SLIDING MODE MODEL BASED PREDICTIVE CONTROL FOR NON MINIMUM PHASE SYSTEMS

Winston García-Gabín[†], Eduardo F. Camacho *

[†] Escuela de Ingeniería Eléctrica Universidad de Los Andes Mérida 5101, Venezuela Telf:+58 2742402902 Fax:+58 2742402903 e-mail: winstong@ing.ula.ve * Dpto.de Ingeniería de Sistemas y Automática Universidad de Sevilla Sevilla 41092, España,

Telf:+34 954487347 Fax:+34 954487340
 e-mail: eduardo@cartuja.us.es

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Abstract

This paper shows the development of a predictive controller. The proposed sliding mode model predictive control (SMPC) algorithm blends the design technique of sliding mode control with model based predictive control. It is demonstrated that an appropriate choice of the tuning parameters of SMPC avoids the instability problems of MPC when applied to non minimum phase systems. It is shown that considerable robustness improvement with respect to MPC can be obtained in the presence of modelling uncertainty, and disturbances, also SMPC showed enhanced ability to handle set point changes in a non linear process. The performance of the controller was judged using a non linear and non minimum phase isothermal Van de Vusse reactor process.

1 Introduction

A discrete system is said to be a non-minimum phase process if at least one of the zeros of the transfer function is located outside the unit circle. These kinds of processes are common in industrial applications and they are characterized by their inverse response. It is well known that non-minimum phase systems present difficulty for applying control strategies because they have an initial inverse response to step input in the opposite direction to the steady state [1]. The presence of unstable zero in a process transfer function is thus identified as being responsible for its difficult dynamic behavior; it is also the source of a considerable amount of difficulty in controller design. Another aspect of controlling a process with unstable zero is the instability problem, which arises in order to achieve high performance when the controller contains an inverse of the process model [2].

Model Based Predictive Control (MPC) has become one of

the most popular control methodologies both in industry and academia. It has been successfully implemented in many industrial applications, showing good performance. The basic idea of MPC is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. The index to be optimized is the expectation of a quadratic function measuring the distance between the predictive system output and some predictive reference sequence over the horizon plus a quadratic function measuring control effort. In order to implement an MPC, a model of the plant is used to predict the future plant outputs. This prediction is based on past and current values of the input and the output of the plant.

The instability problems applying Model Predictive Control to non minimum phase systems have been reported in literature, [3] and [4] show that non-minimum phase systems produce instability when $N_u = N_2 = 1$, while [5] and [6] propose that this can be solved using a control weight parameter. Instability in MPC is also reported by [7], [8] and [9]. In [10] it is shown how MPC can be tuned to have stable behaviour with unstable zeros in the Single Input Single Output (SISO) case. MPC has instability problems because, for non-minimum phase plants, the controller achieves the optimal output by cancelling the plant zeros, including the unstable zeros, which leads to a loss of internal stability of the feedback system.

Sliding Mode Control (SMC) is a technique derived from Variable Structure Control (VSC) which was studied originally by Utkin [11]. For a broad class of systems, this kind of control is particularly appealing due to its ability to deal with nonlinearities, time-variance, as well as uncertainties and disturbances, in a direct manner in the face of modeling imprecisions. In VSC, the control can modify its structure. The design problem consists of selecting the parameters of each structure and defining the traveling logic. The first step in SMC is to define a sliding surface, S(t), along which the process can slide to find its desired final value. In general, the switching surface represents the system behavior during the transient period, therefore, it must be designed to represent a desired system dynamics. The structure of the control system is intentionally altered as its state crosses the sliding surface in the phase plane in accordance with a prescribed control law. Thus, the second step is to design the control law in such a way that any state outside the sliding surface is driven to reach the surface in finite time and stay here.

This article shows how a combination of MPC and SMC results in a control structure that has the main advantages of both SMC and MPC. An algorithm based on variable structure control and generalized predictive control was proposed in [12]. The sliding surface prediction is made only with past values of input and not considering the future control values, furthermore the way in which the sliding surface is computed results in an unstable polynomial. This paper proposes a controller based on the idea of a combination of MPC and SMC that also has the future control movements for predicting the sliding surface, this results in more precise predictions and allows the process to be controlled with dead time; the discontinuous part of the control law is also simple and with fewer parameters and they have a clear meaning for tuning. A dual mode control scheme combining non linear MPC and SMC is presented in [13]. MPC is used to force the state into a terminal region within a finite horizon while it is outside the terminal region and a sliding mode variable structure controller is used while the state is inside the terminal region. The controller proposed is a single mode controller, the main idea is to introduced the prediction of the sliding surface into the control objective.

This article is organized as follows: Section 2 presents the development of SMPC. Section 3 shows the procedure used to obtain SMPC closed loop relationships. Section 4 shows the application of this controller for an isothermal Van de Vusse reaction in a continuously stirred tank reactor with non-minimum phase behaviour. Finally, the conclusions are presented.

2 Sliding mode model based predictive control

Most SISO plants when considering operation around a particular set-point and after linearization can be described by:

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1) + C(z^{-1})\frac{\xi(t)}{\Delta}$$
(1)

Where: y(t) is the output signal process, u(t) is the input signal process, Δ : $1-z^{-1}$, d: is the delay, $\xi(t)$ is the zero mean white noise, $A(z^{-1})$ and $C(z^{-1})$ are monic polynomials and $B(z^{-1})$ is a polynomial that has the zeros of the model. This model is known as the CARIMA Model (Controller Auto-Regressive Integrated Moving-Average). It has been argued that for many industrial applications in which disturbances are non-stationary an integrated CARIMA model is more appropriate [3]. The most usual case $C(z^{-1}) = 1$ has been used because the colouring polynomial are very difficult to estimate with sufficient accuracy in practice [14].

The following new predictive sliding surface is proposed to de-

velop the controller:

$$S_{t+j|t} = P_s(z^{-1})(y(t+j|t) - w(t+j)) + Q_s(z^{-1})\Delta u(t+j-1-d)$$
(2)

where the $P_s(z^{-1})$ and $Q_s(z^{-1})$ are polynomials of degree np and nq respectively, given by,

$$P_s(z^{-1}) = p_{s0} + p_{s1}z^{-1} + \dots + p_{snp}z^{-np}$$
(3)

$$Q_s(z^{-1}) = q_{s0} + q_{s1}z^{-1} + \dots + q_{snq}z^{-nq}$$
(4)

A predictive sliding surface is also presented in [12], however, it does not use the future control signals $\Delta u(t-1+j)$ to predict the future sliding surface values $S_{t+j|t}$, as is done here. Notice that using the future control moves allows for better predictions of the future values of the sliding surface, especially for control process with dead time. The general aim is that the future predictive surface (2) on the considered horizon should be zero and at the same time, the control effort Δu necessary for doing so should be penalized. The expression for the objective function is given by (5).

$$J = \sum_{j=N_1}^{N_2} [\hat{S}(t+j \mid t)]^2 + \sum_{j=1}^{N_u} \lambda(j) [\Delta u(t+j-1)]^2$$
(5)

Where $\hat{S}(t + j \mid t)$ is an optimum *j*-step prediction the of sliding surface on data up to time t, N_1 and N_2 are the minimum and maximum predictive horizons, N_u is the control horizon, and $\lambda(j)$ are weighting sequences. The objective of the controller is to compute the future control sequence in such way that the future surface S(t + j) is driven close to zero. The minimization of the objective function $J(N_1, N_2, N_u)$ produces $\Delta u(t), \Delta u(t+1), \dots, \Delta u(t+N_u)$, but only $\Delta u(t)$ is actually applied. At time t + 1 a new minimization problem is solved. This implementation is called the Receding Horizon controller. The final objective of control is to ensure that the controlled variable is close to its reference value w(t + j) at all times, meaning that e(t) must be zero. The problem of tracking a reference value can be reduced to keeping S(t) at zero. Once the sliding surface has been selected, attention must be turned to designing the control law that satisfies S(t) = 0. The control law, $\Delta u(t)$, consists of two additive parts, a continuous part, $\Delta u_C(t)$, and a discontinuous part, $\Delta u_D(t)$. That is,

$$\Delta u(t) = \Delta u_C(t) + \Delta u_D(t) \tag{6}$$

The continuous part is given by a Model Based Predictive Control algorithm using (5). The discontinuous part, $u_D(t)$, incorporates a nonlinear predictive element that includes the switching element of the control law. This part of the controller is discontinuous across the sliding surface.

$$u_D(t+j) = K_D \frac{S(t+j|t)}{|S(t+j|t)| + \rho}$$
(7)

Where K_D is a gain which is the tuning parameter responsible for the reaching mode, and ρ is a tuning parameter used to reduce the chattering problem [15]. In order to minimize (5), the *j*-step ahead output prediction $\hat{S}(t + j|t)$ for $j = N_1, \dots, N_2$ has been computed based on the information known at time *t* and the future values of the control increments. The following Diophantine equation is considered,

$$1 = E_j(z^{-1})\tilde{A}(z^{-1}) + z^{-j}F_j(z^{-1})$$
(8)

The polynomial $E_j(z^{-1})$ and $F_j(z^{-1})$ are uniquely defined with degrees j - 1 and na respectively, $\tilde{A}(z^{-1}) = \Delta A(z^{-1})$. Combining the plant model (1), and Diophantine equation (8), the follow prediction output equation can be obtained,

$$\hat{y}(t+j) = E_j(z^{-1})B(z^{-1})\Delta u_C(t+j-1)
+ F_j(z^{-1})y(t)$$
(9)

In this expression $\hat{y}(t+j)$ is a function of a known signal value at time t and also of future control inputs which have not yet been computed. Using a second Diophantine equation (10) to distinguish past and future control values,

$$E_j(z^{-1})B(z^{-1}) = G_j(z^{-1}) + z^{-j}\Gamma_j(z^{-1})$$
(10)

The polynomial G_j contains the first j step response parameters of the plant model. The following expression of the prediction is obtained,

$$\hat{y}(t+j) = G_j(z^{-1})\Delta u_C(t+j-1) + \hat{y}(t+j|t)$$
(11)

Where $\hat{y}(t + j|t)$ is the free response prediction of $\hat{y}(t + j)$ assuming that future control increments after time t - 1 will be zero,

$$\hat{y}(t+j|t) = \Gamma_j(z^{-1})\Delta u_C(t-1) + F_j(z^{-1})y(t)$$
(12)

Substituting $E_j(z^{-1})$ of (8) into (10), this yields

$$B(z^{-1}) = z^{-j}\tilde{A}(z^{-1})\Gamma_j + z^{-j}F_j(z^{-1})B(z^{-1}) + \tilde{A}(z^{-1})G_j(z^{-1})$$
(13)

Define the vector f(t), composed of the free response predictions,

$$f(t) = [\hat{y}(t+1|t), \cdots, \hat{y}(t+N_2|t)]^T$$
(14)

the vector of future control increments,

$$\Delta u_C(t) = [\Delta u_C(t), \cdots, \Delta u_C(t+N_u-1)]^T$$
(15)

From prediction (11) the predicted input-output relationship of the plant can be written as,

$$\hat{y}(t) = \mathbf{G}\Delta u_C(t) + f(t) \tag{16}$$

Where matrix **G** is composed of g_k step response parameters of the SISO plant model.

$$\mathbf{G} = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N_2-1} & g_{N_2-1} & \cdots & g_{N_2-N_u} \end{bmatrix}$$
(17)

The prediction of the sliding surface is obtained substituting (16) into (2),

$$\hat{S}(t) = (\mathbf{P_sG} + \mathbf{Q_s})\Delta u_C(t) + \mathbf{P_s}(f_s(t) - w(t))$$
(18)

Where the free response of the sliding surface f_s is given by,

$$f_{s}(t) = \mathbf{F}(z^{-1})y(t) + \Gamma(z^{-1})\Delta u_{C}(t-1) + \mathbf{P_{s}}^{-1}\mathbf{P_{s}}^{*}e(t) + \mathbf{P_{s}}^{-1}\mathbf{Q_{s}}^{*}\Delta u_{C}(t-1)$$
(19)

with the matrices defined as,

$$\mathbf{P_s} = \begin{bmatrix} p_{s0} & 0 & \cdots & 0 \\ p_{s1} & p_{s0} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & p_{s_{np}} & \cdots & p_{s0} \end{bmatrix}$$
(20)

$$\mathbf{P}_{\mathbf{s}}^{*} = \begin{bmatrix} p_{s1} & \cdots & p_{s_{n_{p-1}}} & p_{s_{n_p}} \\ p_{s2} & \cdots & p_{s_{n_p}} & 0 \\ \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$
(21)

$$\mathbf{Q_{s}} = \begin{bmatrix} q_{s0} & 0 & \cdots & 0 \\ q_{s1} & q_{s0} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & q_{s_{nq}} & \cdots & q_{s0} \end{bmatrix}$$
(22)
$$\mathbf{Q_{s}^{*}} = \begin{bmatrix} q_{s1} & \cdots & q_{s_{nq-1}} & q_{s_{nq}} \\ q_{s2} & \cdots & q_{s_{nq}} & 0 \\ \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$
(23)

The objective function (5) can be rewritten as,

$$J = \sum_{j=N_1}^{N_2} [\mathbf{P}_{\mathbf{s}} \mathbf{G} u_C + \mathbf{P}_{\mathbf{s}} (f + \mathbf{P}_{\mathbf{s}}^* + \mathbf{Q}_{\mathbf{s}}^* u_C) - \mathbf{P}_{\mathbf{s}} w + \mathbf{Q}_{\mathbf{s}} u_C]^2 + \sum_{j=1}^{N_u} \lambda(j) [\Delta u_C (t+j-1)]^2$$
(24)

The quadratic minimization of (24) becomes a direct problem of linear algebra, assuming there are no constraints on the control signal, which leads to,

$$\begin{aligned} \Delta u_C(t) &= K_{SMPC}(w(t) - f_s(t)) \\ K_{SMPC} &= [(\mathbf{P_sG} + \mathbf{Q_s})^T (\mathbf{P_sG} + \mathbf{Q_s}) + \lambda I]^{-1} \\ & (\mathbf{P_sG} + \mathbf{Q_s})^T \mathbf{P_s} \end{aligned}$$
(25)

and

$$\Delta u_D(t) = \left[\frac{K_D \hat{S}(t)}{|\hat{S}(t)| + \rho}, \cdots \frac{K_D \hat{S}(t + N_u - 1)}{|\hat{S}(t + N_u - 1)| + \rho}\right]$$
(26)

finally, the control signal is given by,

$$\Delta u(t) = K_{SMPC}(w(t) - f_s(t)) + \Delta u_D(t)$$
 (27)

To summarize, the SMPC has two parts. A discontinuous part, responsible for guiding the system to the sliding surface, and

a continuous part developed like an MPC, which is responsible for keeping the controlled variable on the reference value. Note that choosing $P_s(z^{-1}) = 1$, $Q_s(z^{-1}) = 0$, the objective function is reduced

$$U = \sum_{j=N_1}^{N_2} [\hat{y}(t+j \mid t) - w(t+j)]^2 + \sum_{j=1}^{N_u} \lambda(j) [\Delta u_C(t+j-1)]^2$$
(28)

and the K matrix gain is the usual linear MBPC,

$$K_{MPC} = (\mathbf{G}^T \mathbf{G} + \lambda I)^{-1} \mathbf{G}^T$$
(29)

3 Closed loop relationship

Closed loop relationship has been obtained for the SMPC and MPC in order to compare how the tuning parameters N_1, N_2 , and λ might affect the stability of the controlled plant. SMPC like other MPC use a Receding Horizon concept so, therefore, only the elements α_i of the first row of the matrix K_{SMPC} is considered. Equation (25) can be rewritten as,

$$\Delta u(t) = \sum_{i=1}^{N_2} \alpha_i [-\mathbf{F}(z^{-1})y(t) - \Gamma(z^{-1})\Delta u(t-1) - \mathbf{P_s}^{-1}\mathbf{P_s}^* e(t) - \mathbf{P_s}^{-1}\mathbf{Q_s}^*\Delta u(t-1) + w(t)]$$
(30)

Substituting CARIMA model(1) into (30), the closed loop relationship is obtained,

$$\begin{split} & [\tilde{A} + \sum_{i=1}^{N_2} \alpha_i z^{-1} (\tilde{A} \Gamma_i + \tilde{A} \frac{Q_{s1}}{P_{s0}} + BF_i - B \frac{P_{s1}}{P_{s0}})] y(t) \\ &= B \sum_{i=1}^{N_2} \alpha_i z^{-N_2 + i} (1 + \frac{P_{s1}}{P_{s0}}) w(t + N_2 - 1) \\ &+ (B + B \sum_{i=1}^{N_2} \alpha_i z^{-1} (\Gamma_i + \tilde{A} \frac{Q_{s1}}{P_{s0}}) \xi(t) \end{split}$$
(31)

Where, the characteristic polynomial $M_{CP_{SMPC}}$ can be defined as,

$$\tilde{A} + \sum_{i=1}^{N_2} \alpha_i z^{-1} \left(\tilde{A} \Gamma_i + BF_i + \tilde{A} \frac{Q_{s1}}{P_{s0}} - B \frac{P_{s1}}{P_{s0}} \right)$$
$$\tilde{A} + \sum_{i=1}^{N_2} \alpha_i z^{-1} \left((B - \tilde{A} G_j) z^j + \tilde{A} \frac{Q_{s1}}{P_{s0}} - B \frac{P_{s1}}{P_{s0}} \right)$$
(32)

The polynomial elements of the surface give an extra degree of freedom to assign the closed loop dynamic systems. When the control weight is zero ($\lambda = 0$), (25) can be written as,

$$K_{SMPC} = [(\mathbf{C_s}\mathbf{G} + \mathbf{Q_s})^T (\mathbf{C_s}\mathbf{G} + \mathbf{Q_s})]^{-1}$$
$$(\mathbf{C_s}\mathbf{G} + \mathbf{Q_s})^T \mathbf{C_s}$$
$$= (\mathbf{C_s}\mathbf{G} + \mathbf{Q_s})^{-1} \mathbf{C_s}$$
$$\Delta u = K_{SMPC}(w(t) - f(t))$$
(33)

Furthermore, if the prediction horizon and the control horizon have the same value $N_u = N_2 - N_1 + 1$, the first *n*-rows of K_{SMPC} are given by,

$$\left[\frac{p_{S0}}{p_{S0} g_0 + q_{S0}}, 0, 0, \cdots, 0\right]$$
(34)

Consequently, using (32) and (34) the characteristic polynomial is obtained as,

$$M_{CP} = \tilde{A} + \frac{p_{S0}}{p_{S0} g_0 + q_{S0}} \left(B - \tilde{A} g_0 \right) + \frac{p_{S0}}{p_{S0} g_0 + q_{S0}} \left(z^{-1} \tilde{A} \frac{q_{s1}}{p_{s0}} - z^{-1} B \frac{p_{s1}}{p_{s0}} \right)$$
(35)

The MPC $(P_s(z^{-1}) = 1, Q_s(z^{-1}) = 0)$ under the same conditions has the closed loop polynomial characteristic given by

$$M_{CP_{MPC}}(z^{-1}) = g_0^{-1} B(z^{-1})$$
(36)

Consequently, the poles of the $M_{CP_{MPC}}$ contains the RHP zero of the plant model (1). MPC controller is internally unstable due to the cancellation of unstable zeros of the process with unstable poles of the controller. Note that for the same conditions the SMPC avoids the internal instability problems, because the unstable zeros are not cancelled.

4 Simulation example

The isothermal Van de Vussen reaction systems involve series and parallel reactions. The equations that govern the systems are:

$$\frac{C_a}{dt} = -k_1 C_a - k_3 C_a^2 + (C_{ain} - C_a) \frac{F}{V}
\frac{C_b}{dt} = k_1 C_a - k_2 C_b - C_b \frac{F}{V}$$
(37)

The desired output is the concentration of B, $C_b [mol/l]$, C_a and C_{ain} are the concentrations of A [mol/l] in the reactor and in the feed respectively, the manipulate input, F is the dilution rate [l/min], V is the volume [l], and the rate constants are given by $k_1 = 5/6 [min^{-1}]$, $k_2 = 5/3 [min^{-1}]$, $k_3 = 1/6 [mol/(liter min)]$ [16]. Above no linear, non minimum phase process has been used to show the controller performance.

After linearizing model (37) about the operating point the physical model gives the following transfer function (38). The discretization has been made with a sampling rate $T_s = 0.2$.

$$y(t) = \frac{-0.0939 + 0.1745z^{-1}}{1 - 1.2573z^{-1} + 0.3951z^{-2}}\Delta u(t-1)$$
(38)

Figure 1, shows SMPC and MPC when references changes have been applied. MPC has been tuned to avoid the instability problem shown in (36), it is produced by the cancellation of an unstable zero with an unstable pole. MPC uses the following tuning parameters $N_1 = 1$, $N_2 = 40$, $N_u = 10$, and $\lambda = 1$.

In order to compare both controllers, SMPC has been tuned to have the same closed loop dynamics as MPC. It has the following tuning parameters, $N_1 = 1$, $N_2 = 30$, $N_u = 12 \lambda = 1.5$,



Figure 1: Set point changes

 $P_s = 1 - 0.4z^{-1}$, $Q_s = 0.1(1 - 0.6z^{-1})$, $K_D = 0.1$, and $\rho = 1$. The first set point change is achieved by MPC and SMPC, but in the second one, when the operation point has been considerably changed the MPC has an unstable behaviour owing to no linearities of the process.Figure 2 illustrates the convergence of the sliding surface S(t) to zero, also it can be seen that when S(t) = 0 the tracking error is zero too. SMPC has better robustness than MPC in the presence of non linearities of the process. In order to show the controller robust-



Figure 2: Sliding surface with set point changes

ness, parameter variations with respect to the nominal value are applied to the input concentration of A C_{ain} and the rate constants k_1 , k_2 k_3 . Figure 3 illustrates parameter variations of



Figure 3: Changes in the model

5% and 25%. The controllers use the initial tuning parameters. The parameter variation of 5% has been handled by MPC and SMPC, but in t = 80 min when a parameter variation of 25% was applied MPC has an unstable behaviour, on the other hand SMPC manages the parameter variation with a soft behaviour without oscillations. Figure 4 illustrates the sliding surface, it reaches S(t) = 0 with soft movements. Figure 5 illustrates



Figure 4: Sliding surface with changes in the model

the SMPC and MPC performance when disturbances in a concentration of component A in the input feed have been applied. It has been increased a 5% in t = 10 min, both SMPC and MPC achieve to control the disturbance. In t = 80 min the value of disturbance is increased by 25%, SMPC can keep the controlled variable at its set point without oscillations. MPC cannot reject the disturbance and shows unstable behaviour.



Figure 5: Disturbances in the process

Figure 6 illustrated the sliding surface when disturbances have been applied, the surface is reached with soft movements without oscillations.

5 Conclusions

The proposed SMPC algorithm combines the design technique of SMC and MPC. It has been demonstrated that an appropriate choice of the tuning parameters of SMPC avoids the instability problems of MPC when it is applied to non minimum phase systems. The performance of the controller was judged using a non linear, non minimum phase isothermal Van de Vusse reactor process. It has been shown that considerable improve-



Figure 6: Sliding surface with disturbances in the process

ment in robustness with respect to MPC can be obtained in the presence of modelling uncertainty, and disturbances, enhanced ability is also shown for handling set point changes in a non linear process. The SMPC has the strong points of the two control methods, the robustness features of sliding mode control and the good performance of MPC. The SMPC improves the closed loop behaviour of MPC avoiding the strong control movements of SMC, also the tuning parameters of SMPC has a clear significance. The computational requirements of SMPC are similar to those needed for MPC. It does not require more powerful hardware to be applied in whatever process where MPC is being applied.

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