DISCRETE SLIDING MODE CONTROL USING FAST OUTPUT SAMPLING FEEDBACK

C.M. Saaj*, **B. Bandyopadhyay**†

* Research Scholar, † Professor,
* † Interdisciplinary Programme in
Systems and Control Engineering,
Indian Institute of Technology,
Mumbai-400076, India.
Fax: +91 22 25723480
Tele: +91 22 25767889
e-mail: chakru@ee.iitb.ac.in,
bijnan@ee.iitb.ac.in

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Abstract

In this paper a new algorithm for discrete-time sliding mode control using only output samples is proposed. It is shown that the output feedback gain can be directly obtained using the reaching law of discrete-time sliding mode control. The main contribution of this work is that instead of using the system states, the output samples are used for designing the controller.

1 Introduction

Sliding mode, a particular mode of operation in variable structure control (VSC) systems, is a very powerful tool for control design [1]. In recent years a considerable amount of effort has been put in towards the controller design of digital sliding mode (DSM). The digital implementation of sliding mode control (SMC) for single-input single-output (SISO) systems is done in [2], [3]. In the case of discrete sliding mode, control action can only be activated at sampling instants and the control effort is constant over each sampling period. Also when the state reaches the switching surface, the subsequent discrete-time switching cannot generate the equivalent control to keep the state on the surface. As a result DSM can undergo only quasi-sliding motion i.e., the state of the system can approach the switching surface but cannot generally stay on it. Gao *et al.* [3] has used this reaching law approach to design the controller for discrete-time system using state feedback. Bartoszewicz [4] showed that for uncertain systems the control law introduced by Gao *et al.* [3] will satisfy the reaching condition and guarantee the existence of a quasi-sliding mode only if an additional inequality is satisfied.

Most of the sliding mode control methods require full-state feedback. But in practical situations measurement of all the system states might be neither possible nor feasible. Such situations would demand the need for some observers or dynamic compensators which would make the overall system more complex. Zak and Hui [5] has proposed a new class of variable structure output feedback controllers using the geometric approach. Diong has shown that for sliding mode invariance with stability of the dynamic output feedback SMC, the nominal plant has to be minimum-phase [6]. In [7], Bag *et al.* has proposed a dynamic output feedback strategy to design the sliding mode controller. This method can be used for systems which do not satisfy the 'Kimura-Davison' condition. The static output feedback problem is one of the most investigated problems in control theory and application [8]. It represents the simplest closed loop control that can be realized in practical situations. However no results are available till today which show that complete pole assignment is possible using static output feedback. Output feedback can be realized using fast output sampling feedback [9]. In [9], Werner used the fast output sampling (FOS) feedback which has the features of static output feedback and makes it possible to arbitrarily assign the system poles. Unlike static output feedback, fast output sampling feedback always guarantees the stability of the closed loop system. The application of FOS technique to discrete sliding mode control is a new area of research. The recent developments in the field of fast output sampling sliding mode control (FOSSMC) are reported in [10]-[15].

In [10], a fast output sampling sliding mode controller for nominal plant using the reaching law approach of [3] has been proposed. Here the output feedback gain $\boldsymbol{L}$ is obtained from the state feedback gain $\boldsymbol{F}$ using the relation $\boldsymbol{L}\tilde{\mathbf{C}} = \boldsymbol{F}$, proposed in [9]. If this relation holds good then $\boldsymbol{L}$ realizes the effect of $\boldsymbol{F}$. But in this method the actual output of the system needs to be corrected by a suitable correction factor before being used for feedback purpose. The purpose of this paper is twofold. Firstly, in this work a direct approach to obtain the output feedback gain $\boldsymbol{L}$ from the reaching law is proposed. Thus the method does not use the above mentioned relation connecting $\boldsymbol{L}$ and $\boldsymbol{F}$. Secondly, the requirement of output correction used in [10] is eliminated. Consider a SISO linear time invariant system described in discrete-time and under nominal conditions. It is shown that output feedback gain of the proposed fast output sampling sliding mode control can be directly obtained without using the relation $\boldsymbol{L}\tilde{\mathbf{C}} = \boldsymbol{F}$. The outline of this paper is as follows: a brief review of the preliminary results is introduced first followed by the new fast output sampling sliding mode control methodology. The design procedure is illustrated with a numerical example and finally concluding remarks are made.

2 Preliminary Results

2.1 Discrete Sliding Mode Control

Consider the following SISO linear time invariant system described in discrete-time and under nominal conditions.

$$\left.\begin{array}{lcl} \boldsymbol{x}(k+1) & = & \boldsymbol{\Phi}_\tau \boldsymbol{x}(k) + \boldsymbol{\Gamma}_\tau u(k), \\ y(k) & = & \boldsymbol{C}\boldsymbol{x}(k), \end{array}\right\} \tag{1}$$

where τ is the sampling period, $\boldsymbol{x} \in \mathbf{R}^n$, $u \in \mathbf{R}$, $y \in \mathbf{R}$ and the matrices $\boldsymbol{\Phi}_\tau$, $\boldsymbol{\Gamma}_\tau$, and $\boldsymbol{C}$ are of appropriate dimensions. It is assumed that the pair $(\boldsymbol{\Phi}_\tau, \boldsymbol{\Gamma}_\tau)$ is controllable and the pair $(\boldsymbol{\Phi}_\tau, \boldsymbol{C}\ )$ is observable. Gao *et al.* [3] has shown that a state feedback sliding mode control law for the system (1) can be expressed as

$$u(k) = \boldsymbol{F}\boldsymbol{x}(k) + \gamma sgn(s(k)), \tag{2}$$

where $s(k)$ is the switching function defined as

$$s(k) = \boldsymbol{c}^T \boldsymbol{x}(k) \tag{3}$$

and

$$\begin{array}{lcl} \boldsymbol{F} & = & -(\boldsymbol{c}^T\boldsymbol{\Gamma}_\tau)^{-1}[(\boldsymbol{c}^T\boldsymbol{\Phi}_\tau - \boldsymbol{c}^T\boldsymbol{I} + q\tau\boldsymbol{c}^T)], \\ \gamma & = & -(\boldsymbol{c}^T\boldsymbol{\Gamma}_\tau)^{-1}\epsilon\tau. \end{array}$$

The width of the quasi-sliding mode band, within which the system state remains in steady-state is given by [3], [4]

$$2\delta \leq \frac{2\epsilon\tau}{2 - q\tau}.$$

2.2 Fast Output Sampling Feedback

In fast output sampling technique [9], each sampling period τ is subdivided into N subintervals $\Delta = \tau/N$. Let $(\boldsymbol{\Phi}, \boldsymbol{\Gamma}, \boldsymbol{C})$ be the discrete-time system (1) sampled at rate $1/\Delta$. Let ν denote the observability index of $(\boldsymbol{\Phi}, \boldsymbol{C})$. N is chosen to be greater than or equal to ν. The last N output samples are measured at time instants $t = l\Delta$, $l = 0, 1, \ldots N-1$ and a constant control signal is applied over a period τ. Consider the discrete-time system having at time $t = k\tau$, the fast output samples

$$\boldsymbol{y}_k = \left[\begin{array}{cccc} y(k\tau - \tau) & y(k\tau - \tau + \Delta) & \ldots & y(k\tau - \Delta) \end{array}\right]'. \tag{4}$$

Then a representation for the discrete system (1) is [9]

$$\left.\begin{array}{lcl} \boldsymbol{x}(k+1) & = & \boldsymbol{\Phi}_\tau \boldsymbol{x}(k) + \boldsymbol{\Gamma}_\tau u(k), \\ \boldsymbol{y}_{k+1} & = & \boldsymbol{C}_0 \boldsymbol{x}(k) + \boldsymbol{D}_0 u(k), \end{array}\right\} \tag{5}$$

where $\boldsymbol{C}_0$ and $\boldsymbol{D}_0$ are as defined

$$\boldsymbol{C}_0 = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{C}\boldsymbol{\Phi} \\ \vdots \\ \boldsymbol{C}\boldsymbol{\Phi}^{N-1} \end{bmatrix}, \quad \boldsymbol{D}_0 = \begin{bmatrix} 0 \\ \boldsymbol{C}\boldsymbol{\Gamma} \\ \vdots \\ \boldsymbol{C}\sum_{j=0}^{N-2}\boldsymbol{\Phi}^j\boldsymbol{\Gamma} \end{bmatrix}.$$

Let $\boldsymbol{F}$ be an initial state feedback gain such that the closed loop system matrix $(\boldsymbol{\Phi}_\tau + \boldsymbol{\Gamma}_\tau\boldsymbol{F})$ has no eigenvalues at the origin. Then one can define a fictitious measurement matrix,

$$\tilde{\mathbf{C}}(\boldsymbol{F}, N) = (\boldsymbol{C}_0 + \boldsymbol{D}_0\boldsymbol{F})(\boldsymbol{\Phi}_\tau + \boldsymbol{\Gamma}_\tau\boldsymbol{F})^{-1} \tag{6}$$

which satisfies the fictitious measurement equation

$$\boldsymbol{y}_k = \tilde{\mathbf{C}}\boldsymbol{x}(k). \tag{7}$$

The control signal is constructed as a linear combination of the last N output samples and is given as [6]

$$u(k) = \boldsymbol{L}\boldsymbol{y}_k. \tag{8}$$

where $\boldsymbol{L}$ is derived from $\boldsymbol{F}$ using the relation $\boldsymbol{L}\tilde{\mathbf{C}} = \boldsymbol{F}$. It can be shown that for $N \geq \nu$, generically $\tilde{\mathbf{C}}$ has full column rank so that any state feedback gain can be realized by a fast output sampling gain $\boldsymbol{L}$.

3 Fast Output Sampling Sliding Mode Control

In [10], the following fast output sampling sliding mode control law is proposed

$$\left.\begin{array}{lcllll} u(k) & = & \boldsymbol{L}\bar{\boldsymbol{y}}_k + \gamma & \text{for} & s(k) > 0, \\ & = & \boldsymbol{L}\bar{\boldsymbol{y}}_k - \gamma & \text{for} & s(k) < 0. \end{array}\right\} \tag{9}$$

Here $\bar{\boldsymbol{y}}_k$ is the output samples obtained by correcting the actual output samples $\boldsymbol{y}_k$ of the system by a suitable correction factor. For this a correction algorithm is given in [10]. In this section a different and more direct approach to obtain the output feedback gain $\boldsymbol{L}$ from the reaching law is explained. Also this method eliminates the need for output correction done in [10].

3.1 Reaching phase

Assume that the initial conditions are such that the initial value of the switching function $s(0) > 0$. Then the initial control to be applied is $u(0) = \boldsymbol{F}\boldsymbol{x}(0) + \gamma$. Thereafter to determine the control law for the sampling instants $k = 1, 2, \ldots$ assume that in the reaching phase a control of the form $u(k) = \boldsymbol{F}\boldsymbol{x}(k) + \gamma$ is applied to the system (5). Then the output samples generated is

$$\boldsymbol{y}_k = \tilde{\mathbf{C}}\boldsymbol{x}(k) + \boldsymbol{\alpha}, \tag{10}$$

where

$$\boldsymbol{\alpha} = (\boldsymbol{D}_0 - \tilde{\mathbf{C}}\boldsymbol{\Gamma}_\tau)\gamma.$$

The relation for $s(k)$ in terms of the output samples is

$$s(k) = \boldsymbol{c}^T\tilde{\mathbf{C}}^{-1}(\boldsymbol{y}_k - \boldsymbol{\alpha}). \tag{11}$$

Begin with the incremental change of $s(k)$ which is

$$s(k+1) - s(k) = \boldsymbol{c}^T \tilde{\mathbf{C}}^{-1}(\boldsymbol{y}_{k+1} - \boldsymbol{y}_k). \tag{12}$$

Let

$$\mathbf{g}^T = \boldsymbol{c}^T \tilde{\mathbf{C}}^{-1}.$$

Substituting for $\boldsymbol{y}_{k+1}$ from (5) in (12) gives

$$s(k+1) - s(k) = \mathbf{g}^T(\boldsymbol{C}_0 \boldsymbol{x}(k) + \boldsymbol{D}_0 u(k) - \boldsymbol{y}_k). \tag{13}$$

Substituting for $\boldsymbol{x}(k)$ from (10) and comparing (13) to the reaching law proposed in [3] gives

$$-q\tau s(k) - \epsilon\tau sgn(s(k)) = \mathbf{g}^T(\boldsymbol{C}_0 \tilde{\mathbf{C}}^{-1}(\boldsymbol{y}_k - \boldsymbol{\alpha}) + \boldsymbol{D}_0 u(k) - \boldsymbol{y}_k).$$

Let $\lambda = \epsilon\tau$ and solving for $u(k)$ gives the FOSSMC law as

$$u(k) = \boldsymbol{L}\boldsymbol{y}_k + \eta(k), \tag{14}$$

where

$$\begin{aligned} \boldsymbol{L} &= -(\mathbf{g}^T \boldsymbol{D}_0)^{-1}[\mathbf{g}^T \boldsymbol{C}_0 \tilde{\mathbf{C}}^{-1} - \mathbf{g}^T + q\tau \mathbf{g}^T], \\ \eta(k) &= -(\mathbf{g}^T \boldsymbol{D}_0)^{-1}[-\mathbf{g}^T \boldsymbol{C}_0 \tilde{\mathbf{C}}^{-1} \boldsymbol{\alpha} - q\tau \mathbf{g}^T \boldsymbol{\alpha} + \lambda)]. \end{aligned}$$

This is the control law applied to the system for the time interval, $\tau \le t \le \kappa\tau$, where $\kappa\tau$ is the time instant just before the system trajectory crosses the switching plane first time.

3.2 Switching phase

Case a: $s(k) < 0$
The switching control should cause the system trajectory to recross the switching plane the instant it crosses the plane for the first time and thereafter in every successive sampling period, so as to satisfy the reaching condition. Assume that the control applied at this instant be $u(k) = \boldsymbol{F}\boldsymbol{x}(k) - \gamma$. Then it can be shown that the output samples generated by this control is

$$\boldsymbol{y}_{k+1} = \tilde{\mathbf{C}}\boldsymbol{x}(k+1) - \boldsymbol{\alpha}.$$

The incremental change of $s(k)$ is

$$s(k+1) - s(k) = \boldsymbol{c}^T \tilde{\mathbf{C}}^{-1}(\boldsymbol{y}_{k+1} - \boldsymbol{y}_k + 2\boldsymbol{\alpha}).$$

Proceeding as before the FOSSMC law is derived with $\boldsymbol{L}$ as in (14) and

$$\eta(k) = -(\mathbf{g}^T \boldsymbol{D}_0)^{-1}[-\mathbf{g}^T \boldsymbol{C}_0 \tilde{\mathbf{C}}^{-1} \boldsymbol{\alpha} + 2\mathbf{g}^T \boldsymbol{\alpha} - q\tau \mathbf{g}^T \boldsymbol{\alpha} - \lambda].$$

Case b: $s(k) > 0$
It is assumed that the control applied at the next instant be $u(k) = \boldsymbol{F}\boldsymbol{x}(k) + \gamma$. Then it can be shown that output samples generated by the above control is

$$\boldsymbol{y}_{k+1} = \tilde{\mathbf{C}}\boldsymbol{x}(k+1) + \boldsymbol{\alpha}$$

while the output samples generated by the previous control is

$$\boldsymbol{y}_k = \tilde{\mathbf{C}}\boldsymbol{x}(k) - \boldsymbol{\alpha}.$$

The incremental change of $s(k)$ is

$$s(k+1) - s(k) = \boldsymbol{c}^T \tilde{\mathbf{C}}^{-1}(\boldsymbol{y}_{k+1} - \boldsymbol{y}_k - 2\boldsymbol{\alpha}).$$

Solving for $u(k)$ gives the output feedback control law with $\boldsymbol{L}$ as in (14) and

$$\eta(k) = -(\mathbf{g}^T \boldsymbol{D}_0)^{-1}[\mathbf{g}^T \boldsymbol{C}_0 \tilde{\mathbf{C}}^{-1} \boldsymbol{\alpha} - 2\mathbf{g}^T \boldsymbol{\alpha} + q\tau \mathbf{g}^T \boldsymbol{\alpha} + \lambda].$$

Thereafter in every successive sampling period the control switches between the two values as given in Case a and Case b respectively depending on the sign of $s(k)$ and in steady-state the system trajectory lies within the quasi sliding mode band [4].

4 Illustrative Examples

Example 1

Consider the same second order discrete-time system as in [3] and [10] which is

$$\boldsymbol{\Phi}_\tau = \begin{bmatrix} 1.2 & 0.1 \\ 0.1 & 0.6 \end{bmatrix}, \boldsymbol{\Gamma}_\tau = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Let

$$x_1(0) = 2, x_2(0) = 0,$$
$$\tau = 0.5\,\mathrm{sec}, N = 2, \Delta = 0.25\,\mathrm{sec},$$
$$q = 1,\ \epsilon = 0.05,\ \gamma = -0.01.$$

Switching surface is designed as

$$s(k) = 5x_1(k) + x_2(k).$$

The initial state feedback gain, $\boldsymbol{F} = \begin{bmatrix} -3.6 & -0.6 \end{bmatrix}$. The FOSSMC law equivalent to the state feedback control law,

$$u(k) = \begin{bmatrix} 4.4103 & -5.785 \end{bmatrix} \boldsymbol{y}_k + \eta(k).$$

In the reaching phase $\eta(k) = -0.0079$, in the switching phase $\eta(k) = +0.0421$ for $s(k) > 0$ and $\eta(k) = -0.0421$ for $s(k) < 0$.

Example 2

Consider the following third order continuous-time system

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

Now discretize the above system with $\tau = 0.3\,\mathrm{sec}$

$$\boldsymbol{\Phi}_\tau = \begin{bmatrix} 0.9883 & 0.0505 & 0.1970 \\ -0.3940 & 0.3973 & -0.0505 \\ -0.1009 & 0.2426 & 0.3973 \end{bmatrix}, \quad \boldsymbol{\Gamma}_\tau = \begin{bmatrix} 0.0117 \\ 0.3940 \\ 0.1009 \end{bmatrix}.$$

Let

$$\begin{aligned} x_1(0) &= 2,\ x_2(0) = 0,\ x_3(0) = 0, \\ N &= 3,\ \Delta = 0.1\,\mathrm{sec} \\ q &= 2,\ \epsilon = 0.01,\ \gamma = -0.003. \end{aligned}$$

Switching surface is designed as

$$s(k) = 2.4x_1(k) + 2.0226x_2(k) + 1.7346x_3(k).$$

The initial state feedback gain is

$$\boldsymbol{F} = \left[\begin{array}{ccc} -0.4398 & -0.5364 & -0.366 \end{array} \right].$$

The output feedback gain is

$$\boldsymbol{L} = \left[\begin{array}{ccc} -10.2705 & 25.8638 & -15.737 \end{array} \right].$$

The FOSSMC law equivalent to the state feedback control law is

$$u(k) = \left[\begin{array}{ccc} -10.2705 & 25.8638 & -15.737 \end{array} \right] \boldsymbol{y}_k + \eta(k),$$

where $\eta(k) = -0.0024$ for $\tau \leq t \leq \kappa\tau$. Thereafter in the switching phase $\eta(k) = +0.0036$ for $s(k) > 0$ and $\eta(k) = -0.0036$ for $s(k) < 0$.

4.1 Simulation Results

The simulation results of the second order system are shown in Figure 1. The results are satisfactory. It is observed that the proposed method gives the same results as obtained in state feedback sliding mode control. Once the system trajectory crosses the switching line first time, it will recross the switching plane again in every successive sampling period. Thus the reaching condition is satisfied and quasi sliding mode exists. The width of the quasi sliding mode band is $2\delta \leq 0.0333$. In steady-state $s(k)$ assumes only two values given by $s_1 = 0.0167 < \frac{\epsilon\tau}{2-q\tau}$ and $s_2 = -0.0167 < \frac{-\epsilon\tau}{2-q\tau}$. Similarly the simulation results of the third order system are shown in Figure 2. The width of the quasi sliding mode band is $2\delta \leq 0.0043$. In steady-state $s(k)$ takes values, $s_1 = 0.0021 < \frac{\epsilon\tau}{2-q\tau}$ and $s_2 = -0.0021 < \frac{-\epsilon\tau}{2-q\tau}$. Thus in the steady state the trajectory lies within the quasi sliding mode band.

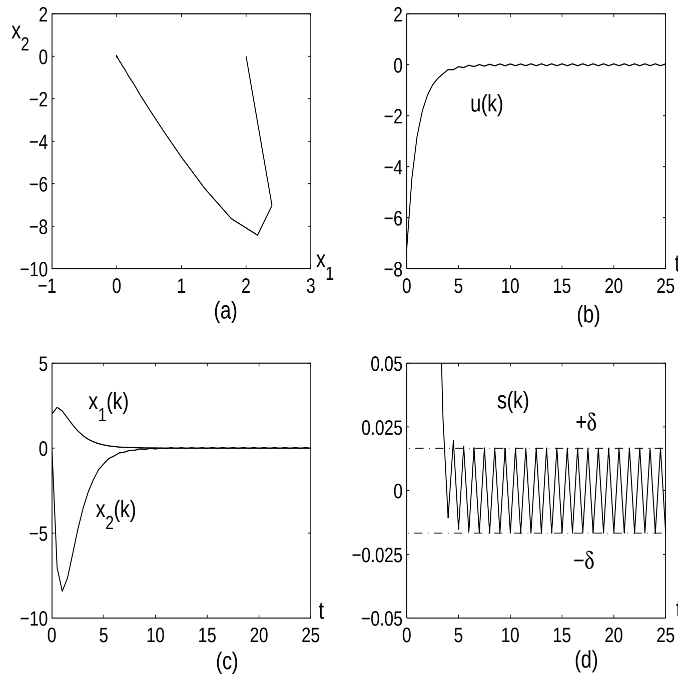

Figure 1: Simulation results of second order system: (a) Phase plot (b) Control input (c) Plant states (d) Switching function

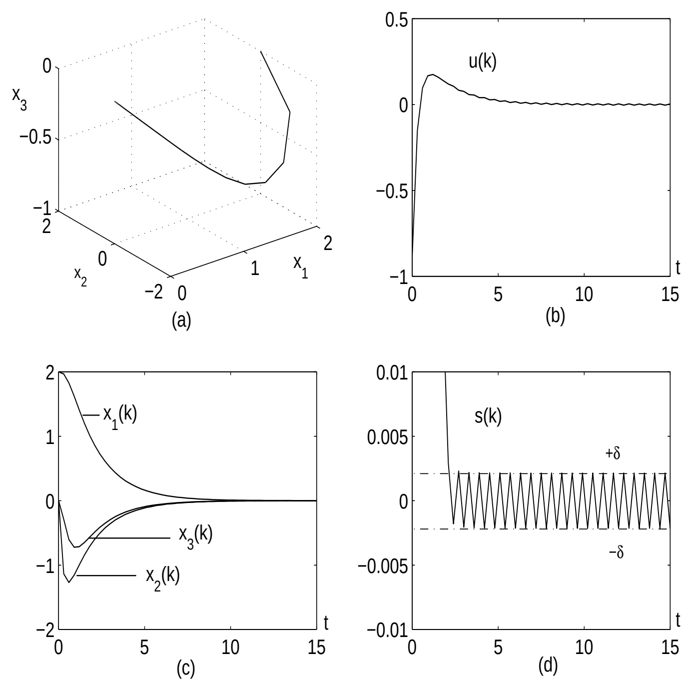

Figure 2: Simulation results of third order system: (a) Phase plot (b) Control input (c) Plant states (d) Switching function

5 Concluding remarks

In this paper, a fast output sampling sliding mode control law is developed in which the output feedback gain is not derived from a state feedback gain but is directly obtained from the reaching law itself. Since the output feedback gain realizes the effect of the state feedback gain, the same results will be obtained as with the state feedback sliding mode control. The requirement of output correction used in [10] is eliminated in this approach. One advantage of this approach is that the designed FOSSMC is static in nature. Therefore there is no need for dynamic compensators. Another advantage of this method is that the system states are used neither for feedback purpose nor for switching function evaluation. Thus it is shown that the FOSSMC technique proposed in this work is a good alternative to state feedback sliding mode control and dynamic output feedback sliding mode control. This technique can also be extended to multivariable systems.

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