# Steering Assistance System for Driver Characteristics using Gain Scheduling Control

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## Abstract

This paper proposes a new driver support system which uses both the steering wheel angle and the steering torque of the vehicle so as to take into account the characteristics of the driver's operation. The proposed method is based on a gain scheduling control to accomplish the cooperation between the driver and the support system. By numerical experiments which simulate obstacle avoidance, it is shown that the proposed method has a good tracking capability and cooperates well with driver's operation.

## 1 Introduction

The prevention of traffic accidents is one of the most important social requests. The accident prevention systems have been developed by a variety of schemes, for example, by adding some intelligence to the vehicles[1]. Among the research on the intelligent vehicles, an automated steering system has been studied[2]~[4]. However, it has many problems such as recognition of driving situation and stabilization of vehicle dynamics.

As a preliminary stage to the automated steering system, driver support systems have been investigated. In order to realize the driver support systems, it is necessary to take into account the characteristics of the driver's operation. In the conventional studies on the support systems[5, 6], the steering torque is managed so that the support system cooperates with the driver's operation. However, the system has a drawback that the road tracking performance was deteriorated.

In order to improve the road tracking performance, this paper proposes a new control method of the support system which uses both the steering wheel angle and the steering torque. The proposed method is based on a gain scheduling control so as to accomplish the cooperation between the driver and the support system. By numerical experiments which simulate an obstacle avoidance, it is shown that the proposed method has a good tracking capability and cooperates well with driver's operation.

## 2 Problem formulation

In this paper, a system configuration[7] shown in Fig.1 is considered. The notation in Fig.1 is briefly explained.

As a steering actuator which drives the front wheels, an Electric Power Steering (EPS)[8] which consists of a DC-motor and a ball screw mechanism is employed. A camera is installed in a vehicle to recognize the road shape. From the camera image, a feature value,  $X_{sum}$  is calculated.

The vehicle model which is a 1-input and 2-outputs model describes the input-output relationship from the actual wheel angle,  $\theta_f$ , to side slip angle,  $\beta$ , and the yaw-rate,  $\gamma$ .  $\beta$  and  $\gamma$  can be considered as the state variables of the vehicle.

The control system contains two controllers. One is a guidance controller, K, to determine a reference wheel angle,  $A_{cmd}$ , using the feature value,  $X_{sum}$ , relating to lane shape. The other is an angle controller, R,



Figure 1: System configuration

to determine the reference current,  $I_{cmd}$ , of the motor which is built in the EPS based on the error between the reference angle and the actual wheel angle. The angle controller is redesigned as a steering controller, which uses both the steering wheel angle and the steering torque.

The performance of the system is examined by numerical experiments which simulates a double-lane change maneuver that recovers straight-line travel after obstacle avoidance. Usually, the maneuver results from the driver's operation.

## 3 System modeling

In this section, two models for controller design are built. One is an integrated model for the guidance controller design, which consists of the vehicle dynamics and the camera characteristics. The other is a model of the EPS for the steering controller, which includes the steering torque characteristics.

### 3.1 Integrated model

The integrated model for designing the guidance controller is described by

$$\begin{aligned} \dot{\boldsymbol{x}}_{\boldsymbol{p}} &= \boldsymbol{A}_{\boldsymbol{p}} \boldsymbol{x}_{\boldsymbol{p}} + \boldsymbol{B}_{\boldsymbol{p}} \boldsymbol{\theta}_{f} + \boldsymbol{D}_{\boldsymbol{p}}, \\ y &= \boldsymbol{C}_{\boldsymbol{p}} \boldsymbol{x}_{\boldsymbol{p}}, \end{aligned}$$
 (1)

where

$$\begin{split} \boldsymbol{x_{p}} &= \left[ \begin{array}{cc} \gamma & \beta & X_{sum} \end{array} \right]^{I}, \\ \boldsymbol{A_{p}} &= \left[ \begin{array}{cc} \frac{-2(K_{f}l_{f}^{2} + K_{r}l_{r}^{2})}{I_{z}} & \frac{-2(K_{f}l_{f} - K_{r}l_{r})}{I_{z}} & 0\\ \frac{-2(K_{f}l_{f} - K_{r}l_{r})}{mV^{2}} - 1 & \frac{-2(K_{f} + K_{r})}{mV} & 0\\ \frac{f}{\lambda} \Sigma_{i=1}^{n} a_{i} & -\frac{V}{H} \Sigma_{i=1}^{n} a_{i} y_{pi} & 0 \end{array} \right], \\ \boldsymbol{B_{p}} &= \left[ \begin{array}{c} \frac{2K_{f}l_{f}}{I_{z}}\\ \frac{2K_{f}}{mV}\\ 0 \end{array} \right], \quad \boldsymbol{C_{p}} = \left[ \begin{array}{cc} 0 & 0 & 1 \end{array} \right], \end{split}$$



Figure 2: Model for steering controller

$$oldsymbol{D}_{oldsymbol{p}} = egin{bmatrix} 0 \ 0 \ rac{1}{H} \Sigma_{i=1}^n a_i y_{pi} T_{xi} \end{bmatrix}$$

 $l_f$  is distance between the center of gravity and the front axle,  $l_r$  is distance between the center of gravity and the rear axle,  $K_f$  is the cornering power of a front tire,  $K_r$  is the cornering power of a rear tire,  $I_z$  is the yaw inertia moment, m is the vehicle weight, V is the vehicle speed, and  $\theta_f$  is the front wheel angle.

### 3.2 Steering model

In order to take the driver's operation applied to the vehicle into account, dynamics of the steering wheel is incorporated into the EPS model, as shown in Fig.2.

Then,

$$\frac{d}{dt} \begin{bmatrix} \boldsymbol{x_m} \\ \boldsymbol{x_s} \end{bmatrix} = \begin{bmatrix} A_m - N_g^2 K_s B_m C_m & N_g K_s B_m C_s \\ K_s N_g B_s C_m & A_s - K_s B_s C_s \end{bmatrix} \begin{bmatrix} x_m \\ x_s \end{bmatrix} + \begin{bmatrix} 0 & B_m K_t \\ B_s & 0 \end{bmatrix} \begin{bmatrix} T_D \\ I_{cmd} \end{bmatrix}$$
(2)

$$\begin{bmatrix} \theta_s \\ T_s \\ \theta_f \end{bmatrix} = \begin{bmatrix} 0 & C_s \\ -K_s N_g C_m & K_s C_s \\ N_t C_m & 0 \end{bmatrix} \begin{bmatrix} x_m \\ x_s \end{bmatrix}$$
(3)

where

 $J_{mt}$  is the inertia moment of the motor,  $B_{mt}$  is the damping constant of the motor,  $J_{sw}$  is the inertia moment of the steering wheel,  $B_{sw}$  is the damping constant of the steering wheel,  $\theta_m$  is the motor rotational angle,  $\dot{\theta}_m$  is the motor rotational rate,  $\theta_s$  is the steering wheel rotational angle,  $\dot{\theta}_s$  is the gear ratio between the motor and the steering wheel,  $N_t$  is the gear ratio between the rotational angle of the motor and of the front wheel,  $K_t$  is the torque constant of the motor,  $K_s$  is the spring constant of torque sensor,  $T_D$  is the steering torque applied by the driver operation.

We suppose that the driver characteristics is time varying parameter described by

$$T_D = \delta(t)q\theta_s, \qquad (4)$$

$$|\delta(t)| < 1. \tag{5}$$

where,  $\delta(t)$  is time varying parameter and q is constant value. From eq.(2) ~ (5), we obtain

$$G_{a} \begin{cases} \frac{d}{dt} \boldsymbol{x}_{\boldsymbol{a}} &= \boldsymbol{A}_{\boldsymbol{a}} \boldsymbol{x}_{\boldsymbol{a}} + \boldsymbol{B}_{\boldsymbol{a}} I_{cmd} \\ \begin{bmatrix} T_{s} \\ \theta_{f} \end{bmatrix} &= \boldsymbol{C}_{\boldsymbol{a}} \boldsymbol{x}_{\boldsymbol{a}} \end{cases}$$
(6)

where,

$$\begin{aligned} \boldsymbol{A}_{\boldsymbol{a}} &= \begin{bmatrix} A_m - N_g^2 K_s B_m C_m & N_g K_s B_m C_s \\ K_s N_g B_s C_m & A_s - K_s B_s C_s - \delta(t) B_s q C_s \end{bmatrix}, \\ \boldsymbol{B}_{\boldsymbol{a}} &= \begin{bmatrix} B_m K_t \\ 0 \end{bmatrix}, \quad \boldsymbol{C}_{\boldsymbol{a}} &= \begin{bmatrix} -K_s N_g C_m & K_s C_s \\ N_t & 0 \end{bmatrix}, \\ \boldsymbol{x}_{\boldsymbol{a}} &= \begin{bmatrix} x_m \\ x_s \end{bmatrix}. \end{aligned}$$

### 3.3 Steering model with driver characteristics

In order to predict the conflict between the driver operation and the system's assistance operation, we defined the following parameter.

$$p(t) = p^{max} \left(1 - \frac{|T_s|}{T_{max}}\right) + p^{min} \frac{|T_s|}{T_{max}}$$
(7)

where  $T_{max}$  is maximum value of steering torque  $T_s$ . Please note that its value is small if the steering torque is big and that its value is big if the steering torque is small. The following frequency weight is introduced in order to limit the control input to the low frequency.

$$W_g(T_s) \begin{cases} \dot{x}_e = a_e x_e + b_e v \\ I_{cmd} = p(t) c_e x_e \end{cases}$$
(8)

where,  $x_e$  and  $v_e$  denote the filter state and the imaginary input, respectively, and  $a_e$ ,  $b_e$ ,  $c_e$  denote the fre-



Figure 3: Block structure of generalized plant

quency weighting constants. Augmenting eq.(6) by using the frequency weight, we obtain

$$\frac{d}{dt} \begin{bmatrix} \boldsymbol{x_a} \\ \boldsymbol{x_e} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A_a} & p(t)\boldsymbol{B_a}\boldsymbol{c_e} \\ \boldsymbol{0} & \boldsymbol{a_e} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_a} \\ \boldsymbol{x_e} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{b_e} \end{bmatrix} \boldsymbol{v}. \quad (9)$$

## 4 Controller design

### 4.1 Guidance controller design

The guidance controller, K, is designed based on the integrated model eq.(1). Due to the space limitation, the detail is omitted, see [9].

#### 4.2 Steering controller design

The angle controller,  $\mathbf{R}$ , shown in Fig.1 is redesigned as the steering controller based on the model depicted in Fig.2.

The control specifications are

- S1: To reduce the influence of the road reaction torque  $d_1$  on the steering torque motor  $\theta_m$ , which corresponds to the disturbance rejection performance.
- S2: To reduce the influence of reference angle  $d_2$  on motor rotational angle  $e_3$ , which corresponds to the reference tracking performance.

Then, we obtain the generalized plant shown in Filg.3, where  $d_3$  are disturbance signals,  $L_1, L_2, L_3$  are constant values for the disturbance signals,  $e_1, e_2, e_3, e_4, e_5$  are evaluated signals, and  $V_1, V_2, V_3, V_4, V_5$  are constant values for the evaluated signals. The generalized plant is described by the following equation.

$$\begin{bmatrix} \boldsymbol{x_{aa}} \\ \boldsymbol{e} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{A}(\boldsymbol{p}(t), \boldsymbol{\delta}(t))} & \boldsymbol{B_1} & \boldsymbol{B_2} \\ \hline \boldsymbol{C_1} & \boldsymbol{D_{11}} & \boldsymbol{D12} \\ \boldsymbol{C_2} & \boldsymbol{D_{21}} & \boldsymbol{D_{22}} \end{bmatrix} \begin{bmatrix} \boldsymbol{d} \\ \boldsymbol{I_{cmd}} \end{bmatrix} (10)$$

where

$$A(p(t), \delta(t)) = A_0 + pA_1 + \delta A_2$$

$$A_0 = \begin{bmatrix} A_m - N_g^2 K_s B_m C_m & N_g K_s B_m C_s & 0 \\ N_g K_s B_s C_m & A_s - K_s B_s C_s & 0 \\ 0 & 0 & 0 & A_e \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & B_m K_t C_e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & qC_s & 0 \\ 0 & qC_s & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ B_e \end{bmatrix}, \quad B_1 = \begin{bmatrix} V_1 K_s N_g C_m & V_1 K_s C_s & 0 \\ -V_2 K_s N_g C_m & V_2 K_s C_s & 0 \\ -V_2 K_s N_g C_m & V_2 K_s C_s & 0 \\ -V_3 N_t C_m & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ B_e \end{bmatrix}, \quad B_1 = \begin{bmatrix} V_1 L_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_5 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad x_{aa} = \begin{bmatrix} x_a & x_e \end{bmatrix}^T.$$

The steering controller is obtained by applying gain scheduling  $H_{\infty}$  control method to the generalized plant eq.(10). Gain scheduling output feedback controllers satisfying

- Internal stability for closed loop system
- $\|T_{ed}\|_{l2} < \gamma$

for all the  $p(t), \dot{p}(t)$  in the generalized plant can be obtained if there exist the symmetric matrices X(p), Y(p) and the four matrices  $A_k(p), B_k(p), C_k(p), D_k(p)$ satisfying the following two LMIs

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \tag{11}$$



Figure 4: Vertices in paramer space

where '\*' denotes

$$\begin{bmatrix} Q & * \\ M & P \end{bmatrix} = \begin{bmatrix} Q & M^T \\ M & P \end{bmatrix},$$
(13)

 $T_{ed}$  is the closed loop transfer function from disturbance signal, d, to controlled signal, e. Notation of t and p is omitted.

Then, we obtain a gain scheduling  $H_\infty$  output feedback controller [10] described by

$$\begin{aligned} \boldsymbol{A_{c}}(p,\dot{p}) &= \{I - X(p)Y(p)\}^{-1}[X(p)\dot{Y}(p)\} \\ &+ A_{k}(p) - X(p)\{A(p) - B_{2}(p)D_{k}(p)C_{2}(p)\}Y(p) \\ &- B_{k}(p)C_{2}(p)Y(p) - X(p)B_{2}(p)C_{k}(p)] \end{aligned}$$
(14)  
$$\boldsymbol{B_{c}}(p) &= \{I - X(p)Y(p)\}^{-1} \end{aligned}$$

$$\{B_k(p) - X(p)B_2(p)D_k(p)\}$$
(15)

$$C_{c}(p) = C_{k}(p) - D_{k}(p)C_{2}(p)Y(p)$$
(16)

$$\boldsymbol{D_c}(\boldsymbol{p}) = D_k(\boldsymbol{p}) \tag{17}$$

In order to solve the LMIs, we suppose the controller is described by

$$\begin{aligned} & \boldsymbol{X}(p) &= \boldsymbol{X}_{0} + p \boldsymbol{X}_{1}, \quad \boldsymbol{Y} = \boldsymbol{Y}_{0} \\ & \boldsymbol{A}_{k}(p) &= \boldsymbol{A}_{k0} + p \boldsymbol{A}_{k1}, \\ & \boldsymbol{B}_{k}(p) &= \boldsymbol{B}_{k0} + p \boldsymbol{B}_{k1}, \\ & \boldsymbol{C}_{k}(p) &= \boldsymbol{C}_{k0} + p \boldsymbol{C}_{k1}, \\ & \boldsymbol{D}_{k}(p) &= \boldsymbol{D}_{k0} + p \boldsymbol{D}_{k1}. \end{aligned}$$

and we design a controller of the structure by solving the LMIs corresponding to the parameters p and  $\delta$  at the vertices in Fig.4.



Figure 5: System structure for numerical experiments

## 5 Numerical Experiments

Numerical experiments were carried out in order to evaluate the performance of the proposed controllers. The parameter values for a compact-class vehicle were used in the experiments. The driver characteristics is modeled as proportional and integral elements whose input is error between the desired wheel angle,  $\theta_R$ , and the actual one. The desired wheel angle is calculated by controller  $\boldsymbol{K}$  based on the feature value, shown in Fig.5. In the numerical experiments, a maneuver which recovers straight-line travel after obstacle avoidance, which is called a double-lane change maneuver shown in Fig.6, was used.

The results of the numerical experiments are shown in Figs, 7 and 8. In order to compare with performance of gain scheduling controller,  $H_{\infty}$  constant gain controller designed at the situation in vertices of  $VTX_1$  and  $VTX_2$  is prepared. It is clear from the dashed line in Fig.7, the vehicle dynamics is unstable when the  $H_{\infty}$  constant gain controller assists for driver's operation. On the other hand, from solid lines in Fig.7, the vehicle dynamics is used driver's assist methods are applied. Especially, it is clear from the solid line in Fig.7, the scheduling steering controller makes the trajectory of the vehicle smooth.

Fig.8 shows the steering wheel torque signals for each controllers. It is clear that the scheduling steering controller generates a more smooth steering wheel signal.

Fig.9 shows the open loop transfer functions in each vertices  $(VTX_1 \sim VTX_4)$ . In a case which employ the gain scheduling controller, In low frequency range less than 1 Hz, the gain scheduling controller has the open loop characteristics in  $VTX_2$ . Therefore, the road tracking performance is carried out by high gain characteristic of the loop transfer function. In middle frequency range from 1 Hz to 20 Hz, the scheduling con-



Figure 6: Manuver for numerical experiments



Figure 7: Trajectory of center of gravity on vehicle

troller has the characteristic in  $VTX_4$ . The robustness for disturbance applied the steering torque signal is performed by low gain characteristic of transfer function in  $VTX_4$ .

From these results and discussions, it is shown that the scheduling steering controller is effective not only from the vehicle stability point of view but also the cooperation with the drive's operation point of view.

## 6 Conclusions

The new driver support system which uses both the steering wheel angle and the steering torque of the vehicle has been proposed. The steering controller was designed by considering the driver's characteristics as uncertainty and was extended to the gain scheduling controller where the scheduling parameter was the steering torque. The effectiveness of the proposed controller was shown by the numerical experiments. The examination of the proposed method by an actual vehicle remains to be done.



Figure 8: Steering torque signals of steering controller



Figure 9: Open loop transfer functions of steering control system

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