IS THE ν-GAP METRIC USEFUL FOR INDUSTRIAL APPLICATIONS?

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Abstract

The ν-gap analysis can be used to analyze the achievable robustness for a control system before designing any controllers. In its unweighted form, the ν-gap promises powerful results. However, for practical applications, dynamic weights must be introduced, and then the results are with respect to this given choice of weights. For this weighted case, H_∞ analysis provides a simpler alternative and yields less conservative results.

1 Introduction

Assessing the robustness of a control system is important. Tools often used for robustness analyses are the H_∞ norm and singular values. More recently, the ν-gap ("nu-gap") has been developed as a tool to estimate the achievable robustness of a control system before actually designing the controller.

The conceptual idea of the ν-gap is best explained with reference to an *a posteriori* H_∞ analysis of the control system with the nominal and a perturbed model of the plant. Once a controller is available, the H_∞ norm of the nominal closed-loop transfer function, possibly weighted with dynamic weights reflecting the performance requirements, can be computed. Replacing the nominal plant model by the perturbed model (but keeping the same weights) gives the H_∞ norm for that case; the two numbers will be different in general. The ν-gap between the nominal and the perturbed plant model is an upper bound of this difference between the two H_∞ norms, hence providing *a priori* information about the achievable robustness.

After some background information on the ν-gap in Section 2, the ν-gap is applied to a simple multivariable plant for which some mild model uncertainty is defined. This illustrates the use of the ν-gap and highlights some problems. Section 4 provides further theoretical analysis and an equivalent, but conceptually simpler procedure to do the same kind of *a priori* robustness analysis. Conclusions are drawn in Section 5.

2 The ν-gap metric

The ν-gap, $\delta_\nu(G_{P1}, G_{P2})$, has been developed by Vinnicombe at Cambridge University. A good tutorial overview can be found in [2], a more detailed treatment in the monograph [11]. The recent paper [3] generalizes the results.

Figure 1 shows the standard feedback configuration for H_∞ controller designs with the McFarlane-Glover approach [8]. G_P is the plant model, possibly weighted for loop shaping, and K the controller. The cost function to be optimized with H_∞ synthesis is the H_∞ norm of the transfer function

$$T_{o_1 i_1} = \begin{bmatrix} S_u K & S_u \\ G_P S_u K & G_P S_u \end{bmatrix} \tag{1}$$

with the input $i_1 = \begin{bmatrix} r \\ v \end{bmatrix}$, the output $o_1 = \begin{bmatrix} u \\ y \end{bmatrix}$, and the sensitivity $S_u = (I + KG_P)^{-1}$. Equivalently, the transfer function

$$T_{o_2 i_2} = \begin{bmatrix} S_e G_P & S_e \\ -KS_e G_P & -KS_e \end{bmatrix} \tag{2}$$

with $i_2 = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$, $o_2 = \begin{bmatrix} y \\ u \end{bmatrix}$, and $S_e = (I + G_P K)^{-1}$ can be optimized. The stability and performance criterion, $b_{G_P, K}$, is defined as

$$b_{G_P, K} = \left\| T_{o_1 i_1} \right\|_\infty^{-1} = \left\| T_{o_2 i_2} \right\|_\infty^{-1}, \tag{3}$$

with $b_{G_P, K} = 0$ if the closed-loop is unstable; $0 \le b_{G_P, K} \le 1$. As seen by (1) and (2), maximizing $b_{G_P, K}$ in a controller design indirectly shapes the sensitivities, S_u and S_e, and the complementary sensitivities, $T_e = G_P S_u K$, $T_u = KS_e G_P$.

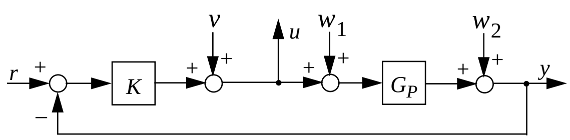

Figure 1: Standard feedback configuration.

If the plant is weighted to reflect the control specifications, the number $b_{G_P, K}$ is a good performance indicator for the control system. A large value ($\gtrsim 0.3$) indicates that the actual loop shape (i.e., including the controller) is not much different from the designed loop shape (i.e., the plant with the weights only).

The ν-gap is the dual to the performance measure $b_{G_P, K}$. It describes the distance between two plants if feedback is applied: The ν-gap $\delta_\nu(G_{P1}, G_{P2})$ is small if a reasonably good controller for one plant shows similar performance (in terms of $b_{G_P, K}$) with the other.

Definition: The ν-gap, $\delta_\nu(G_{P1}, G_{P2})$, for two plants with each m inputs and p outputs is defined as:

$$\delta_\nu(G_{P1}, G_{P2}) = \begin{cases} \left\| \begin{bmatrix} -\tilde{M}_2 & \tilde{N}_2 \end{bmatrix} \begin{bmatrix} N_1 \\ M_1 \end{bmatrix} \right\|_\infty & \begin{array}{l} \text{if } \det\left(\overline{\begin{bmatrix} N_2 \\ M_2 \end{bmatrix}}^T \begin{bmatrix} N_1 \\ M_1 \end{bmatrix} \right)(j\omega) \neq 0 \\ \forall \omega \in (-\infty, \infty) \text{ and} \\ \operatorname{wno}\left(\det\left(\overline{\begin{bmatrix} N_2 \\ M_2 \end{bmatrix}}^T \begin{bmatrix} N_1 \\ M_1 \end{bmatrix} \right)\right) = 0 \end{array} \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

where $G_{Pi} = N_i M_i^{-1} = \tilde{M}_i^{-1} \tilde{N}_i$ are normalized right (left) coprime factorizations of the plants and wno denotes the winding number [2, 11].

The μ Tools [1] for MATLAB provide functions for computing the ν-gap between two systems.

The theorem which relates the ν-gap to the performance measure is the following:

Theorem (Theorem 2.4 in [2]): For any two plants G_{P1}, G_{P2} with m inputs and p outputs and a controller K with p inputs and m outputs,

$$\left| b_{G_{P1}, K} - b_{G_{P2}, K} \right| \leq \delta_\nu(G_{P1}, G_{P2}) . \tag{5}$$

The following interpretation is provided in [2]: "The ν-gap is an effective measure of the important difference between two systems, in terms of closed-loop behaviour when both are controlled by the same, near unity-gain, feedback compensator. When the feedback compensator to be used is not of near unity-gain at all frequencies, it is necessary to weight the system concerned (by the controller, or the expected shape of the controller – as characterised by the weights used in the H_∞ loop-shaping design procedure for example), for such an interpretation to be meaningful."

3 An example

In this section, we follow the procedure implied by the interpretation above: We take a simple plant with a mild uncertainty, and two controllers which have been designed for this plant are used as weights. One of the controllers copes well with the uncertainty, while the other does not control the plant robustly but meets the requirements with the nominal model. For the ν-gap to be a meaningful measure, the computed ν-gap with both "weights" should be the same since we want to get information about the plant and not about the weights. Moreover, the ν-gap should be insensitive to placing the weights at the input or at the output of the plant.

3.1 The plant and two controllers

The plant, taken from [5], is given by

$$G_P = \frac{s + 0.4}{s + 0.07} \begin{bmatrix} 10 & 13 \\ 3.5 & 5 \end{bmatrix} . \tag{6}$$

With a condition number of $\kappa(G_P) = 68$, it is ill-conditioned, i.e., its gain strongly depends on the direction of the input vector. For the disturbed plant, we assume a disturbance of 10 per cent in the plant output, i.e.,

$$G_{Pd} = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix} G_P . \tag{7}$$

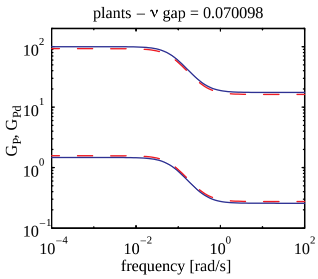

Figure 2: Singular values of G_P and G_{Pd}.

Figure 2 shows the singular values of the plant and the disturbed plant. The ν-gap between G_P and G_{Pd} is rather small: $\delta_\nu(G_P, G_{Pd}) = 0.07$. This number is not meaningful, however, since the plant needs to be controlled with a controller with a gain much larger than unity in order to overcome the small minimum gain of the plant.

In [5], two controllers have been designed for this plant, K_i and K_n; Figure 3 shows their singular values. Both have the same order (6 states) and the same maximum gain of about 100 at low frequencies, but their condition numbers are different.

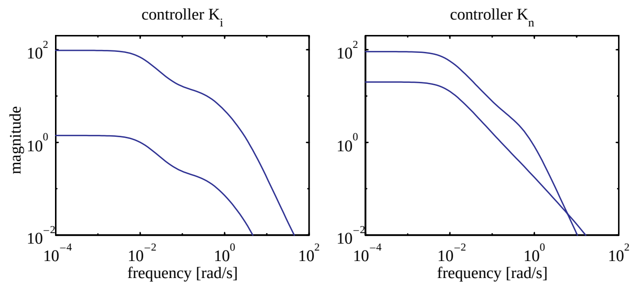

Figure 3: Singular values of the controllers K_i and K_n.

3.2 The ν-gap for the weighted plant

As discussed in Section 2, the plant models need to be weighted in order to obtain meaningful results for the ν-gap. Instead of $\delta_\nu(G_P, G_{Pd})$, we compute

$$\delta_\nu(W_o G_P W_i, W_o G_{Pd} W_i) . \tag{8}$$

W_o and W_i are output and input weights, one of which is chosen to be an identity matrix while the other is set to either K_i or K_n. The series connection of the plant and a controller is the loop gain, and we adopt the notation

$$L_e = G_P K \qquad L_{ed} = G_{Pd} K \tag{9}$$

for the case when the weight (K is either K_i or K_n) is applied at the input of the plant and

$$L_u = K G_P \qquad L_{ud} = K G_{Pd} \tag{10}$$

for the weight at the output. Figure 4 shows the singular values for the resulting loop gain transfer functions; the corresponding ν-gap distances are indicated on top of the plots.

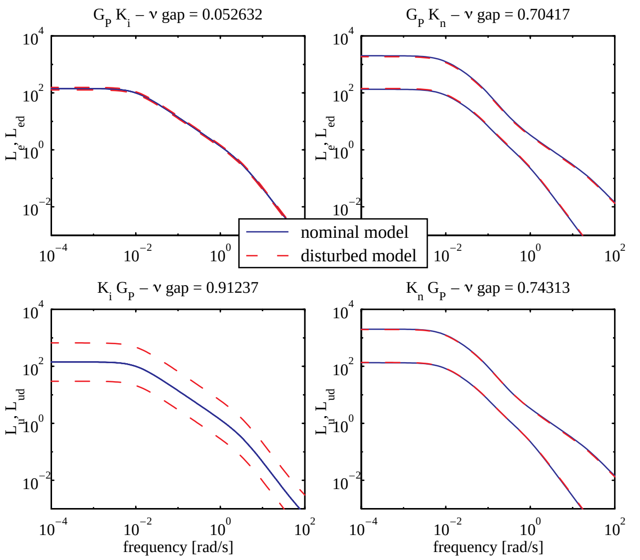

Figure 4: Singular values and ν-gaps for the weighted plant.

The results are rather surprising. With the same controller K_i, the ν-gap can be

$$\delta_\nu(G_P K_i, G_{Pd} K_i) = 0.053 \tag{11}$$

$$\delta_\nu(K_i G_P, K_i G_{Pd}) = 0.91\,, \tag{12}$$

depending on whether the input or the output is weighted. A ν-gap as large as in (12) basically means that there is no hope for finding a controller which robustly controls the plant while the small value in (11) implies that it should be straightforward to find such a controller.

The results with the controller K_n are more consistent.

$$\delta_\nu(G_P K_n, G_{Pd} K_n) = 0.70 \tag{13}$$

$$\delta_\nu(K_n G_P, K_n G_{Pd}) = 0.74 \tag{14}$$

both leave little hope for finding a controller with satisfying performance.

All the expectations seem to be disappointed. The ν-gap heavily depends on the weight used and can be very sensitive to the location of the weight. This is especially true for the weight K_i which, judging from the nominal shape $L_e = G_P K_i = L_u = K_i G_P$, seems to be even more sensible than K_n.

3.3 Analysis of the example

Since the weights used above are controllers, we can add trivial unity-gain feedback compensators and close the loop with an identity matrix. As expected from Figure 4, the closed-loop transfer functions are not affected by the disturbance for the controller K_n; with K_i, they are affected (see also [5]).

The two controllers are both H_∞ controllers. K_i has been designed with an S/T weighting scheme that is known to lead to controllers which invert the plant [10, 5]. Its transfer function is given by

$$K_i = \begin{bmatrix} 10 & 13 \\ 3.5 & 5 \end{bmatrix}^{-1} \frac{s+0.07}{s+0.4} \; \frac{0.0471(s+100)}{(s+0.01)(s+3.314)}\,. \tag{15}$$

The controller K_n is designed with the GS/T weighting scheme [5] which avoids the inversion of the plant; K_n's zeros are at $\{-0.3595, -0.2015, -100, -100\}$.

It has been shown by Freudenberg [6] that it is important to avoid the inversion of the plant by the controller if robust control systems are to be obtained for ill-conditioned plants. For the ν-gap to be a helpful tool for the analysis of ill-conditioned plants, the gap should be smaller with a weight which reflects this requirement of well-conditioned controllers than with an inverting weight.

Why then is the ν-gap with the same weight K_i misleadingly small for one location (if it is placed at the input of the plant) and very high for the other (at the output), correctly predicting that the system will never be robust?

There are three aspects. First, for weighted plants, one does not compute the robustness measure for those transfer functions for which one wanted to originally. Instead of using the inputs w_1, w_2 in Figure 5 and the outputs y, u, the inputs $\bar{w}_1, \bar{w}_2$ are used and the outputs $\bar{y}, \bar{u}$. With the weight of the plant being the controller (and $K = I$), the performance and robustness measure $b_{G_P, K}$ degenerates to $b_{G_P K, I}$ or $b_{K G_P, I}$ (which are not identical!). Hence, instead of investigating the transfer function

$$\begin{bmatrix} I \\ -K \end{bmatrix} (I + G_P K)^{-1} \begin{bmatrix} G_P & I \end{bmatrix} = \begin{bmatrix} S_e G_P & S_e \\ -T_e & -K S_e \end{bmatrix} \tag{16}$$

as intended (2), the transfer functions

$$\begin{bmatrix} I \\ -I \end{bmatrix} (I + G_P K)^{-1} \begin{bmatrix} G_P K & I \end{bmatrix} = \begin{bmatrix} T_e & S_e \\ -T_e & -S_e \end{bmatrix} \tag{17}$$

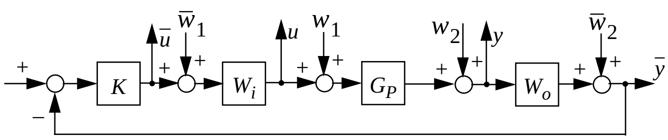

Figure 5: Closed-loop system for robustness analysis.

$$\begin{bmatrix} I \\ -I \end{bmatrix}(I+KG_P)^{-1}\begin{bmatrix} KG_P & I \end{bmatrix} = \begin{bmatrix} T_u & S_u \\ -T_u & -S_u \end{bmatrix} \qquad (18)$$

are used which are lacking the two important cross terms $G_P S_u = S_e G_P$ and $S_u K = K S_e$.

Second, with the weight at the input of the plant, the ill-conditioned gain matrix of the plant is cancelled with K_i, leading to $G_P K_i$ describing two identical harmless SISO systems in parallel which are trivial to control robustly. The plant perturbation does not change this structure. Hence $\delta_\nu(G_P K_i, G_{Pd} K_i)$ is very small. For the weight at the output, the inverted matrix in the weight cannot cancel the matrix of the plant since the perturbation is introduced in between the two. Thus, $\delta_\nu(K_i G_P, K_i G_{Pd})$ becomes large.

Third, only one particular perturbed plant is studied. If the same perturbation of 10 per cent in each channel was applied to the input of the plant as well, the ν-gap

$$\delta_\nu\left(G_P K_i, G_P \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix} K_i\right) = 0.956 \qquad (19)$$

would have revealed that K_i is a bad choice for a weighting. This still would not predict that the plant as such can be controlled robustly if reasonable performance is demanded.

This example undermines the confidence of the ν-gap being a useful analysis tool to predict the achievable robustness of the closed-loop system with ill-conditioned plants. At least in the example used here, the ν-gap seems to tell more about the weight than the plant itself. Moreover, the values obtained with the K_n weighting are so high that one would conclude that robust control of the plant is not possible. K_n, however, is a controller which gives good robustness and reasonable performance.

4 Further analysis and an alternative procedure

In [11] (Section 3.1.1), Vinnicombe provides another interpretation of the ν-gap. Small perturbations ($\delta_\nu(G_{P1}, G_{P2}) \le \beta$) in the plant can be tolerated for a controller which achieves good performance ($b_{G_{P1}, K} > \beta$). On the other hand, a disturbed plant with a ν-gap larger than β will be destabilized by some compensator which stabilizes the nominal plant with a performance criterion of β.

The statement for large ν-gaps is rather vague. All one knows is that there will be at least one controller which performs well with the nominal model but destabilizes G_{P2}. There may or may not be some other controllers which successfully stabilize both models. The conclusion can only be that one cannot discard either of the two models for the further synthesis of controllers. However, the conclusion cannot be that it is impossible to control the family of plant models with a single linear time-invariant (LTI) controller; one simply does not know. Neither does one know how to search for an LTI controller which stabilizes the family of models – in case such a controller existed.

An example shows that the ν-gap can be very conservative (modified version of an example provided by Papageorgiou [9]):

$$G_{P1} = \frac{1}{s+0.1} \qquad\qquad K = 1$$
$$G_{P2} = \frac{s+0.00001}{(s+0.1)(s+0.01)}$$

For the nominal plant, the performance is quite good: $b_{G_{P1}, K} = 0.7064$. The ν-gap, $\delta_\nu(G_{P1}, G_{P2}) = 0.9802$, seems to imply that there is no hope for finding a single LTI controller which would perform adequately with both plants. However, the performance criterion for G_{P2} with the same controller ($K = 1$), $b_{G_{P2}, K} = 0.7064$, is exactly the same as that for the nominal plant: the most simple controller with unity gain does the job. Hence the conclusion that one cannot conclude anything from a large ν-gap.

The result for small ν-gaps looks better. The ν-gap provides information about the family of plant models: all those members for which the ν-gap with the nominal plant is small can be eliminated for further synthesis and analysis because the plant's perturbations influence the performance only marginally. This is explained in Section 3.1.3 of [11].

The reality is not quite as clean, however. As stated in Section 2, for practical applications, weighting functions need to be introduced. Then it is no longer a statement about the plant which results from a ν-gap analysis, but a statement about the weights in combination with the plant. All ν-gap results need to be understood with the restriction that they are valid *for a given choice of weights*.

4.1 Alternative approach: H_∞ analysis

As explained in Section 1 and implied by the theorem in Section 2, the concept of the ν-gap is best understood with reference to an *a posteriori* H_∞ analysis. An alternative approach to the ν-gap can be based on this concept. – It is assumed that the reader is familiar with the basic ideas of H_∞ controller synthesis [1, 4, 7].

Procedure: *A priori* H_∞ robustness analysis
Given a nominal model, G_P, and one or several perturbed models, G_{Pi}, describing the plant.

Step 1: Choose that H_∞ weighting scheme in which the performance criteria can be expressed best.

Step 2: Choose simple weights (i.e., with low-order dynamics) for the scheme selected above.

Step 3: Compute an H_∞ controller, K_∞, for the nominal plant and the associated H_∞ norm of the weighted closed-loop system, $\|T_{zw}(G_P, K_\infty)\|_\infty$. If the performance is acceptable, continue; otherwise go back to Step 2 or 1.

Step 4: Compute the H_∞ norm of the weighted closed-loop system for all perturbed models, $\|T_{zw}(G_{Pi}, K_\infty)\|_\infty$. If the performance is acceptable for all G_{Pi}, a single LTI controller can control the plant robustly; otherwise no final conclusion is possible (except that the controller designed in the previous step is not acceptable).

A few remarks are in place. Firstly, Step 1 – choosing an adequate weighting scheme – should be justified. The scheme presented in Figure 1 leading to the performance criterion $\|T_{zw}(\cdot, \cdot)\|_\infty = 1/b_{\cdot,\cdot}$ is not the only scheme which can be used for H_∞ controller synthesis. The tutorial paper [7] presents another, widely used scheme, the *S*/*T* scheme, (which has some inherent disadvantages, though) and [5] introduces the *GS*/*T* scheme as an alternative. The thesis [4] contains a large number of different schemes for specific applications. For instance, controllers with two degrees of freedom which improve the tracking behavior of a control system cannot be handled with $b_{\cdot,\cdot}$. Other examples are configurations in which the signal to be controlled cannot be measured directly or in which a disturbance input to the plant is modeled.

Secondly, the parallelism to the ν-gap should be highlighted. If the performance criterion $b_{\cdot,\cdot}$ is chosen in Step 1, the second step – choosing weights – is exactly the same for the two approaches, and, in fact, the same weights can be chosen. As discussed in Section 2, the plant models need to be weighted for any practical application of the ν-gap. Step 3, the computation of the nominal $b_{G_P,K}$, is not compulsory for the ν-gap analysis. The last step, however, is parallel: for the ν-gap, $\delta_\nu(W_o G_P W_i, W_o G_{Pi} W_i)$ is computed for all G_{Pi}, while in the alternative procedure $\|T_{zw}(G_{Pi}, K_\infty)\|_\infty$ is evaluated.

Thirdly, the results from the ν-gap or H_∞ analysis are equivalent. In both cases, we get answers with respect to one particular choice of weights. Those models G_{Pi} of the plant for which the performance criterion is satisfactory (i.e., $\delta_\nu(\cdot, \cdot)$ is small or $\|T_{zw}(\cdot, K_\infty)\|_\infty$ is sufficiently close to the nominal $\|T_{zw}(G_P, K_\infty)\|_\infty$) can be neglected for further analysis and synthesis considerations – if "neither the plants nor the controller are overly complex” [11]. If some of the ν-gap/H_∞ norm values are not satisfactory, there may or may not be a single LTI controller which performs adequately with all the models of the plant. One cannot know, and it is not clear how to search for such a controller.

One way of trying to find such a controller is, of course, to choose different weights. These new weights can then be tested with further ν-gap or H_∞ norm analyses but neither method helps directly to choose the weights.

Finally, the question should be answered as to why this alternative procedure based on *a posteriori* H_∞ analysis is called *a priori* robustness analysis. The goal of the controller design may be a structurally constrained controller (e.g., PID type). In such a case, the analysis with the full H_∞ controller from Step 3 of the procedure is done before the final controller, *K*, is designed. The constrained controller, *K*, has to achieve satisfactory performance in terms of $\|T_{zw}(G_P, K)\|_\infty$ as well. The performance calculated with the full H_∞ controller is the bound of what may be achieved with LTI controllers in the best case – just as with the ν-gap analysis where the H_∞ controller implied by the weights achieves the best criterion $b_{\cdot,\cdot}$.

At first thought, μ synthesis [1] may look like a more elaborate alternative to the H_∞-based procedure. However, the effort needed for uncertainty modeling and controller computation is hardly justified for a “quick *a priory* robustness assessment” as intended in this paper.

4.2 H_∞ analysis for the example

The procedure introduced in the previous section is applied to the example discussed in Section 3. In [5], the specifications for the closed-loop system have been defined: Static disturbance rejection and tracking error better than 1 per cent at low frequencies, a minimum bandwidth of the closed-loop system of 0.4 rad/s, and a maximum bandwidth of 4 rad/s.

First, we choose the weighting scheme which has been used in [5] for the design of the controller K_i (Fig. 6). The associated performance criterion is the H_∞ norm of the transfer function

$$T_{zw} = \begin{bmatrix} -W_{\bar{u}} T_u & W_{\bar{u}} K S_e W_d \\ W_u S_u & W_u K S_e W_d \end{bmatrix}. \quad (20)$$

With the weights chosen as in [5], the H_∞ norm is $\|T_{zw}(G_P, K_i)\|_\infty = 0.752$ for the nominal model of the plant; with the perturbed model and the same controller, it is $\|T_{zw}(G_{Pd}, K_i)\|_\infty = 3.30$ which is way above the intended maximum value of 1. Thus, the performance is not achieved robustly, and either the weights or the weighting scheme need to be changed.

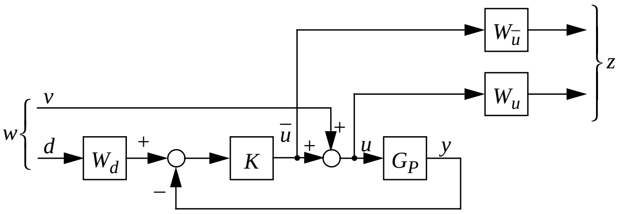

Figure 6: S/T weighting scheme for the design of K_i.

We change the weighting scheme and try the one which had been used in [5] for the design of the controller K_n (Fig. 7). T_{zw} then reads

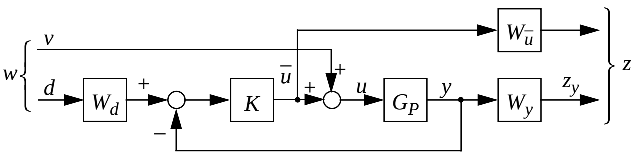

Figure 7: GS/T weighting scheme for the design of K_n.

$$T_{zw} = \begin{bmatrix} -W_{\bar{u}}T_u & W_{\bar{u}}KS_eW_d \\ W_yG_PS_u & W_yT_eW_d \end{bmatrix}; \tag{21}$$

the weights are again chosen as in [5]. For the nominal model, the H_∞ norm achieved is $\|T_{zw}(G_P, K_n)\|_\infty = 0.998$; for the perturbed model, it is $\|T_{zw}(G_{Pd}, K_n)\|_\infty = 1.01$ which is sufficiently close to the intended value of 1. It is concluded that any controller which is sufficiently similar to K_n controls the plant robustly.

The conservatism associated with the ν-gap is eliminated: if the H_∞ controller performs well with all the models of the plant, this is immediately evident. With the ν-gap analysis, the values for K_n were inconclusively high ((13) and (14)).

With both approaches, the results are *with respect to a given choice of weights*, and if the weights are changed substantially, *all* the models have to be tested again because one cannot know whether those models which were close to the nominal model and had been eliminated in earlier iteration steps are still close to the nominal model.

5 Conclusion

The ν-gap has introduced an important new idea to the control community: the *a priori* assessment of the robustness which can be achieved with a plant subject to uncertainty. An impressive mathematical apparatus has been built around the ν-gap and promises rather powerful results. However, for practical problems, the ν-gap cannot be applied in its pure form, but dynamic weights need to be introduced in order to specify the performance requirements. The results then are no longer general, but only valid for the given choice of weights.

The introduction of weights effectively changes the performance criterion from $b_{G_P, K}$ to $b_{G_PK, I}$ or $b_{KG_P, I}$. This negates all robustness guarantees as one changes from (16) to mixed sensitivity problems (17) or (18) which are known to be a bad choice for robustness [10, 5].

Under these limitations, a conceptually simpler alternative provides equivalent results: the H_∞ analysis. The plant is weighted (for instance with the same weights as for the ν-gap analysis), an H_∞ controller is computed for the nominal plant, and this controller is then tested with all available models of the plant. One advantage of this approach is that the result for the one controller implied by the weights is not conservative, as may be the case with the ν-gap.

Acknowledgment

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