DISTURBANCE DECOUPLING PROBLEM WITH STABILITY FOR LPV SYSTEMS

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Abstract

The well-known Disturbance Decoupling Problem with Stability will be investigated in the case of LPV systems and a sufficient condition for its solvability will be given.

By using the concept of parameter varying (A,B)-invariant subspace and parameter varying controllability subspace, this paper investigates the disturbance decoupling problem (DDP) for linear parameter varying (LPV) systems. The parameter dependence in the state matrix of these LPV systems is assumed to be in affine form.

The question of stability is addressed in the terms of Lyapunov quadratic stability by using an LMI technique. If certain conditions for the parameter functions and matrices are fulfilled a sufficient condition is given for the solvability of the DDP problem with stability (DDPS).

1 Introduction

In the so called "geometrical approach" to some fundamental problems of linear time invariant (LTI) control theory, such as the disturbance decoupling problem (DDP), a central role is played by the (A,B)-invariant and (C,A)-invariant subspaces and certain controllability and unobservability subspaces, [18].

As it is well known, for LTI models, a subspace $\mathcal{V}$ is (A,B)-invariant if $A\mathcal{V} \subset \mathcal{V} + ImB$ that is equivalent with the existence of a matrix F such that $(A + BF)\mathcal{V} \subset \mathcal{V}$. The minimal A-invariant subspace containing a given subspace $\mathcal{L}$ will be denoted by $\langle A|\mathcal{L}\rangle$.

This paper deals with the class of linear parameter-varying (LPV) systems that can be described as:

$$\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) \quad (1)$$
$$y(t) = Cx(t) \quad (2)$$

where

$$A(\rho(t)) = A_0 + \rho_1(t)A_1 + \ldots + \rho_N(t)A_N, \quad (3)$$
$$B(\rho(t)) = B_0 + \rho_1(t)B_1 + \ldots + \rho_N(t)B_N. \quad (4)$$

It is assumed that each parameter ρ_i ranges between known extremal values $\rho_i(t) \in [\underline{\rho}_i, \overline{\rho}_i]$ and the parameter set that contains all the $(\rho_1(t), \cdots, \rho_N(t))$, where $t \in [0, T]$ will be denoted by $\mathcal{P}$. For the sake of notational simplicity the time dependency of the matrices will be omitted ($A(\rho) := A(\rho(t))$) where it is possible.

2 Disturbance decoupling problem with stability

Consider the following LTI system:

$$\dot{x} = Ax + Bu + Sq$$
$$y = Cx.$$

The term q represents a disturbance that is not measurable by the controller. The problem is to find a state feedback F, such that q has no influence on the output y and that the closed loop system is stable or equivalently (see [18]): find $F : \mathcal{X} \to \mathcal{U}$ such that $\langle A + BF|\mathcal{S}\rangle \subset \mathcal{C}$ where $\mathcal{C} = KerC$ and $A + BF$ stable.

The necessary and sufficient condition for the solvability of the DDPS is that the maximal element $\mathcal{V}_g^*$ of the class:

$$\{\mathcal{V} \,|\, \exists F : (A + BF)\mathcal{V} \subset \mathcal{V} \subset \mathcal{C}, \text{ and } (A + BF)|_{\mathcal{V}} \text{ stable}\}$$

contains $\mathcal{S}$, [18]. Obviously for the controllability subspace $\mathcal{R}^*$ one has $\mathcal{R}^* \subset \mathcal{V}_g^*$, hence $\mathcal{R}^* \supset \mathcal{S}$ is sufficient for the solvability of DDPS.

This idea will be extended to the parameter varying case by showing that, if some additional technical assumptions hold, the parameter varying controllability subspaces also have a kind of stabilizability property that provides a sufficient condition for the solvability of LPV DDPS problem.

For LTI systems the concept of certain invariant subspaces and the corresponding global decompositions of the state equations induced by these invariant subspaces was one of the main thrusts for the development of geometric methods for solutions to problems of disturbance decoupling or noninteracting control. Nonlinear systems can be studied using tools from differential geometry, when the central role is played by the concept of invariant distributions. From the geometric viewpoint results of the classical linear control can be seen as special cases of more general nonlinear results, [9].

In practical situations however, even in the LPV case, it is quite hard to compute these mathematical objects or it is hard to verify the conditions under which the algorithms provides certain supremal distributions or codistributions. Therefore we would prefer to work with subspaces instead of distributions.

3 Parameter varying invariant subspaces

Linear time varying systems can be viewed as affine nonlinear systems [8], by augmenting the original state space to $\xi := [t, x]^T$. Restricting the investigations to linear subspaces, as special instances of distributions, then a subspace $\mathcal{V}$ of $\mathbb{R}^n$, will be an invariant distribution for system (1) if and only if $A(\rho(t))\mathcal{V} \subset \mathcal{V}$ for all $t \in \mathcal{I}$, where $\mathcal{I}$ is an interval on which the solutions are defined.

This fact motivates the introduction of the following notion for LPV systems:

Definition 1. *A subspace $\mathcal{V}$ is called parameter-varying invariant subspace for the family of the linear maps $A(\rho)$ (or shortly $\mathcal{A}$-invariant subspace) if*

$$A(\rho)\mathcal{V} \subset \mathcal{V} \qquad \text{for all } \rho \in \mathcal{P}. \tag{5}$$

In a similar way one can introduce the extension of the concept of (A,B)-invariant subspace as:

Definition 2. *Let $\mathcal{B}(\rho)$ denote Im $B(\rho)$. Then a subspace $\mathcal{V}$ is called a parameter-varying (A,B)-invariant subspace (or shortly $(\mathcal{A}, \mathcal{B})$-invariant subspace) if any of the following equivalent conditions holds:*

1. *there exists a mapping $F : [0, T] \to \mathbb{R}^{m\times n}$ such that for all $\rho \in \mathcal{P}$:*

$$(A(\rho) + B(\rho)F(\rho))\mathcal{V} \subset \mathcal{V}; \tag{6}$$

2. *for all $\rho \in \mathcal{P}$:*

$$A(\rho)\mathcal{V} \subset \mathcal{V} + \mathcal{B}(\rho). \tag{7}$$

A similar concept was introduced in [3], called *robust controlled invariant* subspace and an algorithm was given in [5] to determine this robust controlled invariant. Since the number of conditions is not finite, the algorithm proposed there, in general, is quite complex. If one sets the gain matrix to be constant then the resulting subspace will be more restrictive, this approach was used in [4] and [13], and was termed as *generalized controllability (A, B)-invariant subspace.* For the dual notion of $(\mathcal{A}, \mathcal{B})$-invariance see [15].

From a practical point of view it is an important question to characterize these parameter-varying subspaces by a finite number of conditions. Assuming the special structure (3) of the matrix $A(\rho)$ it is immediate that if the inclusions holds for all A_i, then they hold also for all $\rho \in \mathcal{P}$. It is not so straightforward under which conditions it is true the reverse implication, too.

In what follows, it will be given a sufficient condition that characterizes property (5) using only a finite number of constraints.

Lemma 1. *If the functions $1, \rho_1, \ldots, \rho_N$ are linearly independent over $\mathbb{R}$ then $A(\rho)\mathcal{V} \subset \mathcal{W} \ \ \forall\rho \in \mathcal{P}$ if and only if*

$$A_i\mathcal{V} \subset \mathcal{W}, \quad i = 0, \ldots, N.$$

The proof is elementary, see [16], hence it is omitted.

We are interested in finding supremal $\mathcal{A}$-invariant subspaces in a given subspace $\mathcal{K}$ or containing a given subspace $\mathcal{L}$. As far as the first purpose is concerned the $\mathcal{A}$-$\mathcal{I}$nvariant $\mathcal{S}$ubspace $\mathcal{A}$lgorithm over $\mathcal{L}$, i.e.:

$$\begin{aligned} \mathcal{AISAL}: \qquad & \mathcal{V}_0 = \mathcal{L} \\ & \mathcal{V}_{k+1} = \mathcal{L} + \sum_{i=0}^{N} A_i\mathcal{V}_k, \qquad k \geq 0, \\ & \mathcal{V}^* = \lim_{k\to\infty} \mathcal{V}_k. \end{aligned} \tag{8}$$

Obviously the algorithm will stop after a finite number of steps, i.e. $\mathcal{V}^* = \mathcal{V}_{n-1}$.

Proposition 1. *The subspace $\mathcal{V}^*$ given by (8) is such that*

$$\mathcal{L} \subset \mathcal{V}^* \tag{9}$$

$$\mathcal{V}^* \text{ is } \mathcal{A}\text{-invariant} \tag{10}$$

and assuming that the condition of Lemma 1. holds, it is minimal with these properties.

Moreover, if the parameter functions are differential algebraically independent, then $\mathcal{V}^*$ coincides with the controlled invariant distribution, [17]. For further reference this property will be called as "persistency" throughout this paper.

Similar to the linear case the subspace $\mathcal{V}^*$ will be denoted by $\langle\mathcal{A}|\mathcal{L}\rangle$.

By duality, one has the $\mathcal{A}$-$\mathcal{I}$nvariant $\mathcal{S}$ubspace $\mathcal{A}$lgorithm in $\mathcal{K}$, i.e.:

$$\begin{aligned} \mathcal{AISAK}: \qquad & \mathcal{W}_0 = \mathcal{K} \\ & \mathcal{W}_{k+1} = \mathcal{K} \cap \bigcap_{i=0}^{N} A_i^{-1}\mathcal{W}_k, \qquad k \geq 0, \\ & \mathcal{W}^* = \lim_{k\to\infty} \mathcal{W}_k. \end{aligned} \tag{11}$$

The subspace $\mathcal{W}^*$ will be denoted by $\langle\mathcal{K}|\mathcal{A}\rangle$. The corresponding version of Proposition 1 follows by duality, and can be stated as:

Proposition 2. *The subspace $\mathcal{W}^*$ given by (11) is such that*

$$\mathcal{W}^* \subset \mathcal{K}$$
$$\mathcal{W}^* \text{ is } \mathcal{A}\text{-invariant}$$

and assuming that the condition of Lemma 1. holds, it is maximal with these properties.

The set of all $(\mathcal{A}, \mathcal{B})$-invariant subspaces contained in a given subspace $\mathcal{K}$, is an upper semilattice with respect to subspace addition. This semilattice admits a maximum which can be computed from the $(\mathcal{A}, \mathcal{B})$-$\mathcal{I}$nvariant $\mathcal{S}$ubspace $\mathcal{A}$lgorithm:

$$\mathcal{ABISA} \qquad \mathcal{V}_0 = \mathcal{K} \tag{12}$$

$$\mathcal{V}_{k+1} = \mathcal{K} \cap \bigcap_{i=0}^{N} A_i^{-1}(\mathcal{V}_k + \mathcal{B}). \tag{13}$$

The limit of this algorithm will be denoted by $\mathcal{V}^*$.

As in the LTI case, if one has to solve the DDP problem with stability, then it is convenient to introduce the concept of the controllability subspace:

Definition 3. *A subspace $\mathcal{R}$ is called parameter-varying controllability subspace if there exists a constant matrix K and a parameter varying matrix $F : [0, T] \to \mathbb{R}^{m\times n}$ such that*

$$\mathcal{R} = \langle \mathcal{A} + B\mathcal{F} | Im\ BK \rangle, \tag{14}$$

where the notation $\mathcal{A} + B\mathcal{F}$ stems for the system $A(\rho) + BF(\rho)$.

As in the classical case, it can be seen that the family of controllability subspaces contained in a given subspace $\mathcal{K}$ is closed under subspace addition. Hence this family has a maximal element which can be computed from the parameter-varying $\mathcal{C}$ontrollability $\mathcal{S}$ubspace $\mathcal{A}$lgorithm:

$$\mathcal{CSA}: \qquad \mathcal{R}_0 = 0$$
$$\mathcal{R}_{k+1} = \mathcal{V}^* \cap \left(\sum_{i=0}^{N} A_i \mathcal{R}_k + \mathcal{B} \right)$$
$$\mathcal{R}^* = \lim_{k\to\infty} \mathcal{R}_k$$

where $\mathcal{V}^*$ is computed by $\mathcal{ABISA}$.

Proposition 3. *The subspace $\mathcal{R}^*$ is the largest parameter–varying controllability subspace in $\mathcal{C}$.*

A useful characterization of parameter–varying controllability subspaces is the following:

Proposition 4. *$\mathcal{R}$ is a parameter–varying controllability subspace if and only if*

$$\mathcal{R} = \langle \mathcal{A} + B\mathcal{F} | \mathcal{B} \cap \mathcal{R} \rangle.$$

4 Stability concepts

Let us consider the following ordinary differential equation:

$$\dot{x}(t) = f(t, x(t)), \qquad t \geq t_0$$
$$x(t_0) = x_0.$$

and let us recall some basic stability concepts.

Let us suppose that $\mathbf{0}$ is an equilibrium point i.e., $f(t, \mathbf{0}) = \mathbf{0}$ for $t \geq 0$. Denote by $s(t, t_0, x_0)$ the solution of the above equation, i.e.

$$\frac{\partial}{\partial t} s(t, t_0, x_0) = f(t, s(t, t_0, x_0)), \qquad t \geq t_0$$
$$s(t_0, t_0, x_0) = x_0.$$

Definition 4. *The equilibrium $\mathbf{0}$ is said to be stable (uniformly stable) if for all $\varepsilon > 0, t_0 > 0$ there exists $\delta = \delta(\varepsilon, t_0) (= \delta(\varepsilon))$, such that*

$$\|x_0\| < \delta \quad \Longrightarrow \quad \|s(t, t_0, x_0)\| < \varepsilon, \quad t \geq t_0.$$

Definition 5. *The $\mathbf{0}$ is attractive (uniformly attractive) equilibrium point if for all $t_0 > 0$ there exists $\eta = \eta(t_0) > 0$ ($\eta > 0$) such that*

$$\|x_0\| < \eta \quad \Longrightarrow \quad \lim_{t\to\infty} \|s(t + t_0, t_0, x_0)\| = 0,$$

uniformly in x_0 and t_0.

Definition 6. *The $\mathbf{0}$ is (uniformly) asymptotically stable equilibrium point if it is (uniformly) stable and (uniformly) attractive.*

Definition 7. *The $\mathbf{0}$ is exponentially stable equilibrium point if there exist $r, a, b > 0$, such that*

$$\|s(t_0 + t, t_0, x_0)\| \leq a\|x_0\|e^{-bt}$$

for all $t, t_0 \geq 0$ and $\|x_0\| < r$.

For global stability definitions see [19].

Note that for LTV systems $\mathbf{0}$ is uniformly asymptotically stable (uni.as.st.) if and only if it is exponentially stable (exp.st.).

In what follows, if it is not specified explicitly, the term stable will refer to any of these specific types of stability. A system is said to be stable if the identically zero solution of the corresponding differential equation is stable.

Considering LPV systems of the form

$$\dot{x}(t) = A(\rho(t))x(t) \tag{15}$$

one can use the following concepts, [1, 7]:

Definition 8. *The LPV system (15) is quadratically stable if there exists $X > 0$ such that*

$$A^T(\rho)X + XA(\rho) < 0$$

for all $\rho \in \mathcal{P}$.

A slightly milder restriction is introduced in the next definition that as the previous one, also implies exponential stability:

Definition 9. *The LPV system (15) is affinely quadratically stable if there exists $X_0, \ldots, X_N$ such that*

$$X(\rho) := X_0 + \rho_1 X_1 + \ldots + \rho_N X_N > 0$$
$$A^T(\rho)X(\rho) + X(\rho)A(\rho) + \frac{\partial X(\rho)}{\partial t} < 0$$

for all $\rho \in \mathcal{P}$.

For the LTV system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t).$$

one can define stabilizability as the property, that there exists a possible time dependent state feedback $u(t) := F(t)x(t)$ such that the system

$$\dot{x}(t) = (A(t)) + B(t)F(t))x(t),$$

is stable. In this sense, in the context of LPV systems, one can investigate the stablizability of the pair $(\mathcal{A}, \mathcal{B})$.

5 DDPS for LPV systems

Let us consider the following LPV system:

$$\dot{x} = A(\rho)x + B(\rho)u + S(\rho)q$$
$$y = Cx$$

where q represents a disturbance and the matrix $S(\rho)$ has the same affine parameter dependent structure as (3).

We would like to design a state feedback gain which depends affinely on ρ in order to remove the effect of the disturbance on the output and which is stabilizing, i.e., the following problem will be considered:

LPV DDPS: find a subspace $\mathcal{V}$ in $\mathbb{R}^n$ which contains $\mathcal{S}$ and $F : [0, T] \to \mathbb{R}^{m \times n}$ such that

$$(A(\rho) + BF(\rho))\mathcal{V} \subset \mathcal{V} \subset \mathcal{C} \quad \text{for all } \rho \in \mathcal{P} \tag{16}$$
$$A(\rho) + BF(\rho) \quad \text{stable ,} \tag{17}$$

where $ImB = \sum_{\rho \in \mathcal{P}} B(\rho)$ and $ImS = \sum_{\rho \in \mathcal{P}} S(\rho)$.

It is possible to give a sufficient condition for the solvability of the problem using the notion of asymptotically stabilizability property of parameter varying controllability subspaces.

In order to solve the LPV DDPS problem one can prove, as in the LTI case, that it is enough to find a subset $\mathcal{V}$ and $F(\rho)$ for which $\mathcal{S} \subset \mathcal{V}$, condition (16) holds and $(A(\rho) + BF(\rho))|_{\mathcal{V}}$ is asymptotically stable.

Theorem 1. *Let us consider an asymptotically stabilizable pair ($\mathcal{A}$,$\mathcal{B}$) and an ($\mathcal{A}$,$\mathcal{B}$)-invariant subspace $\mathcal{V}$ contained in $\mathcal{C}$ such that there is an $F_0(\rho)$ that $(A(\rho) + BF_0(\rho))_{|\mathcal{V}}$ is asymptotically stable. Then there exists an $F(\rho)$ such that $(A(\rho)+BF(\rho))\mathcal{V} \subset \mathcal{V}$ and $A(\rho) + BF(\rho)$ is asymptotically stable.*

The following lemmas have been used to prove Theorem 1.

Lemma 2. *If $(\mathcal{A}, \mathcal{B})$ is asymptotically stabilizable then $(\mathcal{A}+B\mathcal{F}_0, \mathcal{B})$ is asymptotically stabilizable for any feedback $\mathcal{F}_0$.*

Lemma 3. *Let $\mathcal{A}$ be asymptotically stabilizable and let $\mathcal{V}$ be $\mathcal{A}$-invariant. Denote by $\tilde{\mathcal{A}}$ the induced map on $\mathcal{V}$. Then $\tilde{\mathcal{A}}$ is asymptotically stabilizable.*

Lemma 4. *Let $(\mathcal{A}, \mathcal{B})$ be asymptotically stabilizable and let $\mathcal{V}$ be $\mathcal{A}$-invariant. Denote by $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}})$ the induced pair on $\mathcal{V}$. Then $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}})$ is asymptotically stabilizable.*

Proof (theorem): Let $Q : \mathcal{X} \to \mathcal{X}/\mathcal{V}$ be the canonical projection to $\mathcal{X}/\mathcal{V}$ and

$$\tilde{A}_0(\rho) = (A(\rho) + BF_0(\rho))|_{\mathcal{X}/\mathcal{V}}, \qquad \tilde{B}_0 = QB,$$

where $\mathcal{X}/\mathcal{V}$ denotes the factor space with respect to $\mathcal{V}$. Combining Lemma 2 and 4 one has that the pair $(\tilde{\mathcal{A}}_0, \tilde{\mathcal{B}}_0)$ is asymptotically stable, hence there exists $\tilde{F}(\rho) : \mathcal{X}/\mathcal{V} \to \mathcal{U}$ such that $\tilde{A}_0 + \tilde{B}_0\tilde{\mathcal{F}}$ is asymptotically stable. Extend $\tilde{F}(\rho)$ to $\mathcal{V}$ arbitrarily (for instance let $0 \in \mathcal{U}$ for all $v \in \mathcal{V}$, $\rho \in \mathcal{P}$ and denote it by $\hat{F}(\rho)$) and define

$$F(\rho) := F_0(\rho) + \hat{F}(\rho)Q.$$

This $F(\rho)$ is suitable, see [18]. ■

The theorem above simplifies the solvability of LPV DDPS. From $\mathcal{ABISA}$ one can compute $\mathcal{V}^*$, i.e. the maximal $(\mathcal{A}, B)$-invariant subspace in $\mathcal{C}$. Obviously $\mathcal{V}^*$ satisfies (16). Furthermore, if $\mathcal{V}^*$ satisfies the condition of Theorem 1. with an appropriate feedback gain then a sufficient condition for the solvability of DDPS is simply

$$\mathcal{V}^* \supset \mathcal{S}$$

provided that the pair $(\mathcal{A}, \mathcal{B})$ is asymptotically stabilizable.

If this is not the case (i.e. $(\mathcal{A} + B\mathcal{F})|_{\mathcal{V}^*}$ is not as.st.) then consider the maximal parameter-varying controllability subspace $\mathcal{R}^*$ in $\mathcal{C}$ which can be computed from $\mathcal{CSA}$. There exists $\mathcal{F}_0$ (see Proposition 4) for which

$$\mathcal{R}^* = \langle \mathcal{A} + B\mathcal{F}_0 | \mathcal{B} \cap \mathcal{R}^* \rangle.$$

Let $\mathcal{B}_0 = \mathcal{B} \cap \mathcal{R}^*$ and $A_0(\rho) = A(\rho) + BF_0(\rho)|_{\mathcal{R}^*}$. Then

$$\langle \mathcal{A}_0 | \mathcal{B}_0 \rangle = \mathcal{R}^*,$$

In the case of LTI systems the necessary and sufficient condition for the solvability of DDPS was that $\mathcal{V}_g^*$ (which contains $\mathcal{R}^*$) must contain $\mathcal{S}$. In the proof, the pole allocation property of controllability subspaces (which characterize them) were used. If one has to solve LPV DDPS one has to find the analogy of pole allocation in this context. We found that asymptotic stabilizability is applicable for this purpose, however quadratic stabilizability is not, in general.

Theorem 2. *Suppose that ρ_i are persistently exciting, see [17], and $\langle\mathcal{A}|\mathcal{B}\rangle = \mathbb{R}^n$. Then the pair $(\mathcal{A}, \mathcal{B})$ is asymptotically stabilizable.*

Proof. From the assumption and [16] it follows that the system is completely controllable in the sense of Kalman (see [10]). From Theorem 6.10 of [10] we get that we can find a state feedback with which the system will be as.st. ■

Corollary 1. *Assume the persistency property of the ρ_is. Then there exists $\mathcal{F}$:*

$$\mathcal{R}^* = \langle\mathcal{A} + B\mathcal{F}|Im\ BK\rangle = \langle\mathcal{A} + B\mathcal{F}|\mathcal{B} \cap \mathcal{R}^*\rangle$$

and $(\mathcal{A} + B\mathcal{F})|_{\mathcal{R}^*}$ is asymptotically stable.

Proof. Restrict our attention to the subspace $\mathcal{R}^*$, where the induced pair satisfies the assumption of Theorem 2. ■

This is a possible analogy between pole allocation property of controllability subspaces for LTI systems and asymptotic stabilizability of LPV systems.

Remark 1. *The most important question is that what kind of stabilizability can be drawn from the assumption $\langle\mathcal{A}|\mathcal{B}\rangle = \mathbb{R}^n$. One might guess that quadratic stabilizability which is more efficiently computable, can be guaranteed. Unfortunately, this is not the case:*

$$A(\rho) = \begin{bmatrix} -1 & \rho \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \rho(t) \in [-1, 1]$$
$$\nexists X > 0, F(\rho) \quad \text{such that for all } \rho \in \mathcal{P}_0 :$$
$$(A^T(\rho) + F^T(\rho)B^T)X + X(A(\rho) + BF(\rho)) < 0.$$

In this example (which is a simplification of [6] pp. 55.) $\langle A(0)|\mathcal{B}\rangle \neq \mathbb{R}^2$. One can put the following question: if for all $\rho \in \mathcal{P}$, $\langle A(\rho)|\mathcal{B}\rangle = \mathbb{R}^n$, is the pair $(\mathcal{A}, B)$ quadratically stabilizable ?

The counterexample above can be overcome by affine parameter dependent Lyapunov matrix, which gives another conjecture: $\langle\mathcal{A}|B\rangle = R^n$? $\Rightarrow$? affinely quadratic stabilizability. The question of quadratic stabilizability arose also in the context of switching systems [11]. Here the mildest assumption from which quadratic stability can be derived is the following:

Theorem 3. *Consider the Levy decomposition of the Lie algebra $\mathfrak{g} = \mathfrak{r} \oplus \mathfrak{s}$ generated by the stable matrices $\{A_i : i = 1, 2, \ldots, N\}$, where $\mathfrak{r}$ is the radical and $\mathfrak{s}$ is a semisimple subalgebra. If $\mathfrak{s}$ is compact then the A_i matrices share a common quadratic Lyapunov function.*

As far as quadratic stabilizability is concerned it is hard to see the relationship between controllability and the Lie-algebraic property above. For $n = 2$ there exist necessary and sufficient conditions for the existence of a common quadratic Lyapunov function, see [12].

Our main result is the following:

Theorem 4. *Suppose the persistency of ρ_is and $\langle\mathcal{A}|\mathcal{B}\rangle = \mathbb{R}^n$. If furthermore*

$$\mathcal{R}^* \supset \mathcal{S}$$

then DDPS for LPV systems is solvable with asymptotic stability.

Proof. It follows from the assumptions and from Corollary 1 that there exists $F_1(\rho)$ such that

$$\mathcal{R}^* = \langle\mathcal{A} + B\mathcal{F}_1|\text{Im } BK\rangle = \langle\mathcal{A} + B\mathcal{F}_1|\mathcal{B} \cap \mathcal{R}^*\rangle$$

and $(\mathcal{A} + B\mathcal{F}_1)|_{\mathcal{R}^*}$ as.st. The conditions of Theorem 2 are also fulfilled hence the pair $(\mathcal{A}, \mathcal{B})$ asym.ste. Now applying Theorem 1 with the choices of $\mathcal{V} := \mathcal{R}^*$ and $\mathcal{F}_0 := \mathcal{F}_1$ one can get the simplified solvability condition for LPV DDPS considering that $\mathcal{R}^* \supset \mathcal{S}$. ■

In practical situations the construction of an asymptotically stabilizing feedback could be hard. One can try to automatize the solution of the problem by LMI techniques (quadratic stability, affine quadratic stability). However, in this case our theorems does not guarantee the existence of such solutions. A sufficient condition which implies the existence of a Lyapunov function is the following [11]:

Theorem 5. *If the system $\dot{x}(t) = A(t)x(t)$ is uniformly exponentially stable, it has a strictly convex, homogeneous common Lyapunov function of a quasiquadratic form*

$$V(x) = x^T L(x)x,$$

where $L(x) = L^T(x) = L(\tau x)$ for all nonzero $x \in \mathbb{R}^n$ and all $\tau \neq 0$

6 Example

Let

$$A(\rho) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & \rho \\ \rho & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \rho \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad S(\rho) = \begin{bmatrix} 0 \\ 1 + \rho \\ \rho \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

where the parameter ρ is varying in the interval $[1, 2]$. It can be seen that $\mathcal{C} = \text{Ker } C$ is $\mathcal{A}$ invariant, hence $(\mathcal{A}, \mathcal{B})$ either i.e. $\mathcal{V}^* = \mathcal{C}$. By solving linear matrix inequalities we get that for the induced pair

$$\tilde{F}(\rho) = [1.8225\ 1.2957] + \rho[-8.0181 - 2.5257]$$

quadratically stabilizes the system on $\mathcal{V}^*$ and satisfies the requirements of Theorem 1.

7 Conclusion

By using the concept of parameter varying (A,B)-invariant subspace and parameter varying controllability subspace, this paper investigated the disturbance decoupling problem (DDP) with the requirement of the stability for linear parameter varying (LPV) systems. The parameter dependence in the state matrix of these LPV systems was assumed to be in affine form.

If certain conditions for the parameter functions and matrices are fulfilled a sufficient condition was given for the solvability of the DDP problem with stability (DDPS).

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