H_∞ CONTROL OF DESCRIPTOR SYSTEMS: AN EXAMPLE FROM BINARY DISTILLATION CONTROL

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Abstract

In this paper the H_∞ control problem for descriptor systems is considered. This problem can efficiently be solved by specialization of a recent solution of the general quadratic performance control problem to the H_∞ case. The solution is given in terms of strict linear matrix inequality (LMI) conditions. Contrary to previous solutions of the descriptor H_∞ control problem, these synthesis conditions easily can be evaluated by standard LMI solvers. The presented synthesis result is applied to a S/KS H_∞ control problem from binary distillation control. The process model of the underlying separation process is given by means of a phenomenological descriptor model which describes the movement of concentration profiles in rectifying and stripping section of the distillation column.

1 Introduction

Descriptor systems (sometimes also referred to as singular, semistate or differential-algebraic equation (DAE) systems) describe a broad class of systems which are not only of theoretical interest but also have great practical significance. Models of chemical processes for example typically consist of differential equations describing the dynamic balances of mass and energy while additional algebraic equations account for thermodynamic equilibrium relations, steady-state assumptions, empirical correlations, etc. [3]. In mechanical engineering descriptor systems result from holonomic and non-holonomic constraints [12]. Also in electronics and even in economics modeling in terms of descriptor systems frequently is encountered [5].

Descriptor systems are able to describe system behaviors, that cannot be captured by "non-descriptor" systems (i.e. systems governed only by differential equations) [1]. Therefore index reduction techniques (i.e. reduction of a descriptor system to an ODE) necessarily are connected to a loss of information for high index systems. Due to this fact in recent years much work has focused on analysis and design techniques for high index descriptor systems (see [4] for an overview).
For linear systems many of the standard design techniques for state-space systems have been extended to descriptor systems. Especially there has been a focus on LMI synthesis techniques which guarantee bounds on induced vector norms (e.g. H_2, H_∞-norm) for input-output descriptions of the form

$$\begin{aligned} E\dot{\boldsymbol{\xi}}(t) &= A\boldsymbol{\xi}(t) + B\boldsymbol{w}(t),\ \ t \geq 0,\ \boldsymbol{\xi}(0^-) = \boldsymbol{\xi}_0^- \\ \boldsymbol{z}(t) &= C\boldsymbol{\xi}(t) + D\boldsymbol{w}(t). \end{aligned} \tag{1}$$

Here $\boldsymbol{\xi}(t) \in I\!R^{n_\xi}$ denote the descriptor variables, $\boldsymbol{w}(t) \in I\!R^{n_w}$ the external input variables, and $\boldsymbol{z}(t) \in I\!R^{n_z}$ the external output variables. E, A, B, C, D are constant system matrices of appropriate dimensions with E being a possibly singular $n_\xi \times n_\xi$ matrix with $n_\xi \geq \mathrm{rank}(E) =: r$. Usually the LMI approaches to this kind of problems (e.g. [6, 10]) assume an E-matrix in SVD form, i.e.

$$E = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}, \quad \Sigma = \Sigma^{\mathrm{T}} \in I\!R^{r\times r}. \tag{2}$$

Theoretically there is no loss of generality connected to this assumption since a transformation to an E matrix of the form (2) is always possible. However, this transformation may be ill conditioned. This is especially the case for mechanical descriptor descriptions where point masses of extremely different magnitudes are involved. Furthermore the approaches based on (2) result in synthesis LMIs with all occurring system matrices partitioned according to (2). This is not only notational inconvenient but in fact means, that the standard case (regular E matrix) is not included. These shortcomings are overcome for the general quadratic performance (GQP) output feedback control problem for descriptor systems in [11].

In this paper the GQP synthesis result is specialized to the most important subproblem, namely the descriptor H_∞ control problem. The solution of the controller synthesis problem is based on congruence transformation of a corresponding analysis result in descriptor form. The analysis result basically is an LMI based test (the generalized bounded real lemma) which allows for a given closed loop system to decide whether or not a prescribed H_∞ norm bound is met or not. This test is given here for convenience of the reader. The transition to the controller synthesis solution is only briefly outlined. Details can be found in [11]. The focus here is to show the applicability of the descriptor H_∞ controller synthesis result to realistic control problems in process control. To our knowledge, this is the first application of a descriptor H_∞ controller synthesis result to a realistic control setup.

2 The Generalized Bounded Real Lemma

In contrast to state space system descriptions a descriptor system may allow non-unique solutions which possibly contain impulses. This certainly does not fit into the internal stability requirement which goes along with the H_∞-norm bound requirement in the standard H_∞ control problem. As a generalization one therefore considers regular (i.e. descriptor systems with a unique solution) and impulse-free descriptor systems. Descriptor systems which additionally are stable are termed *a*dmissible [6]. An LMI based characterization of admissible descriptor systems (E, A, B, C) (i.e. descriptor systems (1) with $D = 0$) which are H_∞-norm bounded is given in the following proposition:

Proposition 2.1 *(Generalized bounded real lemma, GBRL) A system (E,A,B,C) is a stable index one system with*

$$\|G\|_\infty < \gamma, \quad G(s) := C(sE - A)^{-1}B \tag{3}$$

iff there exists a matrix X with

$$E^{\mathrm{T}}X = X^{\mathrm{T}}E \geq 0 \tag{4}$$

$$\mathcal{B}(\gamma, X) := \begin{bmatrix} A^{\mathrm{T}}X + XA & X^{\mathrm{T}}B & C^{\mathrm{T}} \\ B^{\mathrm{T}}X & -\gamma I & 0 \\ C & 0 & -\gamma I \end{bmatrix} < 0. \tag{5}$$

Proof. See [11]. □

Remark 1. The consideration of the case $D = 0$ in the previous proposition is not restrictive since every descriptor system (1) can be reformulated as a descriptor system with $D = 0$ if additional descriptor variables with $\boldsymbol{\xi}_{add}(t) := D\boldsymbol{w}(t)$ are introduced.

Remark 2. The LMI (4) is non-strict. The key towards a strict inequality is the symmetry constraint $E^{\mathrm{T}}X = X^{\mathrm{T}}E$ expressed in (4). All X fulfilling this constraint can be parameterized in terms of the fundamental subspaces of E as

$$X = \tilde{X}E + E^{\perp}W, \quad \tilde{X} = \tilde{X}^{\mathrm{T}} \tag{6}$$

with $E^{\perp}$ denoting a full rank matrix such that $E^{\mathrm{T}}E^{\perp} = 0$ and $\tilde{X}$, U being matrices of appropriate dimensions. The parameterization (6) in $\tilde{X}$, W is valid since we may write (4) as $VE^{\mathrm{T}}U^{\mathrm{T}}U^{-\mathrm{T}}XV^{\mathrm{T}} = VX^{\mathrm{T}}U^{-1}UEV^{\mathrm{T}}$ with $E_{svd} := UEV^{\mathrm{T}}$ being a SVD decomposition of E. With $X' := U^{-\mathrm{T}}XV^{\mathrm{T}}$ we get $E_{svd}^{\mathrm{T}}X' = X'^{\mathrm{T}}E_{svd}$, i.e. $X' = \begin{bmatrix} X_1' & 0 \\ X_3' & X_4' \end{bmatrix}$ with a block structure corresponding to E_{svd}. This X' clearly can be parameterized as in (6). Finally we observe that the (1,1)-element in (5) implies the regularity of X. In view of (4) the parameterization (6) can be strengthen by $\tilde{X} > 0$. A strict inequality characterization of a H_∞-norm bound γ then can be derived by substituting (6) into (5) and replacing (4) by $\tilde{X} > 0$.
Note that the matrix X is over-parameterized by (6) with respect to the variables not affected by the positive definiteness requirement in (4). This may be used to put further constraints on $\tilde{X}$ in (6).
The previous remark shows how to check H_∞-norm bounds with standard strict LMI solvers as e.g. the LMI toolbox in MatLab. However, the main importance of this remark will become clear in the context of the corresponding H_∞ controller synthesis problem for DAE systems which is addressed in the next section.

$$
\begin{bmatrix} AY_1 + Y_1^{\mathrm{T}}A^{\mathrm{T}} + B_2\hat{C}_K + \left(B_2\hat{C}_K\right)^{\mathrm{T}} & \left(A + B_2\hat{D}_K C_2\right) + \hat{A}_K^{\mathrm{T}} & B_1 & Y_1^{\mathrm{T}}C_1^{\mathrm{T}} \\ \left(A + B_2\hat{D}_K C_2\right)^{\mathrm{T}} + \hat{A}_K & A^{\mathrm{T}}X_1 + X_1^{\mathrm{T}}A + \hat{B}_K C_2 + C_2^{\mathrm{T}}\hat{B}_K^{\mathrm{T}} & X_1^{\mathrm{T}}B_1 & C_1^{\mathrm{T}} \\ B_1^{\mathrm{T}} & B_1^{\mathrm{T}}X_1 & -\gamma I & 0 \\ C_1 Y_1 & C_1 & 0 & -\gamma I \end{bmatrix} < 0, \tag{7}
$$

$$
\begin{array}{rclll} Y_1 & := & RE^{\mathrm{T}} + E^{\mathrm{T}\perp}W_Y, & R > 0, & \\ X_1 & := & SE + E^{\perp}W_X, & S > 0, & \end{array} \quad \begin{bmatrix} R & E^+ \\ E^{\mathrm{T}+} & S \end{bmatrix} > 0 \tag{8}
$$

3 The H_∞ Control Problem for Linear Descriptor Systems

Consider a generalized plant Σ_E that is a descriptor system

$$
\Sigma_E : \quad \begin{array}{rcl} E\dot{\boldsymbol{x}}(t) & = & A\boldsymbol{x}(t) + B_1\boldsymbol{w}(t) + B_2\boldsymbol{u}(t) \\ \boldsymbol{z}(t) & = & C_1\boldsymbol{x}(t) \\ \boldsymbol{y}(t) & = & C_2\boldsymbol{x}(t) \end{array} \tag{9}
$$

where $\boldsymbol{x}(t) \in I\!R^{n_x}$ denotes the descriptor variables, $\boldsymbol{u}(t) \in I\!R^{n_u}$ the control input, $\boldsymbol{w}(t) \in I\!R^{n_w}$ the external input, $\boldsymbol{z}(t) \in I\!R^{n_z}$ the external output, and $\boldsymbol{y}(t) \in I\!R^{n_y}$ the measured output. A, B_i, C_i are constant matrices of appropriate dimension and E is a possibly singular matrix having the same dimension as A. Notice that there is no loss of generality in the descriptor setup in neglecting a direct fed-through of control/external input to the measured/external output since such a dependency also can be expressed by means of an augmented descriptor vector $\boldsymbol{x}$ [6].
The control problem is to find a linear output feedback controller such that the undisturbed closed loop ($\boldsymbol{w} \equiv \boldsymbol{0}$) is an admissible system and such that the transfer matrix from the external input $\boldsymbol{w}$ to the external output $\boldsymbol{z}$ is H_∞-norm bounded by a prescribed number $\gamma > 0$.

With a controller K_E,

$$
K_E : \quad \begin{array}{l} E\dot{\boldsymbol{\zeta}}(t) = A_K\boldsymbol{\zeta}(t) + B_K\boldsymbol{y}(t) \\ \boldsymbol{u}(t) = C_K\boldsymbol{\zeta}(t) + D_K\boldsymbol{y}(t), \quad \boldsymbol{\zeta}(t) \in I\!R^{n_x} \end{array} \tag{10}
$$

parametrized by A_K, B_K, C_K, D_K the closed loop system is given by

$$
\begin{array}{ll} E_{cl}\dot{\boldsymbol{\xi}}(t) = A_{cl}\boldsymbol{\xi}(t) + B_{cl}\boldsymbol{w}(t) & \\ \boldsymbol{z}(t) = C_{cl}\boldsymbol{\xi}(t), & \boldsymbol{\xi}(t) \in I\!R^{2n_x}, \end{array} \tag{11}
$$

$$
E_{cl} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \; A_{cl} = \begin{bmatrix} A + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix},
$$

$$
B_{cl} = \begin{bmatrix} B_1 \\ 0_{n_x \times n_w} \end{bmatrix}, \; C_{cl} = \begin{bmatrix} C_1 & 0_{n_z \times n_x} \end{bmatrix}. \tag{12}
$$

Then all controllers K_E solving the H_∞ control problem for descriptor systems are characterized by the following theorem:

Theorem 3.1 *Consider a plant (9) and a controller (10). There exists a controller parameterization* A_K, B_K, C_K, D_K *such that the undisturbed closed loop system (11) is admissible with* $\|G_{cl}\|_\infty < \gamma$ *(with* $G_{cl}(s) := C_{cl}(sE_{cl} - A_{cl})^{-1}B_{cl}$*) if and only if the LMIs (7), (8) at the top of the page*[1] *admit a solution* $\{R, S, W_Y, W_X, \hat{A}_K, \hat{B}_K, \hat{C}_K, \hat{D}_K\}$.

Proof. The Theorem is a special case of the GQP result in [11]. Here only a brief sketch of the proof is imparted.

Application of the generalized bounded real lemma (Proposition 2.1) to the closed loop system matrices (12) renders the necessary and sufficient LMI/BMI conditions

$$
E_{cl}^{\mathrm{T}}X = X^{\mathrm{T}}E_{cl} \geq 0, \tag{13}
$$

$$
\begin{bmatrix} A_{cl}^{\mathrm{T}}X + XA_{cl} & X^{\mathrm{T}}B_{cl} & C_{cl}^{\mathrm{T}} \\ B_{cl}^{\mathrm{T}}X & -\gamma I & 0 \\ C_{cl} & 0 & -\gamma I \end{bmatrix} < 0. \tag{14}
$$

This matrix inequality is clearly nonlinear due to products of unknown controller matrices with the matrix X. The idea in the following is to introduces new matrix variables ("linearizing change of variables") such that (13), (14) can be replaced by LMIs. This is not possible directly but with an intermediate step, i.e. an congruence transformation of (13), (14). Then, new

[1] Here E^+ denotes any generalized inverse with the property $EE^+E = E$.

variables can be introduced such that we get synthesis LMIs. These LMIs are constructive since the new variables parameterize a system of linear equations which uniquely can be solved for the controller matrices. With $Y := X^{-1}$ and

$$X = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix}, \quad Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix},$$
$$X_i, Y_i \in I\!R^{n_x \times n_x}. \tag{15}$$

non-singular transformation matrices

$$\Pi_1 := \begin{bmatrix} Y_1 & I \\ Y_3 & 0 \end{bmatrix}, \quad \Pi_2 := \begin{bmatrix} I & X_1 \\ 0 & X_3 \end{bmatrix} \tag{16}$$

can be defined such that $X\Pi_1 = \Pi_2$ holds true. Since Π_1 is non-singular, a non-singular congruence transformation

$$\Pi_1^{\mathrm{T}} E_{cl}^{\mathrm{T}} X \Pi_1 = \Pi_1^{\mathrm{T}} X^{\mathrm{T}} E_{cl} \Pi_1 \geq 0 \tag{17}$$

$$\Psi_{\Pi_1}^{\mathrm{T}} \begin{bmatrix} A_{cl}^{\mathrm{T}} X + X A_{cl} & X^{\mathrm{T}} B_{cl} & C_{cl}^{\mathrm{T}} \\ B_{cl}^{\mathrm{T}} X & -\gamma I & 0 \\ C_{cl} & 0 & -\gamma I \end{bmatrix} \Psi_{\Pi_1} < 0$$
$$\text{with } \Psi_{\Pi_1} := \mathrm{diag}(\Pi_1, I, I) \tag{18}$$

of (13), (14) is possible. The matrix inequality (18) together with the linearizing changes of variables

$$\begin{aligned} \hat{D}_K &:= D_K \\ \hat{C}_K &:= C_K Y_3 + D_K C_2 Y_1 \\ \hat{B}_K &:= X_3^{\mathrm{T}} B_K + X_1^{\mathrm{T}} B_2 D_K \\ \hat{A}_K &:= X_1^{\mathrm{T}}(A + B_2 D_K C_2) Y_1 + X_3^{\mathrm{T}} A_K Y_3 + \\ &\quad + X_3^{\mathrm{T}} B_K C_2 Y_1 + X_1^{\mathrm{T}} B_2 C_K Y_3 \end{aligned} \tag{19}$$

leads to (7). Inequality (17) becomes

$$\begin{bmatrix} E & 0 \\ 0 & E^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} R & E^{+} \\ E^{\mathrm{T}+} & S \end{bmatrix} \begin{bmatrix} E^{\mathrm{T}} & 0 \\ 0 & E \end{bmatrix} \geq 0, \tag{20}$$

with $R > 0$, $S > 0$. The strict inequality in (8) can be ensured by means of the degrees of freedom in R, S (see Remark 2).

To show sufficiency an inversion of the congruence transformation (17), (18) has to be established. More precisely the validity of $X\Pi_1 = \Pi_2$ with non-singular matrices Π_1, Π_2 as in (16) has to be shown. Some lengthy calculations show that this condition always can be established if

$$X_1 Y_1 + X_2 Y_3 = I \tag{21}$$
$$X_3 Y_1 + X_4 Y_3 = 0 \tag{22}$$

hold true with non-singular matrices X_3, Y_3 (these equations correspond to the block matrices of X, Y in (15) together with the symmetry constraints

$$E^{\mathrm{T}} X_2 = X_3^{\mathrm{T}} E, \;\; E Y_2 = Y_3^{\mathrm{T}} E^{\mathrm{T}}, \;\; E^{\mathrm{T}} X_4 = X_4^{\mathrm{T}} E. \tag{23}$$

A detailed analysis shows that (21), (22) always can be established provided the synthesis LMIs (7), (8) admit an solution. □

The preceding (conceptual) proof is constructive: with a solution of the LMIs (7), (8) it is possible to establish (21), (22) by simple factorization techniques. Then the linear equations (19) can be solved for the controller matrices $D_K, C_K\ B_K, A_K$.

4 Descriptor Control of a Binary Distillation Column

We consider separation of a binary mixture in a 40 tray distillation column with one feed stream. A schematic representation of the process is given in Fig. 1 (a). Exemplary we consider the separation of two alcohols (Methanol,n-Propanol). The mixture is fed in the column with the feed flow rate F. Feed flow rate F and feed composition x_F (molar fraction) are determined by upstream processes. The stationary feed flow rate and feed composition are corrupted by disturbances. The feed stream separates the column into rectifying- (upper part of the column) and stripping section (lower part of the column). Separation is achieved due to intensive heat and mass transfer between liquid flow and countercurrently rising vapor flow. At the bottom of the column the liquid flow splits up into a liquid product stream which is removed with flow rate B from the column and a stream which is, after being heated in the reboiler, recirculated back to the column as vapor flow with flow rate V. At the top of the column the vapor flow with the accumulated more volatile product is completely condensed in the condenser. The condensate is partly pumped back in the column with a flow rate L (reflux stream) and is partly removed as the distillate product with a flow rate D [2]. We consider the distillation column in “LV” configuration, that is: liquid flow rate L and vapor flow rate V are considered to be control inputs. Measured variables are the concentrations on trays 14 and 28.

The control objective is to stabilize the product concentrations at the top and bottom of the column at their stationary values. The control relevant dynamics of the

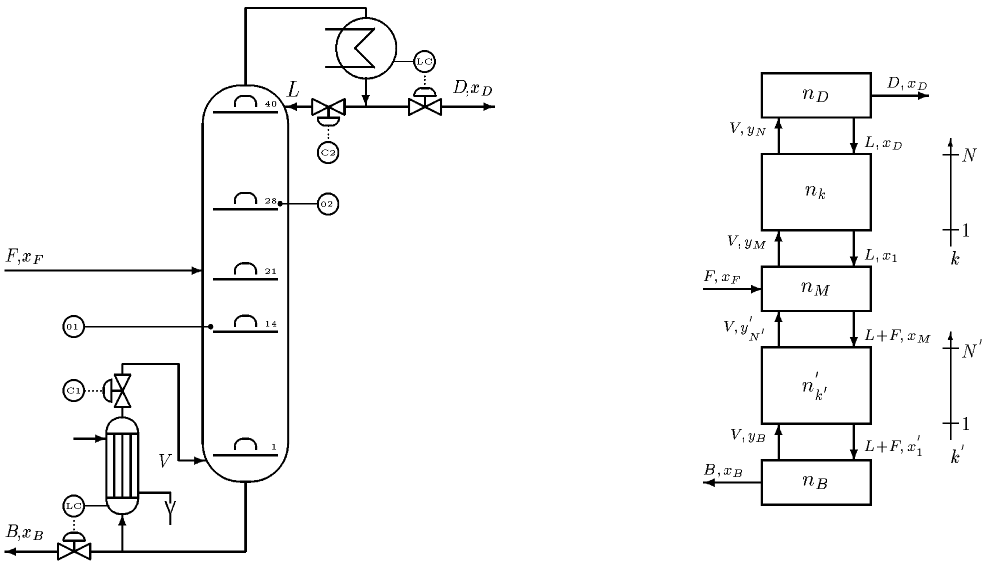

Figure 1: (a) Distillation column (scheme) (b) Subsystems of the column

$$
\begin{bmatrix} * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & 0 & * & 0 & 0 \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * \end{bmatrix}
\begin{bmatrix} \frac{d\Delta x_B}{dt} \\ \frac{d\Delta s_r}{dt} \\ \frac{d\Delta x_M}{dt} \\ \frac{d\Delta s_s}{dt} \\ \frac{d\Delta x_D}{dt} \end{bmatrix}
=
\begin{bmatrix} * & * & * & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & * & * & * \end{bmatrix}
\begin{bmatrix} \Delta x_B \\ \Delta s_r \\ \Delta x_M \\ \Delta s_s \\ \Delta x_D \end{bmatrix}
+
\begin{bmatrix} 0 & * \\ 0 & * \\ * & * \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\begin{bmatrix} \Delta x_F \\ \Delta F \end{bmatrix}
+
\begin{bmatrix} * & * \\ * & * \\ * & * \\ * & * \\ 0 & * \end{bmatrix}
\begin{bmatrix} \Delta L \\ \Delta V \end{bmatrix}
$$
$$
\begin{bmatrix} \Delta x_{14} \\ \Delta x_{28} \end{bmatrix}
=
\begin{bmatrix} * & * & * & 0 & 0 \\ 0 & 0 & * & * & * \end{bmatrix}
\left[\Delta x_B, \Delta s_r, \Delta x_M, \Delta s_s, \Delta x_D\right]^{\mathrm{T}} \tag{24}
$$

process can be captured by a reduced model of the distillation column [8]. This model assumes that the concentrations of the lighter component (molar fractions, denoted by x in the following) in the rectifying and stripping section can be described by the movement of a concentration profile. A descriptor model with concentration x_B in the reboiler, position of profile s_r in the rectifying section, concentration x_M for the feed tray, position of profile s_s in the stripping section, and concentration x_D in the condenser as descriptor variables is given in (24). Here "$*$" denotes numerical entries. A detailed derivation of the model and numerical values are given in [7].

4.1 S/KS Mixed Sensitivity Problem Setup

The control problem is solved in terms of a mixed sensitivity problem depicted in Fig. 2 with G representing the plant, K the controller, and W_1, W_2, V frequency dependent weighting matrices. Controller design by "loop shaping" requires a selection of the weighting matrices such that the solution of the H_∞ control problem

$$
\left\| \begin{matrix} W_1(I+GK)^{-1}V \\ -W_2K(I+GK)^{-1}V \end{matrix} \right\|_\infty \overset{!}{<} \gamma \tag{25}
$$

results in a well behaved closed loop system. In this

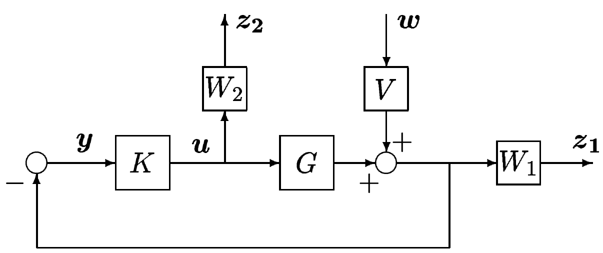

Figure 2: A mixed sensitivity configuration

setup V can be interpreted as a filter which models the disturbance considered to be relevant for the problem at hand. With $S(s) := (I+GK)^{-1}$ being the sensitivity matrix of the closed loop the expression (25) with $\gamma = 1$ suggests to choose W_1 to be approximately the inverse of the wanted behavior for $S(s)$ and analogously W_2 to be the inverse of $K \cdot S$. General indications on selecting these weighting matrices can be found in [13].

In case of the distillation control problem at hand an indirect approach is taken: with stabilizing the measured concentrations x_{14}, x_{28} also the stationary profiles are fixed and thus approximately also the product concentrations. In order to realize this idea the descriptor S/KS H_∞ control problem depicted in Figure 2 (with G being the descriptor model (24)) is solved by the outlined descriptor H_∞ synthesis procedure. The synthesis LMIs are jointly optimized with respect to γ. A final value of $\gamma = 1.01$ shows that the control objectives are approximately met. The resulting controller is tested in simulation with a first principles model of the distillation process and shows a good control performance even for large input disturbances.

5 Conclusions

We presented a constructive solution to the descriptor H_∞ control problem. Synthesis conditions are given as numerically feasible strict LMI conditions. The resulting controller computation is successfully applied to a realistic control problem from chemical process control. To our knowledge this is in fact the first application of descriptor H_∞ control to a control problem with real physical background. In the final version of the paper we will additionally include a robustness evaluation along the line of [9] for the presented descriptor H_∞ controller synthesis of the distillation column.

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