best choice of interpolation points? 2) Is it possible to derive a bound on the approximation error? 3) What is the best choice for μ in the Cayley transformation? Future work will include a study of these questions as well as a far more exhaustive set of test examples.

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