Adaptive Robust H_∞ Control for Nonlinear Systems with Parametric Uncertainties and External Disturbances

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Abstract

This paper proposes a novel design method for the adaptive robust H_∞ control problem of a class of nonlinear systems with parametric uncertainties and external disturbances, which combines adaptive control and robust H_∞ control techniques. By the use of the parameter projection method in adaptive control, the adaptive control laws are derived. Based on Hamilton-Jacobi inequalities, the adaptive robust H_∞ controllers are designed. A numerical simulation demonstrates the correctness of the proposed design method.

1 Introduction

In practical control systems, there exist many classes of uncertainties, such as parameter variations, external disturbances and model errors. Robust control for uncertain nonlinear systems has widely been studied in the last decade [1~11]. Nonlinear H_∞ control, as an important branch of nonlinear robust control, has attracted much attention since 1990s. With dissipative theory and differential game, nonlinear H_∞ control can be equivalent to the solvability of Hamilton-Jacobi equalities or inequalities [1,2]. Robust H_∞ control for nonlinear systems was further studied based on the above results [3~8].

In recent years, a combination of adaptive control and robust control receives more and more attention and a large number of research results have been obtained. The adaptive tracking problem for a class of SISO nonlinear systems is discussed by the use of exact linearization, and the effect of an external disturbance on the tracking error was measured [12]. The robust adaptive control for a class of strict-feedback nonlinear systems is studied in [13]. In [14], the adaptive tracking problem with disturbance attenuation of a class of parametric strict-feedback nonlinear systems is reduced to nonlinear H_∞ control problem. Based on Hamilton-Jacobi inequalities and the backstepping method, a robust adaptive controller is designed. The literature [15] discusses the adaptive H_∞ tracking for a class of MIMO nonlinear systems represented by input-output model. By the use of the parameter projection method and Riccati inequalities, controllers are designed to ensure all signals in the closed-loop systems are bounded and the tracking error with H_∞ performance is uniformly bounded. The adaptive robust control of a class of nonlinear systems of semi-strict feedback form is considered in [16, 17]. In the case where there exist unknown parameters and unknown nonlinear functions, the adaptive robust controllers are designed to ensure the trajectory tracking and transient performance is satisfactory. References [12-15] measure the effect of external disturbances on tracking error with L_2-gain. However, all of the above mainly discuss the tracking problem and H_∞ control problem defined in [5] is not studied.

This paper considers the adaptive robust H_∞ control problem of a class of nonlinear systems with parametric uncertainties and external disturbances. It proposes a novel design method of adaptive robust controllers. The design of the adaptive laws exploits the idea of the parameter projection method in [15,18]. The robust H∞ control problem is solved using Hamilton-Jacobi inequalities. The proposed design method combines adaptive control and robust H∞ control techniques. Compared with the past research [14,15], this paper attacks the nonlinear H∞ control problem which is similar to the one in [5] and also the uncertain nonlinear systems to be considered are more general.

2 Problem Formulation

Consider a nonlinear system

$$\begin{aligned}\dot{x} &= f(x)+[g(x)+\Delta g(x)]u+g_w(x)w \\ &= f(x)+[g(x)+\sum_{i=1}^{r}\theta_i p_i(x)]u+g_w(x)w\end{aligned} \tag{1a}$$

$$z = h(x)+k(x)u \tag{1b}$$

where $x \in X \subseteq R^n$, $w \in R^l$, $u \in R^m$, $z \in R^s$ represent the state, disturbance input, control input and control output vectors, respectively, $f(x) \in R^{n\times 1}$, $g(x) \in R^{n\times m}$, $g_w(x) \in R^{n\times l}$, $k(x) \in R^{s\times m}$ are the smooth matrix functions, $\Delta g(x)$ is the parametric uncertainty, $p_i(x) \in R^{n\times m}$ ($1 \le i \le r$) is the known matrix function, and $\theta = [\theta_1,\theta_2,\cdots,\theta_r]^T$ is the unknown parameter vector.

For nonlinear system (1), suppose that x_0 is the initial state, and $f(x_0)=0$, $g(x_0)=0$, $g_w(x_0)=0$, $k(x_0)=0$, $p_i(x_0)=0$, for $1 \le i \le r$. From now on, $\hat{\theta}$ denotes the estimated value of θ . First, the definition of the 'zero-state observable' concept is introduced.

Definition 1: For nonlinear system $\{f(x),h(x)\}$, *i.e.*

$$\begin{aligned}\dot{x} &= f(x) \\ z &= h(x)\end{aligned} \tag{2}$$

if $h(t)\equiv 0$ implies $x(t)\equiv x_0$, then system $\{f(x),h(x)\}$ is zero-state observable.

The following assumptions are made on system (1):

A1: The system $\{f(x),h(x)\}$ is zero-state observable.

A2: The unknown parameter vector θ satisfies $\|\theta\|^2 \le \rho$, where ρ is a positive number, and $\|\bullet\|$ denotes Euclidean norm.

A3: $k^T(x)[h(x) \quad k(x)] = [0 \quad I]$ (3)

Assumption A1 is to ensure the internal stability. Assumption A2 guarantees the parameter vector within a known region. Assumption A3 simplifies the considered model. Assumptions A1 and A2 are often made for nonlinear H∞ control in the literature.

The adaptive robust H∞ control problem t to be discussed in this paper is defined as follows:

Definition 2: For given positive numberes γ and ε , construct the controller

$$\begin{aligned}\dot{\hat{\theta}} &= \psi(x,\hat{\theta}) \\ u &= \alpha(x,\hat{\theta})\end{aligned} \tag{4}$$

such that the closed-loop system described by (1) and (4) satisfies the conditions below:

(i) The following L_2-gain is finite.

$$\int_0^T \|z\|_2^2 dt \le \gamma^2 \int_0^T \|w\|_2^2 dt + \varepsilon \tag{5}$$

where $T \ge 0$.

(ii) It is asymptotically stable, that is, when $w(t)\equiv 0$ the system is asymptotically stable at the point x_0 .

Remark 1: Definition 2 is slightly different from the definition on the robust H∞ control problem in [5,6], where equation (5) in definition 2 is replaced by

$$\int_0^T \|z\|_2^2 dt \le \gamma^2 \int_0^T \|w\|_2^2 dt \tag{6}$$

The difference is that the condition (i) in Definition 2 includes a positive number ε , while this is not in [5,6]. However, because the number ε can be chosen arbitrarily, it can be small enough to guarantee the robust performance of the closed-loop systems. And similar functions exist in [12,13,15], but the literatures only discuss the tracking problem.

3 Main Results

The adaptive law is designed by the use of the idea of the parameter projection method in [15,18]. From a practical perspective, $\hat{\theta}$ is usually required to be within a pre-assigned region. Let $\Omega_1 = \{\hat{\theta} \mid \hat{\theta}^T\hat{\theta} \le \rho\}$ and $\Omega_2 = \{\hat{\theta} \mid \hat{\theta}^T\hat{\theta} \le \rho+\delta\}$, where $\delta > 0$. Then a smooth projection algorithm can be obtained as

$$\mathrm{Proj}(\phi,\hat{\theta}) = \begin{cases}\phi - \dfrac{(\|\hat{\theta}\|^2-\rho)\hat{\theta}^T\phi}{\delta\|\hat{\theta}\|^2}\hat{\theta} & \text{if } \|\hat{\theta}\|^2 > \rho \text{ and } \hat{\theta}^T\phi \ge 0 \\ \phi & \text{otherwise}\end{cases} \tag{7}$$

where ϕ is a smooth function. Let

$$\dot{\hat{\theta}} = \mu \mathrm{Proj}(\phi,\hat{\theta}) \tag{8}$$

where μ is an adaptive gain, $\mu > 0$. With projection function (7), if $\hat{\theta}(0) \in \Omega_1$, $\hat{\theta} \in \Omega_2$ for any $t \geq 0$.

The theorem below describes a sufficient condition to solve the adaptive robust H_∞ control problem of nonlinear system (1) and provides an adaptive robust controller design method.

Theorem 1: For nonlinear system (1) with assumptions A1~A3 and given positive numbers γ, ε, ρ and δ, if there exist a positive number λ and a positive definitive function $V(x)$, $V(x_0)=0$, such that the following Hamilton-Jacobi inequality

$$\frac{\partial V}{\partial x} f(x) - \frac{1}{2}(1-\lambda)\frac{\partial V}{\partial x} g(x) g^T(x) \frac{\partial V^T}{\partial x} - \frac{1}{2}(1-\frac{1}{\lambda})(\rho+\delta)\frac{\partial V}{\partial x}[\sum_{i=1}^{r} p_i(x) p_i^T(x)]\frac{\partial V^T}{\partial x} + \frac{1}{2}\gamma^{-1}\frac{\partial V}{\partial x} g_w(x) g_w^T(x)\frac{\partial V^T}{\partial x} + \frac{1}{2}h^T h \leq 0 \tag{9}$$

holds, then the following controller u can solve adaptive robust H_∞ control problem and guarantee $\hat{\theta} \in \{\hat{\theta} \mid \hat{\theta}^T\hat{\theta} \leq \rho+\delta\}$.

$$u = -g^T(x)\frac{\partial^T V}{\partial x} - [\sum_{i=1}^{r}\hat{\theta}_i p_i(x)]^T \frac{\partial^T V}{\partial x} \tag{10}$$

with

$$\dot{\hat{\theta}} = \mu \mathrm{Proj}(\phi, \hat{\theta}) \tag{11}$$

$$\phi = \begin{bmatrix} \frac{\partial V}{\partial x} p_1 \\ \cdots \\ \frac{\partial V}{\partial x} p_r \end{bmatrix} u \tag{12}$$

where $\mu = \frac{2\rho+\delta}{\varepsilon}$ is the gain of the adaptive law.

Proof: In order to prove the theorem , it needs to ensure the conditions (i) and (ii) in Definition 2 are satisfied. Choose a Lyapunov function as

$$W(x) = V(x) + \frac{1}{2\mu}\tilde{\theta}^T\tilde{\theta} \tag{13}$$

where $\tilde{\theta} = \hat{\theta} - \theta$. The time derivative of $W(x)$ with respect to x is

$$\dot{W}(x) = \frac{\partial V}{\partial x}\{f(x) + [g(x) + \Delta g(x)]u + g_w(x) w\} + \tilde{\theta}^T\dot{\hat{\theta}} \tag{14}$$

So

$$\begin{aligned}
&\dot{W}(x) - \gamma\frac{1}{2}\|w\|^2 + \frac{1}{2}\|z\|^2 \\
&= \frac{\partial V}{\partial x}\{f(x) + [g(x) + \Delta g(x)]u + g_w(x) w\} + \frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}} - \frac{1}{2}\gamma\|w\|^2 + \frac{1}{2}\|z\|^2 \\
&= \frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x}[g(x) + \sum_{i=1}^{r}\theta_i p_i(x)]u + \frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}} + \frac{\partial V}{\partial x} g_w(x) w - \frac{1}{2}\gamma\|w\|^2 + \frac{1}{2}u^T u + \frac{1}{2}h^T h \\
&= \frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x}[g(x) + \sum_{i=1}^{r}\hat{\theta}_i p_i(x)]u + \frac{\partial V}{\partial x} g_w(x) w + \frac{\partial V}{\partial x}[\sum_{i=1}^{r}(\theta - \hat{\theta}_i) p_i(x)]u + \frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}} - \frac{1}{2}\gamma\|w\|^2 + \frac{1}{2}u^T u + \frac{1}{2}h^T h \\
&= \frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x}[g(x) + \sum_{i=1}^{r}\hat{\theta}_i p_i(x)]u - \tilde{\theta}^T \begin{bmatrix} \frac{\partial V}{\partial x} p_1 \\ \cdots \\ \frac{\partial V}{\partial x} p_r \end{bmatrix} u + \frac{\partial V}{\partial x} g_w(x) w + \frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}} - \frac{1}{2}\gamma\|w\|^2 + \frac{1}{2}u^T u + \frac{1}{2}h^T h
\end{aligned} \tag{15}$$

Substituting (10), (11) and (12) into (15) yields

$$\begin{aligned}
&\dot{W}(x) - \gamma\frac{1}{2}\|w\|^2 + \frac{1}{2}\|z\|^2 \\
&\leq \frac{\partial V}{\partial x} f(x) - \frac{1}{2}u^T u + \frac{1}{2}\gamma^{-1}\frac{\partial V}{\partial x} g_w(x) g_w^T(x)\frac{\partial^T V}{\partial x} - \tilde{\theta}^T\phi + \frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}} + \frac{1}{2}h^T h \\
&\leq \frac{\partial V}{\partial x} f(x) - \frac{1}{2}(1-\lambda)\frac{\partial V}{\partial x} g(x) g^T(x)\frac{\partial V^T}{\partial x} - \frac{1}{2}(1-\frac{1}{\lambda})(\rho+\delta)\frac{\partial V}{\partial x}[\sum_{i=1}^{r} p_i(x) p_i^T(x)]\frac{\partial V^T}{\partial x} \\
&\quad + \frac{1}{2}\gamma^{-1}\frac{\partial V}{\partial x} g_w(x) g_w^T(x)\frac{\partial V^T}{\partial x} - \tilde{\theta}^T\phi + \frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}} + \frac{1}{2}h^T h
\end{aligned} \tag{16}$$

From adaptive law (11), we have

$$-\tilde{\theta}^T\phi + \frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}} \leq 0$$

Thus, application of (9) to (16) results in

$$\dot{W}(x)-\frac{1}{2}\gamma\|w\|^2+\frac{1}{2}\|z\|^2\le 0 \quad (17)$$

which implies that

$$W(x(T),\hat{\theta}(T))-W(x(0),\hat{\theta}_0)\le\int_0^T(\frac{1}{2}\gamma\|w\|^2-\frac{1}{2}\|z\|^2)dt \quad (18)$$

As a result,

$$\begin{aligned}W(x(T),\hat{\theta}(T))&\le\int_0^T(\frac{1}{2}\gamma\|w\|^2-\frac{1}{2}\|z\|^2)dt+W(x(0),\hat{\theta}_0)\\&=\int_0^T(\frac{1}{2}\gamma\|w\|^2-\frac{1}{2}\|z\|^2)dt+\frac{1}{\mu}(\hat{\theta}_0-\theta)^T(\hat{\theta}_0-\theta)\\&\le\int_0^T(\frac{1}{2}\gamma\|w\|^2-\frac{1}{2}\|z\|^2)dt+\frac{1}{\mu}(2\rho+\delta) \quad (19)\end{aligned}$$

Therefore, condition (i) in definition 2 is satisfied.

Next, consider system (1) with $w(t)\equiv 0$. It is clear that the following system

$$\begin{aligned}\dot{x}&=f(x)+[g(x)+\sum_{i=1}^r\theta_i p_i(x)]u\\z&=\begin{bmatrix}h(x)\\u\end{bmatrix}\end{aligned} \quad (20)$$

is zero-state observable. For $w\equiv 0$,

$$\begin{aligned}\dot{W}(x)&=\frac{\partial V}{\partial x}f(x)+\frac{\partial V}{\partial x}[g(x)+\sum_{i=1}^r\hat{\theta}_i p_i(x)]u-\tilde{\theta}^T\begin{bmatrix}\frac{\partial V}{\partial x}p_1\\\cdots\\\frac{\partial V}{\partial x}p_r\end{bmatrix}u+\frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}}\\&=\frac{\partial V}{\partial x}f(x)-u^Tu-\tilde{\theta}^T\begin{bmatrix}\frac{\partial V}{\partial x}p_1\\\cdots\\\frac{\partial V}{\partial x}p_r\end{bmatrix}u+\frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}}\\&=\frac{\partial V}{\partial x}f(x)-\frac{1}{2}u^Tu+\frac{1}{2}\gamma^{-1}\frac{\partial V}{\partial x}g_w(x)g_w^T(x)\frac{\partial^T V}{\partial x}+\frac{1}{2}h^Th\\&\quad-\tilde{\theta}^T\phi+\frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}}-\frac{1}{2}\gamma^{-1}\frac{\partial V}{\partial x}g_w(x)g_w^T(x)\frac{\partial^T V}{\partial x}-\frac{1}{2}u^Tu-\frac{1}{2}h^Th\\&\le\frac{\partial V}{\partial x}f(x)-\frac{1}{2}(1-\lambda)\frac{\partial V}{\partial x}g(x)g^T(x)\frac{\partial V^T}{\partial x}\\&\quad-\frac{1}{2}(1-\frac{1}{\lambda})(\rho+\delta)\frac{\partial V}{\partial x}[\sum_{i=1}^r p_i(x)p_i^T(x)]\frac{\partial V^T}{\partial x}\\&\quad+\frac{1}{2}h^Th-\tilde{\theta}^T\phi+\frac{1}{\mu}\tilde{\theta}^T\dot{\hat{\theta}}-\frac{1}{2}\gamma^{-1}\frac{\partial V}{\partial x}g_w(x)g_w^T(x)\frac{\partial^T V}{\partial x}\\&\quad-\frac{1}{2}u^Tu-\frac{1}{2}h^Th\\&\le-\frac{1}{2}u^Tu-\frac{1}{2}\gamma^{-1}\frac{\partial V}{\partial x}g_w(x)g_w^T(x)\frac{\partial^T V}{\partial x}-\frac{1}{2}h^Th \quad (21)\end{aligned}$$

which results in $\dot{W}(x,\hat{\theta})\le 0$. Also, $\dot{W}(x,\hat{\theta})=0$ implies $u=0, h(x)=0$. It can be concluded that $x=x_0$. Therefore, condition (ii) in Definition 2 is satisfied as well.

Remark 2: In the previous literatures on robust H$_\infty$ control, the sufficient conditions and controllers are obtained based on the solutions of Hamilton-Jacobi inequalities. However, the forms of the controllers are fixed since they are designed, so they can not exploit the information obtained in the control. This results some level of conservatism in robust control. In term of Theorem 3 in the literature [7], the sufficient condition to guarantee the robust H$_\infty$ control problem to be solvable is the following Hamilton-Jacobi inequality

$$\begin{aligned}&\frac{\partial V}{\partial x}f(x)-\frac{1}{2}(1-\lambda)\frac{\partial V}{\partial x}g(x)g^T(x)\frac{\partial V^T}{\partial x}\\&+\frac{1}{2}\frac{1}{\lambda}\rho\frac{\partial V}{\partial x}[\sum_{i=1}^r p_i(x)p_i^T(x)]\frac{\partial V^T}{\partial x}\\&+\frac{1}{2}\gamma^{-2}\frac{\partial V}{\partial x}g_w(x)g_w^T(x)\frac{\partial V^T}{\partial x}+\frac{1}{2}h^Th\le 0\end{aligned} \quad (22)$$

holds, and a robust controller is

$$u=-g^T(x)\frac{\partial^T V}{\partial x} \quad (23)$$

Compared the above Hamilton-Jacobi inequality with (9), it indicates that when the following inequality

$$\begin{aligned}&\frac{1}{2}\frac{1}{\lambda}\rho\frac{\partial V}{\partial x}[\sum_{i=1}^r p_i(x)p_i^T(x)]\frac{\partial V^T}{\partial x}>\\&-\frac{1}{2}(1-\frac{1}{\lambda})(\rho+\delta)\frac{\partial V}{\partial x}[\sum_{i=1}^r p_i(x)p_i^T(x)]\frac{\partial V^T}{\partial x}\end{aligned} \quad (24)$$

holds, the Theorem 1 in this paper is less conservative. That means, when (9) holds, the above Hamilton-Jacobi inequality may not hold, so the robust H$_\infty$ control problem perhaps can not be solved. However, using Theorem 1 in this paper, a suitable controller can be designed to solve the adaptive H$_\infty$ control problem, and the parameter can be adjusted according to the adaptive law.

4 A Simulated Example

Consider nonlinear system (1) with

$$f(x)=\begin{bmatrix}-0.5x_1+2x_2-5x_2^2\\ -1.5x_2+5x_1x_2\end{bmatrix} \quad (25)$$

$$g_w(x)=\begin{bmatrix}\sqrt{2}/2\cos(x_1) & -\sqrt{2}/2\cos(x_1)\\ \sqrt{2}/2\sin(x_2) & \sqrt{2}/2\sin(x_2)\end{bmatrix} \quad (26)$$

$$g(x)=[2 \quad 1]^T \quad (27)$$

$$h(x)=[x_1-x_2 \quad 0]^T \qquad k(x)=[0 \quad 1]^T \quad (28)$$

$$p_1(x)=[1 \quad 0]^T \qquad p_2(x)=[0 \quad 1]^T \quad (29)$$

$$x=[x_1 \quad x_2]^T \qquad x_0=[0 \quad 0]^T$$

Clearly, system $\{f(x),h(x)\}$ above is zero-state observable, *i.e.* assumption A1 is satisfied; and from (28), system (1) satisfies assumption A3. Let $\rho=0.9$, $\|\theta\|\le 0.9$ and then assumption A2 is also satisfied. Choose a non-negative function

$$V(x)=\frac{1}{2}(x_1^2+x_2^2) \quad (30)$$

Obviously, $V(x_0)=0$. Let $\gamma=1$, $\varepsilon=0.1$, $\delta=0.1$ and $\lambda=0.5$. Equation (8) can be rewritten as

$$\frac{\partial V}{\partial x}f(x)-\frac{1}{2}(1-\lambda)\frac{\partial V}{\partial x}g(x)g^T(x)\frac{\partial V^T}{\partial x}$$
$$-\frac{1}{2}(1-\frac{1}{\lambda})(\rho+\delta)\frac{\partial V}{\partial x}[\sum_{i=1}^{r}p_i(x)p_i^T(x)]\frac{\partial V^T}{\partial x}$$
$$+\frac{1}{2}\frac{\partial V}{\partial x}\gamma^{-1}g_w(x)g_w^T(x)\frac{\partial V^T}{\partial x}+\frac{1}{2}h^Th$$
$$=(-0.5+0.25\cos^2 x_1)x_1^2+(-0.75+0.25\sin^2 x_2)x_2^2\le 0 \quad (31)$$

In terms of Theorem 1, the adaptive robust controller is given by

$$u=-(2x_1+x_2)-\hat{\theta}_1x_1-\hat{\theta}_2x_2 \quad (32)$$

$$\dot{\hat{\theta}}=19\mathrm{Proj}(\phi,\hat{\theta}) \quad (33)$$

$$\mathrm{Proj}(\phi,\hat{\theta})=\begin{cases}\phi-\dfrac{(\|\hat{\theta}\|^2-\rho)\phi^T\hat{\theta}}{0.1\|\hat{\theta}\|^2}\hat{\theta} & \text{if } \|\hat{\theta}\|^2>\rho \text{ and } \phi^T\hat{\theta}>0\\ \phi & \text{otherwise}\end{cases} \quad (34)$$

$$\phi=-\begin{bmatrix}2x_1^2+x_1x_2\\ 2x_1x_2+x_2^2\end{bmatrix} \quad (35)$$

With controller (32)-(35), the system (1) achieves the robust performance

$$\int_0^\infty\|z\|_2^2dt\le\int_0^\infty\|w\|_2^2dt+0.1 \quad (36)$$

and the system is asymptotically stable at $x_0=[0 \quad 0]^T$.

In term of Theorem 3 in the literature [7], the sufficient condition to guarantee the robust H∞ control problem to be solvable is the following Hamilton-Jacobi inequality

$$\frac{\partial V}{\partial x}f(x)-\frac{1}{2}(1-\lambda)\frac{\partial V}{\partial x}g(x)g^T(x)\frac{\partial V^T}{\partial x}$$
$$-\frac{1}{2}\frac{1}{\lambda}\rho\frac{\partial V}{\partial x}[\sum_{i=1}^{r}p_i(x)p_i^T(x)]\frac{\partial V^T}{\partial x} \quad (37)$$
$$+\frac{1}{2}\frac{\partial V}{\partial x}\gamma^{-1}g_w(x)g_w^T(x)\frac{\partial V^T}{\partial x}+\frac{1}{2}h^Th\le 0$$

holds. However, by computation

$$\frac{\partial V}{\partial x}f(x)-\frac{1}{2}(1-\lambda)\frac{\partial V}{\partial x}g(x)g^T(x)\frac{\partial V^T}{\partial x}$$
$$-\frac{1}{2}\frac{1}{\lambda}\rho\frac{\partial V}{\partial x}[\sum_{i=1}^{r}p_i(x)p_i^T(x)]\frac{\partial V^T}{\partial x}$$
$$+\frac{1}{2}\frac{\partial V}{\partial x}\gamma^{-1}g_w(x)g_w^T(x)\frac{\partial V^T}{\partial x}+\frac{1}{2}h^Th$$
$$=(-0.1+0.25\cos^2 x_1)x_1^2+(-0.35+0.25\sin^2 x_2)x_2^2 \quad (38)$$

So it can not assure that the inequality (37) holds. This means that, in term of Theorem 3 in the previous literature [7], robust controller can not be obtained to guarantee the robust performance of the system (1) in the case. Therefore, we can say, the conclusion in this paper decreases the conservatism of robust control for nonlinear systems to some level.

In order to illustrate the correctness of the conclusions made in the paper, a simulation was carried out using Matlab, Simulink, and Matlab Toolboxes. In the simulation, disturbance inputs w_1 and w_2 were impulse signals, as shown in Figure 1 and Figure 2. The closed-loop system states x_1 and x_2 are shown in Figure 3 and Figure 4, and the L_2 norms of the disturbance and control output are shown in Figure 5 and Figure 6. The simulation results show that the closed-loop system is internal stable, and the L_2-gain from disturbance w to output z is less than the given positive scale $\gamma = 1$. Thus, the conclusions in the paper are correct.

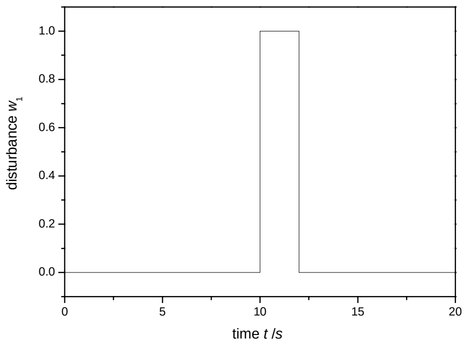

Figure 1： External disturbance w_1

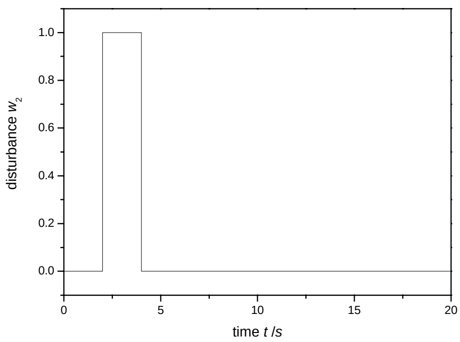

Figure 2： External disturbance w_2

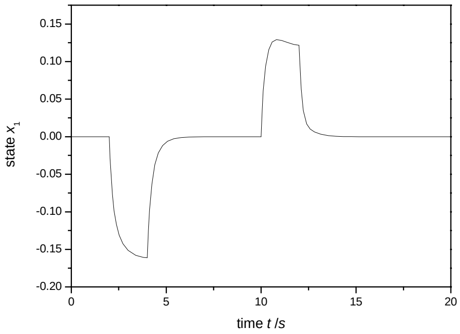

Figure 3： State x_1

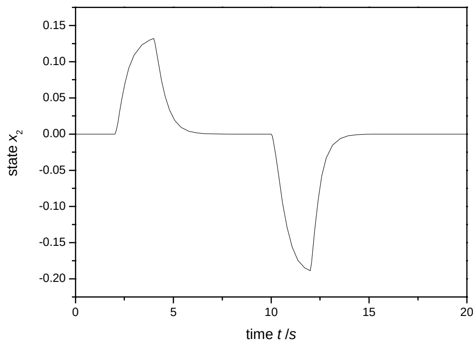

Figure 4： State x_2

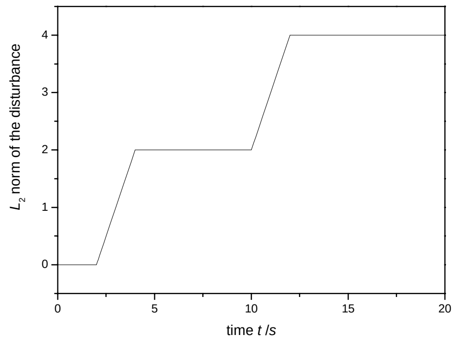

Figure 5 ：L_2 norm of the external disturbance

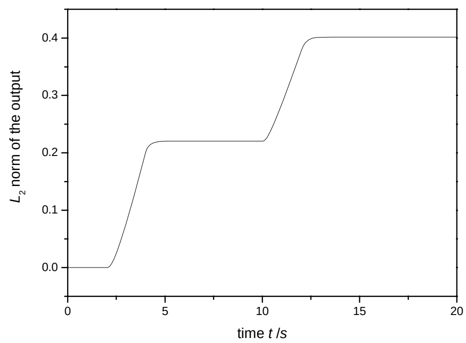

Figure 6： L_2 norm of the control output

5 Conclusions

This paper has considered a class of nonlinear systems with parametric uncertainties and external disturbances. Using the parameter projection algorithm and Hamilton-Jacobi inequality, it has proposed a new design method for the adaptive robust H_∞ control problem, which combines adaptive control and robust H_∞ control. Compared with the past results on combining adaptive control and robust control, this paper successfully applied the parameter projection algorithm to nonlinear H_∞ control problem. The numerical simulation shows the design method is effective. Further research is needed to solve the problem: how to guarantee the estimation of the unknown parameters converges to an arbitrarily small region around the real parameters.

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