MIMO 2-SLIDING CONTROL DESIGN

A. Levant

School of Mathematical Sciences, Tel-Aviv University, Ramat-Aviv, 69978 Tel-Aviv, Israel
E-mail: levant@post.tau.ac.il . Tel.: 972-3-6408812, Fax: 972-3-6407543

Keywords: Variable Structure Control, Robust Control, Multivariable Control, Tracking Systems

Abstract

Multi-Input-Multi-Output (MIMO) tracking problem under uncertainty conditions is considered. The proposed vector 2-sliding control design preserves the main Single-Input-Single-Output 2-sliding control features: the control is finite-time convergent and chattering-free, the tracking is exact. With discrete sampling it provides for the tracking accuracy proportional to the sampling step squared. The design procedure requires non-singularity of the control matrix.

1 Introduction

Control under heavy uncertainty conditions remains one of the main research fields of the modern control theory. One of the most simple and effective ways to withstand the uncertainty is based on the sliding-mode technique [18, 19]. Sliding modes keep equality of some output variable σ to zero. With σ being the deviation of some real-time given signal from the output, the standard sliding mode provides actually for full output control in the case when the relative degree is 1 (i.e. the control appears explicitly already in the first total derivative of σ). The idea is to react immediately to any deviation of σ from zero, making it move to 0 by a sufficiently-energetic control effort. Such sliding modes feature finite-time convergence, high accuracy and robustness with respect to a large class of disturbances. Unfortunately, the standard sliding mode features also high-frequency control switching which may cause possibly dangerous system vibrations (the so-called chattering effect [18, 8]).

A number of methods were proposed to overcome these difficulties. In particular, high-gain control with saturation approximates the sign-function and diminishes the chattering, while on-line estimation of the so-called equivalent control [18] is used to reduce the discontinuous-control component [17], the sliding-sector method [9] is suitable to control disturbed linear time-invariant systems. Yet, the sliding-mode order approach [10, 4, 11, 1, 3, 14] seems to be more comprehensive, for it allows to remove all the above restrictions, while preserving the main sliding-mode features and improving its accuracy. Independently developed dynamical [16] and terminal [15] sliding modes are closely related to this approach.

Let first σ be a scalar output. Suppose that $\sigma \equiv 0$ is kept by a discontinuous dynamic system. While successively differentiating σ along trajectories, a discontinuity will be encountered sooner or later in the general case. Thus, sliding modes $\sigma \equiv 0$ may be classified by the number r of the first successive total derivative $\sigma^{(r)}$ which is not a continuous function of the state space variables or does not exist due to some reason like trajectory nonuniqueness. That number is called sliding order [11, 3, 14]. The standard sliding mode on which most variable structure systems (VSS) are based is of the first order ($\dot{\sigma}$ is discontinuous). Let now σ be a vector. Then each scalar component of σ may have its own sliding order. As a result a vector sliding order is achieved.

While the standard sliding mode precision is proportional to the sampling time interval or to the switching delay, r-sliding mode realization provides for up to the rth order of sliding precision with respect to the measurement interval [11]. Properly used, higher-order sliding modes (HOSM) totally remove the chattering effect and feature finite-time convergence.

Scalar HOSM are already well studied, and a number of applications were reported [7, 13]. In particular, arbitrary-order sliding mode controllers [14] provide for full output control of any uncertain smooth Single-Input-Single-Output (SISO) minimum-phase dynamic system with known relative degree r. The auxiliary-constraint construction is avoided, the convergence time is finite and may be made arbitrarily small, while only one scalar parameter needs to be adjusted. The control can be made arbitrarily-smooth in time, totally removing the chattering effect and providing for ultimate accuracy in realization. An output-feedback version of the same controller is also available.

At the same time Multi-Input-Multi-Output (MIMO) applications of HOSM are actually still “terra incognita”. The only known result in this field was obtained by Bartolini et al. [2]. The classical chattering-removing MIMO VSS problem is considered there: a vector output of an uncertain system has well defined relative degree (1, ..., 1), and the problem is to make it vanish in finite time by means of continuous control. It is shown in [2] that hierarchical 2-sliding control is possible if the control matrix has a dominant diagonal, or the matrix is positive-definite. In the latter case only asymptotic convergence is attained, and the above-mentioned second-order sliding accuracy is lost.

The approach of the present paper generalizes the classical hierarchical MIMO sliding-mode design [18] to the 2-sliding case. The main SISO 2-sliding control features are preserved: the control is finite-time convergent and chattering-free, the tracking is exact. With discrete sampling the sliding accuracy is proportional to the sampling step squared. The design procedure requires non-singularity of the control matrix and is simple and straight-forward. The approach is demonstrated by computer simulation.

2 Preliminaries: SISO 2-sliding control

Only the chattering removal problem is considered here. The standard VSS feedback contains a relay with output taking on values U_M, $-U_M$. That feedback provides for keeping some constraint $\sigma = 0$ in a 1-sliding mode. Let relay output be a control variable u. The idea is to install continuous output of some dynamic subsystem instead of relay output. Let for simplicity the dynamic system be given by an equation linearly dependent on u:

$$\dot{x} = a(t,x) + b(t,x)\, u\,. \qquad (1)$$

where $x \in \mathbf{R}^n$, u, $\sigma \in \mathbf{R}$, t is time, a, b are a C^1-functions. Let $\sigma(t,x)$ be a C^2-function. Any solution of (1) is assumed to be infinitely extendible in t, provided $u(t)$ is continuous and $|u(t)| \le U_M$ for each t. The goal is to force the constraint function σ to vanish in finite time by means of a control continuously dependent on time.

Let $u_{eq}(t,x) = \sigma'_x a \,/\, \sigma'_x b$ (the equivalent control [18]), K_m, K_M, C_0 be positive constants, $K_m < K_M$, and assume that

$$|\, u_{eq}(t,x)| \le u_0 < U_M,\ 0 < K_m \le \sigma'_x b \le K_M,$$
$$|\, \sigma'_x(\dot{a} + \dot{b}\ u) + \sigma''_{tx}(a + b\,u)\,| \le C.$$

The latter inequality means that $\dot{u}_{eq}$ is bounded. That makes it possible to approximate u_{eq} by a Lipschitzian control. A more general statement of the problem without linear dependence on control u can be found in [11].

The controllers considered in the paper have the form

$$\dot{u} = \begin{cases} -u \ \text{ with } |u| > U_M\,, \\ \varphi(\sigma(\cdot), \dot{\sigma}(\cdot)) \ \text{ with } |u| \le U_M\,. \end{cases} \qquad (2)$$

The function φ may depend here on the histories $\dot{\sigma}(\cdot)$ and $\sigma(\cdot)$ of $\dot{\sigma}$ and σ measurements. The solutions are understood in the Filippov sense [6]. Only few traditional 2-sliding controllers are considered here, though all the results are valid for any 2-sliding controller from [3]. The so-called twisting controller [3, 10, 11] and the convergence conditions are given by

$$\varphi = -(r_1 \operatorname{sign} \sigma + r_2 \operatorname{sign} \dot{\sigma}),\ r_1 > r_2 > 0, \qquad (3)$$
$$(r_1 + r_2)K_m - C > (r_1 - r_2)K_M + C,\ \ (r_1 - r_2)K_m > C.$$

A particular case of the controller with prescribed convergence law [5, 11] is given by

$$\varphi = -\alpha \operatorname{sign}(\dot{\sigma} + \lambda|\sigma|^{1/2} \operatorname{sign} \sigma), \qquad (4)$$
$$\alpha, \lambda > 0,\ \ \alpha K_m - C > \lambda^2/2.$$

Controller (4) is close to terminal sliding mode controllers [15]. The so-called sub-optimal controller [1, 2, 3] is given by

$$\varphi = -r_1 \operatorname{sign}(\sigma - \sigma^*/2) + r_2 \operatorname{sign} \sigma^*,\ \ r_1 > r_2 > 0, \qquad (5)$$
$$2[(r_1 + r_2)K_m - C\,] > (r_1 - r_2)K_M + C,\ (r_1 - r_2)K_m > C,$$

where σ^* is the current value of σ detected at the closest time when $\dot{\sigma}$ was 0. The initial value of σ^* is 0. Any computer implementation of this controller requires successive measurements of $\dot{\sigma}$ or σ with some time step. Usually the detection of the moments when $\dot{\sigma}$ changes its sign is performed. The control value u depends here actually on the history of $\dot{\sigma}$ and σ measurements, i.e. on $\dot{\sigma}(\cdot)$ and $\sigma(\cdot)$.

Theorem 1 [11, 1]. 2-*sliding controllers* (3), (4) *and* (5) *provide for finite-time convergence of any trajectory of* (1), (2) *to* 2-*sliding mode* $\sigma \equiv 0$. *The convergence time is a locally bounded function of the initial conditions*.

Let the measurements be carried out at times t_i with constant step $\tau > 0$, $\sigma_k = \sigma(t_k, x(t_k))$, $\Delta\sigma_k = \sigma_k - \sigma_{k-1}$, $t \in [t_k, t_{k+1})$. Substituting σ_k for σ, sign $\Delta\sigma_k$ for sign $\dot{\sigma}$, and sign($\Delta\sigma_k - \lambda\tau|\sigma_k|^{1/2}$sign σ_i) for sign($\dot{\sigma} - \lambda|\sigma|^{1/2}$sign σ) achieve discrete-sampling versions of the controllers.

Theorem 2 [11, 1]. *Discrete-sampling versions of controllers* (3), (4), (5) *provide for the establishment of the inequalities* $|\sigma| < \mu_0\tau^2$, $|\dot{\sigma}| < \mu_1\tau$ *for some positive* μ_0, μ_1.

The following theorem establishes robustness of the controllers with respect to small model imperfections.

Theorem 3. *Let under the conditions of Theorem* 1 *system* (1) *be disturbed by a small function* ω *so that*

$$\dot{x} = a(t,x) + \omega(t,x,u) + b(t,x)\, u\,,$$

where $|\sigma'_x \omega \,/\, \sigma'_x b| \le \varepsilon$, $u_0 + \varepsilon < U_M$. *Then the convergence is provided to the set defined by the inequalities* $|\sigma| < \mu_0\varepsilon^2$, $|\dot{\sigma}| < \mu_1\varepsilon$ *for some positive* μ_0, μ_1. *The same is true with sufficiently small sampling step.*

Theorem 3 was proved in [4] for the twisting controller. The controllers (4) and (5) are similarly considered. The main idea is to consider the motion in the coordinates σ and $\xi = u - u_{eq}$, $\dot\sigma = \sigma'_x b\cdot(\xi + \sigma'_x \omega / (\sigma'_x b))$.

Remark. With negative $\sigma'_x b$, $0 < K_m \le -\sigma'_x b \le K_M$, the function φ has to be replaced in (2) by -φ.

The listed controllers depend on few constant parameters. These parameters are to be tuned in order to control the whole class of processes and constraint functions defined by the concrete values of U_M, K_M, K_m, C. Increasing the constants U_M, K_M, K_m, C, we enlarge the controlled class too. Such algorithms are obviously insensitive to any model perturbations and external disturbances which do not stir the dynamic system from the given class.

3 MIMO control design

Let the system to control be given by (1) but now with $u, \sigma \in \mathbf{R}^m$. Suppose that the relative degree is (1, ..., 1), in other words, that the matrix $\sigma'_x b$ is nonsingular. Consider an auxiliary formal system of linear equations

$$Gu = F, \; G = \sigma'_x b = (g_{ij}(t, x)),$$

where F is any vector. Suppose that $g_{i_1, j_1} \neq 0$, then u_{j_1} may be excluded from the other equations subtracting equation i_1 with the appropriate coefficient:

$$g_{ij} := g_{ij} - g_{i_1,j} g_{i,j_1} / g_{i_1,j_1}, \; i \neq i_1.$$

Take now any equation number $i_2 \neq i_1$ and take any $j_2 \neq j_1$ such that the element of the modified matrix $g_{i_2,j_2} \neq 0$ and similarly exclude u_{j_2} from the rest equations (i.e. from the equations with numbers $i \neq i_1, i_2$). That is the well-known Gauss procedure of variable exclusion. It can be successfully carried out till the end for any nonsingular G. After the procedure finish and the corresponding enumeration of the controls the obtained matrix gets the upper-triangular form.

Definition. The correspondence $\begin{pmatrix} i_1 & \dots & i_m \\ j_1 & \dots & j_m \end{pmatrix}$ is called a well-defined output-input assignment, if the corresponding Gauss procedure can be performed for any t, x, and the corresponding elements $\tilde g_{i(j),j}$ of the resulting modified matrix $\tilde G$ are uniformly separated from zero. Thus, each control component u_j is associated with the corresponding component $\sigma_{i(j)}$ of σ. The number $\zeta_j = \operatorname{sign} \tilde g_{i(j),j}$ is called the influence sign.

Assume that the matrix $\sigma'_x b$ is nonsingular and bounded, the equivalent control $u_{eq}(t,x) = -(\sigma'_t + \sigma'_x a)(\sigma'_x b)^{-1}$ is bounded together with its total derivative, and $\begin{pmatrix} i_1 & \dots & i_m \\ j_1 & \dots & j_m \end{pmatrix}$ is a well-defined output-input assignment. Then the controller is

$$\dot u_j = \begin{cases} -u_j & \text{with} \; |u_j| > U_{jM}, \\ \varsigma_j \varphi_j(\sigma_{i(j)}(\cdot), \dot\sigma_{i(j)}(\cdot)) & \text{with} \; |u_j| \le U_{jM} \end{cases}, \quad j = 1,..., m \qquad (6)$$

with φ_j chosen in one of forms (3) - (5). The form of φ_j can be chosen independently for each j.

Theorem 4. *Let the parameters of* φ_{j_k} *and* U_{j_k} *be chosen sufficiently large in the reverse order* $k = m, ..., 1$. *Then controller* (6) *provides for the finite-time convergence to the vector* 2*-sliding mode* $\sigma \equiv 0$.

Proof. Let for simplicity $j_k = m - k + 1$. Apply the induction with respect to m. The case $m = 1$ was considered in the previous section. Let now reduce the case m to $m - 1$. As follows from (1)

$$\dot\sigma = \sigma'_t(t,x) + \sigma'_x(t,x) a(t,x) + G(t,x)\, u. \qquad (7)$$

Denote $u = (\hat u, u_m)^t$, where $\hat u = (u_1, ..., u_{m-1})^t$, and $\hat g = (g_{1m}, ..., g_{1\, m-1})$. The mth equation takes on the form

$$\dot\sigma_m = \sigma'_t + \sigma'_{m\,x} a + \hat g \; \hat u + g_{mm} u_m. \qquad (8)$$

Taking $\dot\sigma_m = 0$ (to be still provided), obtain the function

$$u_{m\,eq} = -(\sigma'_{m\,t} + \sigma'_{m\,x} a + \hat g \; \hat u) / g_{mm}. \qquad (9)$$

Substituting $u_{m\,eq}$ for u_m in the m-1 first equations of the vector equation (7) obtain a new system with (m-1)-dimensional vector control $\hat u$ and output $\hat\sigma$. Its control matrix coincides with the first m-1 columns and lines of the matrix G after the first step of the above Gauss procedure. This system satisfies all conditions of the Theorem. Hence, 2-sliding control design is available for it.

Apply the resulting controls (6) for $j = 1, ..., m - 1$, and consider dynamic system (1) as a SISO system with control u_m and output σ_m. Due to the boundedness of $\hat u$ and $\dot{\hat u}$, it satisfies the conditions of Theorem 1. Therefore, taking appropriate (sufficiently large) parameters of φ_m and U_{mM}, finite-time convergence to the 2-sliding mode $\sigma_m = 0$ is provided. Thus, after finite time $\dot\sigma_m \equiv 0$, which means that also the identity $u_m \equiv u_{m\,eq}$ is kept. Now the rest of controls provide for the finite-time vanishing of the whole σ. ■

Let the measurements be carried out at times t_i with constant step $\tau > 0$, $\sigma_{ik} = \sigma(t_k, x(t_k))$, $\Delta\sigma_{ik} = \sigma_{ik} - \sigma_{i,k-1}$, $t \in [t_k, t_{k+1})$. Substituting σ_i for σ, sign $\Delta\sigma_{ik}$ for sign $\dot\sigma_i$, and sign($\Delta\sigma_{ik} - \lambda\tau|\sigma_{ik}|^{1/2}$sign σ_{ik}) for sign($\dot\sigma_k - \lambda|\sigma_k|^{1/2}$sign σ_k) achieve discrete-sampling versions of controller (6).

Theorem 5. *Discrete-sampling versions of controllers* (6) *provide for the establishment of the inequalities* $\|\sigma\| < \mu_0\tau^2$, $\|\dot\sigma\| < \mu_1\tau$ *for some positive* μ_0, μ_1.

Theorem 6. *Let under the conditions of Theorem* 4 *system* (1) *be disturbed by a small vector function* ω *so that*

$$\dot x = a(t,x) + \omega(t,x,u) + b(t,x)\, u \,,$$

where $\|\sigma'_x\,\omega\, /\, (\sigma'_x\, b)^{-1}\| \le \varepsilon$. *Then with control parameters chosen as in Theorem 4, the convergence is provided to the set defined by the inequalities* $\|\sigma\| < \mu_0\varepsilon^2$, $\|\dot\sigma\| < \mu_1\varepsilon$ *for some positive* μ_0, μ_1. *The same is true with sufficiently small sampling step.*

Proof of Theorems 5, 6. Similarly to the proof of Theorem 4 the proof is carried out according to the induction principle. Theorems 5, 6 are true with $m = 1$ (Theorems 2, 3). Consider now any $m > 1$. The mth equation is

$$\dot\sigma_m = \sigma'_t + \sigma'_{m\,x}\, a + \omega_m + \hat g\, \hat u + g_{mm}\, u_m.$$

Let the sub-controllers with $j = 1, ..., m$ be chosen as in the proof of Theorem 4. Then, due to Theorem 3, $|\sigma_m| \sim \varepsilon^2$, $|\dot\sigma_m| \sim \varepsilon$, which means that also $|\, u_m - u_{m\,eq} \,| \sim \varepsilon$. Thus, the problem is reduced to the (m - 1)-dimensional case, which proves Theorem 6 both for the continuous and discrete sampling.

Let now $\omega = 0$, the sampling step be τ_0. The same considerations show that the relations $|\sigma_m| \sim \tau_0^2$, $|\dot\sigma_m| \sim \tau_0$, $|u_m - u_{m\;eq}| \sim \tau_0$ are established in finite time. Thus, the deviation of u_m from $u_{m\,eq}$ is felt by the (m - 1)-dimensional system as a small disturbance of the order of τ_0. Hence, due to the discrete-sampling version of Theorem 6 for the (m - 1)-dimensional case, relations $\|\sigma\| \le \varepsilon^2$, $\|\dot\sigma\| \le \varepsilon$ are established for some small ε (Theorem 6 cannot provide here for the full proof of Theorem 5, for τ_0 is required to be small with respect to the disturbance).

It is easy to check that differentiating (7) achieve with discrete sampling in the above small vicinity of the 2-sliding mode that

$$\ddot\sigma \in \mathrm{B} + \Gamma\Phi(\sigma(t_k),\Delta\sigma_k),\ \mathrm{B} = (\beta_j),\ \Gamma = (\gamma_{ij}),\ \Phi = (\zeta_j\, \tilde\varphi_j), \quad (10)$$

where $\tilde\varphi_j\,(\sigma_j(t_k),\Delta\sigma_{jk})$ is the discrete version of the corresponding controller (3) - (7), $t \in [t_k,\ t_{k+1})$. B is a column and Γ is a diagonal matrix with elements

$$\beta_j = [-\,\underline{\beta}_j\,,\ \underline{\beta}_j],\ \underline{\beta}_j > 0,\ \gamma_{jj} = [\underline{\gamma}_{jm}\,,\ \underline{\gamma}_{jM}\,],\ \underline{\gamma}_{jM} > \underline{\gamma}_{jm} > 0;$$

the set operations are understood in the natural way. The corresponding constants $\underline{\beta}_j$, $\underline{\gamma}_{jM}$, $\underline{\gamma}_{jm}$ are easily found from the Theorem conditions. It is easy to see that the set of trajectories of (10) is invariant with respect to the combined time-coordinate-parameter transformation

$$H_\kappa:\quad (t,\sigma,\ \dot\sigma\,,\ \tau) \mapsto (\,\kappa t,\ \kappa^2\sigma,\ \kappa\dot\sigma\,,\ \kappa\tau).$$

Hence, with $\kappa = \tau/\tau_0$ achieve that with any arbitrary sufficiently-small sampling step τ the trajectories are concentrated after finite time in the set $\|\sigma\| \le (\varepsilon/\tau_0)^2\,\tau^2$, $\|\dot\sigma\| \le (\varepsilon/\tau_0)\,\tau$. ■

Output-feedback control. As follows from (7) $\ddot\sigma$ is uniformly bounded, which allows successful feedback application of m robust exact differentiators [12] without disturbing the statements of the Theorems. Thus, the usage of finite differences can be avoided.

The listed controllers depend on constant parameters. These parameters determine a class of processes and constraint functions which may be successfully controlled by the designed controller. The parameters being increased, the controlled class is also enlarged. Such algorithms are obviously insensitive to any model perturbations and external disturbances which do not stir the dynamic system from the given class.

4. Numeric example

A problem of the rigid body angular orientation and tracking is considered. The body is moved by means of 3 jet pairs. The following system is a disturbed model from [7] (also the control matrix was changed):

$$\begin{aligned}
\dot x_1 &= -x_2x_3 + \omega_1(t) + \rho_1(t,u) + u_1 + 1.2u_2 + 1.5u_3,\\
\dot x_2 &= x_1x_3 + \omega_2(t) + \rho_2(t,u) + 1.5u_1 + u_2 + 1.2u_3, \quad (11)\\
\dot x_3 &= -\tfrac{1}{3}\,x_1x_2 + \omega_3(t) + \rho_3(t,u) + 1.2u_1 + 1.5u_2 + u_3.
\end{aligned}$$

Here x_j, u_j are the angular velocities and jet torques respectively, the "uncertain" disturbances are as follows:

$\omega_1(t) = \cos t\ (1 + 0.05 \sin 4t + 0.1 \cos t)$,
$\omega_2(t) = \sin t \cos t\ (1 + 0.05 \sin 4t + 0.1 \cos t)$,
$\omega_3(t) = \sin^2 t\ (1 + 0.05 \sin 4t + 0.1 \cos t)$;

$\rho_1(t, u) = 0.01 \sin(t + 2.1)\ (u_1 - 0.5\, u_3)$,
$\rho_2(t, u) = 0.01 \cos t\ (-0.2\, u_2 + 0.8\, u_3)$,
$\rho_3(t, u) = 0.01 \cos(t + 1.3)\ (-0.2\, u_1 - u_2 + 0.7\, u_3)$.

The task is to track a given in the real time vector-function of time by x. The right-hand side of (11) is not bounded with $u = 0$. Thus, the conditions of Theorems 4-6 are satisfied only in some vicinity of $x = 0$, and the designed controller will be also only locally valid. For the simulation the signal x_c to be tracked was taken

$$x_{1c} = 1 + \sin 0.5t\ ,$$
$$x_{2c} = 0.5 \cos 0.5t\ \cos t\ ,$$
$$x_{3c} = 0.5 \cos 0.5t \sin t\ .$$

Denote by $\sigma = x - x_c$ the vector output to be nullified. Apply the Gauss procedure to the nominal control matrix in (11).

Excluding u_3 from the first 2 equations and u_2 from the first one, achieve the matrix

$$\begin{pmatrix} -0.88 & 0 & 0 \\ 0.06 & -0.8 & 0 \\ 1.2 & 1.5 & 1 \end{pmatrix}.$$

The corresponding influence signs are (-1, -1, 1). It is easily seen that the disturbance ρ does not interfere with this procedure. Thus, the assignment $\begin{pmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ is well defined and the 2-sliding controller is chosen based on the twisting controller (3) as follows:

$$\dot{u}_1 = \begin{cases} -u_1 & \text{with } |u_1| > 3, \\ 5\,\mathrm{sign}\,\sigma_1(t_k) + 3\,\mathrm{sign}\,\Delta\sigma_{1k} & \text{with } |u_1| \le 3; \end{cases}$$

$$\dot{u}_2 = \begin{cases} -u_2 & \text{with } |u_2| > 10, \\ 20\,\mathrm{sign}\,\sigma_2(t_k) + 15\,\mathrm{sign}\,\Delta\sigma_{2k} & \text{with } |u_2| \le 10; \end{cases}$$

$$\dot{u}_3 = \begin{cases} -u_3 & \text{with } |u_3| > 80, \\ -100\,\mathrm{sign}\,\sigma_3(t_k) - 60\,\mathrm{sign}\,\Delta\sigma_{3k} & \text{with } |u_3| \le 80. \end{cases}$$

The initial control values were taken $u_1 = u_2 = u_3 = 0$. The integration was carried out by the Euler method, which is the only reliable method for the sliding-mode simulation.

The trajectory on the plane σ_3, $\dot{\sigma}_3$ is presented in Fig. 1. That is the most fast and dominating process, therefore the obtained convergence curve is the standard for the SISO twisting controller. It is seen from Fig. 2 that the convergence to the second 2-sliding mode $\sigma_2 = \dot{\sigma}_2 = 0$ starts only after $\sigma_3 \equiv 0$ is obtained. Convergence to $\sigma_1 = \dot{\sigma}_1 = 0$ requires $\sigma_3 \equiv 0$ and $\sigma_2 \equiv 0$ (Fig. 3). The tracking results are demonstrated in Fig. 4. The controls are shown in Fig. 5. It is seen that after finishing the convergence to $\sigma_3 = \dot{\sigma}_3 = 0$ the control component u_3 successfully compensates for the transients of u_2 and u_1.

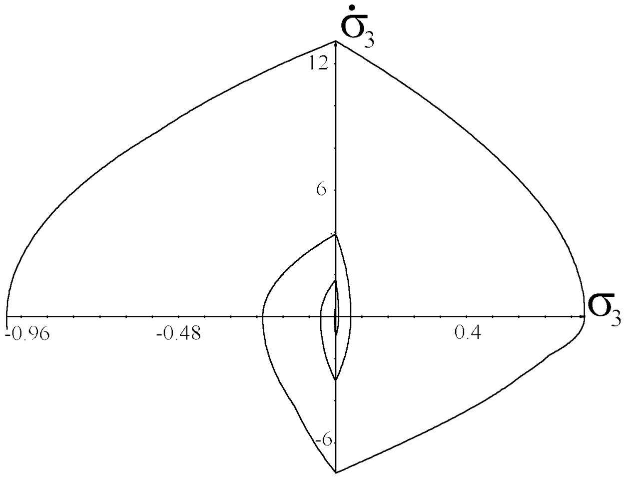

Fig. 1: The trajectory on the plane σ_3, $\dot{\sigma}_3$

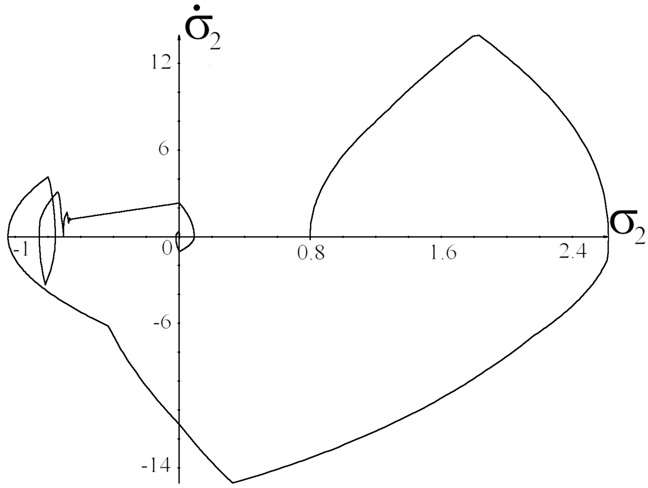

Fig. 2: The trajectory on the plane σ_2, $\dot{\sigma}_2$

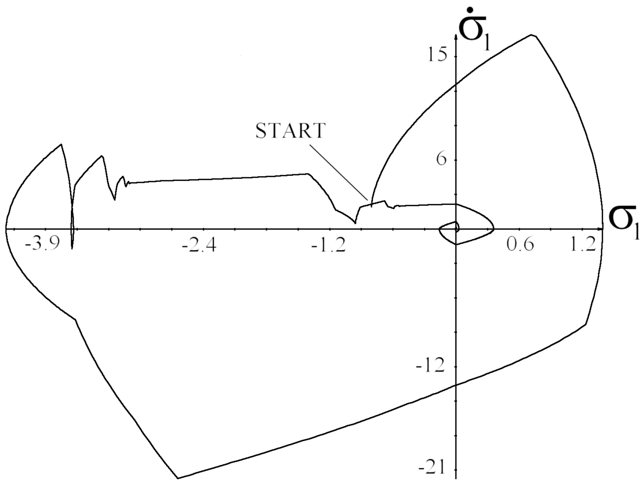

Fig. 3: The trajectory on the plane σ_1, $\dot{\sigma}_1$

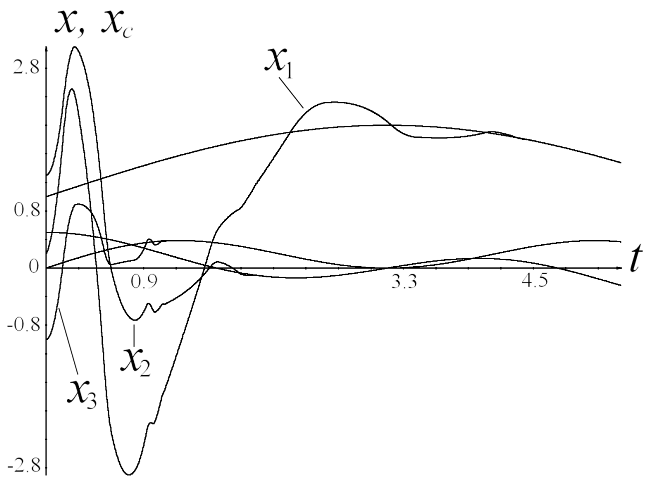

Fig. 4: Tracking results

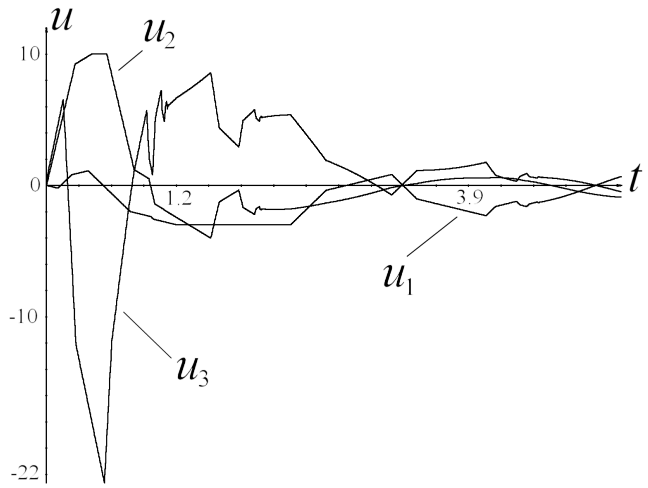

Fig. 5: 2-sliding controls

The resulting accuracies were $\|\sigma\| \le 3.5\cdot 10^{-4}$ and $\|\dot\sigma\| \le 0.23$ after the transient time $t = 5$ with the sampling step $\tau = 10^{-4}$. After the sampling step was changed to $\tau = 10^{-5}$, the accuracies changed to $\|\sigma\| \le 4.7\cdot 10^{-6}$ and $\|\dot\sigma\| \le 0.024$, which generally corresponds to Theorem 5.

5. Conclusions

A simple procedure of 2-sliding MIMO control design is proposed which requires only nonsingularity of the control matrix. The procedure is effective with relative degree 1 which means that the 2-sliding mode can be used instead of the standard MIMO 1-sliding mode totally removing the chattering, preserving the finite-time-convergence and improving the sliding accuracy.

A number of problems still remain. Though in practice the proposed approach is sufficient, global convergence with known functional bounds of $\sigma'_x a$ and $\sigma'_x b$ is still needed to be assured. While output-feedback control can be designed here, using robust exact first-order sliding differentiators with finite-time convergence [12, 14], the differentiation is better to be avoided. In other words a MIMO super-twisting controller [11, 3] is to be developed.

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