On Stabilization of First-Order Plus Dead-Time Unstable Processes Using PID Controllers

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Abstract

Abstract This paper considers the problem of stabilizing first-order plus dead-time (FOPDT) unstable processes using PID controllers. The D-partition technique is applied to characterize the stability domain in the in the space of system and controller parameters. Moreover, analytical expressions are derived for describing the stability domain boundaries. These analytical expressions can be used to construct the complete set of stabilizing PID controller parameters for both open-loop unstable time-delay processes. They can also be used to investigate the effect of time delay on the stabilizability of the process.

Keywords: PID controllers, time delay systems, unstable processes, stabilization, stability boundary, D-partition technique.

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1. Introduction

Despite continual advances in control theory and development of advanced control strategies, the proportional, integral, and derivative (PID) control algorithm still finds wide applications in industrial process control systems. It has been reported in [1] that 98% in of the control loops in the pulp and paper industries are controlled by proportional-integral controllers. Moreover, as reported in [2], more than 95% of the controllers used in process control applications are of the PID type [2]. The popularity among industrial practitioners stems from the facts that the PID control structure is simple and its principle is easy to be understand and that the PID controllers are deemed to be satisfactory and robustness for a vast majority of processes. The primary problem associated with the use of PID controllers is tuning, that is, the determination of PID controller parameters to produce satisfactory control performance. Due to the longstanding use of PID controllers in a variety of industries, there exist many different methods to find suitable controller parameters [3]-[6]. The methods differ in complexity, flexibility, and in the amount of process knowledge used.

With the advances in computational and optimization techniques and the stringent performance requirement for control, the class of optimization-based PID controller tuning methods has been receiving increasing attention [7]-[14]. In the optimal PID tuning, a process model is required and the optimal controller parameters are searched to minimize a certain integral performance criterion. Since the primary requirement of the candidate PID controller parameters is that of making the closed-loop system stable, it is often desired to construct the complete set of stabilizing PID parameters. With the complete set of stabilizing PID controller parameters being available for a given process, it can avoid the time-consuming stability check for each set of PID controller parameters in the search process and thereby to save the controller tuning time. However, the construction of the complete set of stabilizing PID controller parameters is not a trivial task. This is particularly the case when the process contains a time delay since the corresponding closed-loop system has an infinite number of poles which make the analytical stability analysis extremely difficult.

The boundaries of the stabilizing PID controller parameter region can be determined by the technique of D-partition [15-18]. The boundaries of the stability region are defined by the equations $P(0;\mathbf{k})=0, P(\infty;\mathbf{k})=0$ and $P(\pm j\omega;\mathbf{k})=0$, where $P(s;\mathbf{k})$ is the characteristic function of the closed-loop system and $\mathbf{k}$ is the vector of controller parameters. The boundary defined by $P(\pm j\omega;\mathbf{k})=0$ is parameterized by the frequency ω and the range of ω corresponding to the true stability boundary has to be identified with the aid of Nyquist stability criterion. Recently, a method based on using a version of Hermite-Biehler theorem applicable to quasipolynomials has been used to determine the complete set of stabilizing PID controller gain parameters for first-order time-delay systems [19]-[22]. The method involves finding the zeros of a transcendental equation to determine the range of stabilizing gains. It is noted that both the D-partition technique and the method of using Hermite-Biehler theorem do not provide an explicit characterization of the boundary of the stabilizing PID parameter region.

In this paper, we consider the problem of stabilizing first-order plus dead time (FOPDT) unstable systems using a PID controller. The main objective is to present a novel approach to derive analytical expressions for describing the boundaries of the stability domain in the space of system and controller parameters. These expressions can be used to construct the complete set of stabilizing PID controller parameters. Also, they can be used to investigate the stabilizability of PID feedback control for time delay processes.

2. The Basic Idea

The closed-loop characteristic function of a feedback control system with time-delay process can be generally written in the form

$$\begin{aligned} F(s;h,\mathbf{p}) &= [s^l + a_{l-1}(\mathbf{p})s^{l-1} + \cdots + a_0(\mathbf{p})] + [b_m(\mathbf{p})s^m + b_{m-1}(\mathbf{p})s^{m-1}) + \cdots + b_0(\mathbf{p})]e^{-hs} \\ &\equiv A(s;\mathbf{p}) + B(s;\mathbf{p})e^{-hs} \end{aligned} \tag{1}$$

where $a_k(\mathbf{p})$ and $b_k(\mathbf{p})$ are real continuous functions of n controller parameters $\mathbf{p}=(p_1,p_2,\cdots,p_n)$ and $h\geq 0$ is the time delay. For given n-parameter vector $\mathbf{p}$ and delay time h, the closed-loop system with characteristic function $F(s;h,\mathbf{p})$ is said to be asymptotically stable if $F(s;h,\mathbf{p})$ is analytic on the closed right half of the complex s-plane, i.e., the quasipolynomial $F(s;\mathbf{p})$ has no zeros with positive real part. Let $h \in \mathbf{H} \subset \mathbf{R}$ and $\mathbf{p} \in \mathbf{P} \subset \mathbf{R}^n$ with $\mathbf{H} \times \mathbf{P}$ be connected compact subset of $\mathbf{R}^{n+1}$, where $\mathbf{R}$ denotes the set of real numbers. The stability domain $\mathbf{S} \subset \mathbf{H} \times \mathbf{P}$ is defined to be the region such that for $(h, \mathbf{p}) \in \mathbf{S}$ all the solutions to the characteristic equation (1) lie in the open left half of the complex s-plane. Determination of stability domain $\mathbf{S}$ plays a central role in the optimal design of controllers with fixed order and structure as well as in the investigation of the effects of time delay and uncertain parameters on the closed-loop stability.

It is noted that if the system is delay free, i.e., $h = 0$, the characteristic function $F(s; 0, \mathbf{p})$ is an algebraic polynomial which has a finite number of zeros. For $h > 0$, the characteristic function $F(s; h, \mathbf{p})$ is a quasipolynomial which has an infinite number of zeros. However, it has been shown by Krall [23] that if $l > m$, or $m = l$ and $0 < |b_l(\mathbf{p})| < 1$, then the quasipolynomial $F(s; h, \mathbf{p})$ has only a finite number of zeros with positive real part. Under the assumption that $l \geq m$ and $|b_m(\mathbf{p})| < 1$ for $m = l$, the D-partition method can be used to construct the stability domain $\mathbf{S}$. Since the zeros of the characteristic quasipolynomial $F(s; h, \mathbf{p})$ are continuous functions of the parameters $\mathbf{p}$ and the delay time h, the space $\mathbf{H} \times \mathbf{P}$ can be divided into regions by hypersurfaces, the points of which correspond to quasipolynomials having at least one zeros on the imaginary axis or at $s = \infty$. Such a decomposition is called the D-partition of the space $\mathbf{H} \times \mathbf{P}$. The points of each region of such a D-partition obviously correspond to quasipolynomials with the same number of zeros with positive real parts, since under the continuous variation of h and $\mathbf{p}$, the number of zeros with positive real parts change only if at least one zero passes across the imaginary axis, that is, if the point in $\mathbf{H} \times \mathbf{P}$, the space of delay and parameters, passes across the boundary of a region of the D-partition.

The set of D-partition boundaries can be defined as follows:

$$\partial \mathbf{D} = \partial D_0 \cup \partial D_\omega \cup \partial D_\infty \tag{2}$$

where

$$\partial D_0 \equiv \{(h, \mathbf{p}) \in \mathbf{H} \times \mathbf{P} : F(0; h, \mathbf{p}) = 0\} \tag{3a}$$

$$\partial D_\omega \equiv \{(h, \mathbf{p}) \in \mathbf{H} \times \mathbf{P} : F(\pm j\omega; h, \mathbf{p}) = 0, \quad \forall \omega \in (0, \infty)\} \tag{3b}$$

$$\partial D_\infty \equiv \{(h, \mathbf{p}) \in \mathbf{H} \times \mathbf{P} : l = m,\ |b_l(\mathbf{p})| = 1\} \tag{3c}$$

It is noted that the D-partition boundary ∂D_∞ exists when $m = l$ and $|b_l(\mathbf{p})| = 1$. Also noted is that in literature the D-partition boundary ∂D_ω is constructed from ω-parameterized equations $F(\pm j\omega; h, \mathbf{p}) = 0$ by sweeping ω from 0 to ∞. In the following, we shall apply the elimination method of Walton and Marshall [24] to obtain an ω-free defining equation of the D-partition boundary ∂D_ω.

For a point $(h, \mathbf{p})$ on the D-partition boundary ∂D_ω, there exists an $\omega \in (0, \infty)$ such that $F(\pm j\omega; h, \mathbf{p}) = 0$, i.e.,

$$A(j\omega; \mathbf{p}) + B(j\omega; \mathbf{p})e^{-j\omega h} = 0 \tag{4a}$$

$$A(-j\omega; \mathbf{p}) + B(-j\omega; \mathbf{p})e^{j\omega h} = 0 \tag{4b}$$

Eliminating the exponential terms, we have

$$\Delta(\pm j\omega; \mathbf{p}) = 0 \tag{5}$$

where

$$\Delta(s; \mathbf{p}) \equiv A(s; \mathbf{p})A(-s; \mathbf{p}) - B(s; \mathbf{p})B(-s; \mathbf{p}) \tag{6}$$

It is seen that the polynomial $\Delta(s; \mathbf{p})$ is even and of finite degree, and, hence, the number of its root branches on the imaginary axis is finite. Also seen is that $\Delta(s; \mathbf{p})$ is independent of the delay time h. Thus, when $\Delta(s; \mathbf{p})$ is an even s-polynomial with degree less than six, symbolic expression for its pure imaginary roots $s = \pm j\omega(\mathbf{p})$ exist. With substitution of $s = j\omega(\mathbf{p})$ into the characteristic quasipolynomial $F(s; h, \mathbf{p})$, we can obtain ω-free defining equation for the D-partition boundary ∂D_ω.

3. Feedback Stabilization of Unstable FOPDT Systems

In this section, using the D-partition technique, we investigate the stabilization of the unity feedback control system shown in Fig. 1 for first-order plus dead-time (FOPDT) unstable systems using P, PI, PD, and PID controllers. More specially, we investigate the effect of the delay on the stabilizability of the feedback control system and determine the entire sets of stabilizing P, PI, PD, and PID controllers.

Let the FOPDT unstable process $G_p(s)$ in Fig. 1 be given by

$$G_p(s) = \frac{Ke^{-ds}}{\tau s - 1} \tag{7}$$

By scaling the time variable t to t/τ, we have the normalized transfer function

$$G_p(s) = \frac{Ke^{-hs}}{s - 1} \tag{8}$$

where $h = d/\tau$. Hence, if the $G_c(s)$ in Fig. 1 is a PID controller whose normalized transfer function is given by

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \tag{9}$$

where K_p, K_i and K_d denote the proportional, integral and derivative gains of the controller, the closed-loop transfer function of the feedback control system is given by

$$G_{CL}(s) = \frac{(k_d s^2 + k_p s + k_i)e^{-hs}}{s(s-1) + (k_d s^2 + k_p s + k_i)e^{-hs}} \tag{10}$$

where $k_p = KK_p, k_i = KK_i$, and $k_d = KK_d$. In the sequel, k_p, k_i, and k_d are assumed to be nonnegative.

3.1 Stabilization of FOPDT Unstable Systems Using P Controllers

Letting $k_i = 0$ and $k_d = 0$ in (10), we have the closed-loop characteristic equation:

$$F(s; h, k_p) = (s-1) + k_p e^{-hs} \equiv A_0(s) + A_1(s)e^{-hs} = 0 \tag{11}$$

Substituting $s = 0$ into the above equation, we have the following D-partition boundary

$$\partial D_0 : \quad k_p = 1 \tag{12}$$

Since the degree of $A_0(s)$ is higher than that of $A_1(s)$, there is no ∂D_∞ boundary. To obtain the defining equation of the D-partition boundary ∂D_ω, we construct the Δ polynomial as

$$\Delta(s) = (s+1)(-s+1) - k_p^2 = -s^2 + (1 - k_p^2) \tag{13}$$

Notice that if $k_p > 1$ the polynomial $\Delta(s)$ has a pair of complex-conjugate pure imaginary roots at

$$s = \pm j\sqrt{k_p^2 - 1}, \quad k_p > 1 \tag{14}$$

This pair of roots are also the zeros of the characteristic equation (11) if the following two equations are simultaneously satisfied:

$$F_r = -1 + k_p \cos(h\sqrt{k_p^2 - 1}) = 0 \tag{15a}$$

$$F_i = \sqrt{k_p^2 - 1} - k_p \sin(h\sqrt{k_p^2 - 1}) = 0 \tag{15b}$$

which are obtained by substituting (14) into (11). The above two equations are equivalent to the equation

$$F_r^2 + F_i^2 = 2k_p \left(k_p - \cos(h\sqrt{k_p^2 - 1}) - \sqrt{k_p^2 - 1}\sin(h\sqrt{k_p^2 - 1})\right) = 0 \tag{16}$$

Since $k_p > 1$, we obtain from (16) the defining equation of the D-partition boundary ∂D_ω as

$$\partial D_\omega : \quad f(h, k_p) = k_p - \cos(h\sqrt{k_p^2 - 1}) - \sqrt{k_p^2 - 1}\sin(h\sqrt{k_p^2 - 1}) = 0 \tag{17}$$

Based on (15) and (17), we plot the D-partition boundaries ∂D_0 and ∂D_ω in Fig. 2. In this figure, solid and dash curves are defined by $F_r = 0$ and $F_i = 0$, respectively, and the overlap of these curves are described by $f = 0$. It can be seen from Fig. 2 that the maximum normalized delay time h_{max} of a FOPDT unstable process that can be stabilized using a P controller is $h_{max} = 1$. This result has been previously pointed by several authors [15, 25].

3.2 Stabilization of FOPDT Unstable Systems Using PD Controllers

The closed-loop characteristic equation for PD-stabilization of the FOPDT unstable system is given by

$$F(s; h, k_p, k_d) = (s-1) + (k_d s + k_p)e^{-hs} = 0 \tag{18}$$

The corresponding Δ polynomial is

$$\Delta(s) = (k_d^2 - 1)s^2 + (1 - k_p^2) \tag{19}$$

It follows from (18) that the D-partition boundaries ∂D_0 and ∂D_∞ are described by

$$\partial D_0 : \quad k_p = 1 \tag{20}$$

$$\partial D_\infty : \quad k_d = 1 \tag{21}$$

To derive the defining equation of the D-partition boundary ∂D_ω, we solve (19) for $x = s^2$:

$$x = \frac{1 - k_p^2}{1 - k_d^2} \tag{22}$$

Hence, if $0 \leq k_d < 1$ and $k_p > 1$, $\Delta(s)$ has a pair complex-conjugate pure-imaginary zeros:

$$s = \pm j\omega, \quad \omega = \sqrt{-x} \tag{23}$$

Notice that this pair of two zeros are also the zeros of the characteristic equation (18) if the following two equations are satisfied simultaneously:

$$F_r = -1 + k_p \cos(h\omega) + \omega k_d \sin(h\omega) = 0 \tag{24a}$$

$$F_i = \omega - k_p \sin(h\omega) + k_d\omega \cos(h\omega) = 0 \tag{24b}$$

These two equations is equivalent to the single one:

$$F_r^2 + F_i^2 = \frac{2(k_d + k_p)(k_d - k_p + (1 - k_p k_d)\cos(h\omega) + \omega(1 - k_d^2)\sin(h\omega)}{k_d^2 - 1} \tag{25}$$

Since under the conditions of $0 \leq k_d < 1$ and $k_p > 1$, the factors $(k_d + k_p)$ and $(k_d^2 - 1)$ never vanish, (25) is further equivalent to

$$f(h, k_p, k_d) = k_d - k_p + (1 - k_p k_d)\cos(h\omega) + (1 - k_d^2)\omega \sin(h\omega) = 0 \tag{26}$$

This equation is the desired defining equation of the D-partition boundary ∂D_ω.

Using the D-partition boundary equations (20) and (26), we construct stability boundaries in the k_p–h plane for various values of the derivative gain k_d. The constructed stability domains are plotted in Fig. 3. The stability domains in k_p–k_d plane for various values of the delay time h are shown in Fig. 4.

It is observed form Fig. 3 that the maximum allowable time delay of FOPDT unstable systems that can be stabilized by a PD controller is $1 + k_d$. To prove this, we note from (24) that the time delay h can be represented as

$$h = \frac{1}{\omega} \tan^{-1}\Big(\frac{\omega(k_d + k_p)}{k_p - k_d\omega^2}\Big) \tag{27}$$

Since h is a monotonically decreasing function of $k_p \in [1, \infty)$, h attains its maximum of $1 + k_d$ as $k_p \to 1$.

3.3 Stabilization of FOPDT Unstable Systems Using PI Controllers

Substituting $k_d = 0$ into (10), we have the characteristic equation for the PI-controlled system as follows:

$$F(s; h, k_p, k_i) = s(s - 1) + (k_p s + k_i)e^{-hs} \equiv A_0(s) + A_(s)e^{-hs} = 0 \tag{28}$$

It can be seen that the D-partition boundary ∂D_0 is described by the equation

$$\partial D_0 : \quad k_i = 0 \tag{29}$$

Since the degree of $A_0(s)$ is greater than that of $A_1(s)$, the D-partition boundary ∂D_∞ does not exist.

The Δ polynomial associated with the characteristic equation (28) is given by

$$\Delta(s) = s^4 + (k_p^2 - 1)s^2 - k_i^2 \tag{30}$$

Solving the equation $\Delta(s) = 0$ for $x = s^2$, we have

$$x_\pm = \frac{-(k_p^2 - 1) \pm \sqrt{4k_i^2 + (k_p^2 - 1)^2}}{2} \tag{31}$$

It is seen that, for $k_i > 0$, the solution x_- is negative real while x_+ is positive real. Hence, polynomial $\Delta(s)$ and $F(s; h, k_p, k_i)$ have common pure-imaginary zeros at

$$s = \pm j\omega, \quad \omega = \sqrt{-x_-} \tag{32}$$

provided that the following two equations are satisfied simultaneously:

$$F_r = k_i \cos(h\omega) - \omega^2 + k_p\omega \sin(h\omega) = 0 \tag{33a}$$

$$F_i = -k_i \sin(h\omega) - \omega + k_p\omega \cos(h\omega) = 0 \tag{33b}$$

These two equations are equivalent to the following one:

$$\begin{aligned} f(h, k_p, k_i) &= F_r^2 + F_i^2 \\ &= 2k_i^2 + k_p^2(k_p^2 - 1 + \sqrt{4k_i^2 + (k_p^2 - 1)^2}) - 2(k_i + k_p)\omega^2 \cos(h\omega) + 2(k_i - k_p\omega^2)\omega \sin(h\omega) \\ &= 0 \end{aligned} \tag{34}$$

This equation defines the D-partition boundary ∂D_ω.

By tracing the curves defined by (34) for various values of the delay time h, we plot the stability regions in the k_p–k_i plane in Fig. 5. To investigate the effect of time delay on the stabilizability of unstable FOPDT processes, the stability boundaries in the k_p–h plane for various values of the integral gain k_i are plotted in Fig. 6. It is seen from this figure that the maximum delay time h_{max} of unstable FOPDT systems that can be stabilized by a PI controller depends on the integral gain k_i.

To obtain the dependency of h_{max} on the proportional gain k_i, it is helpful to have the following two equations from (33):

$$\sin(h\omega) = \frac{k_p\omega^3 - k_i\omega}{k_i^2 + k_p^2\omega^2} \tag{35a}$$

$$\cos(h\omega) = \frac{k_i\omega^2 + k_p\omega^2}{k_i^2 + k_p^2\omega^2} \tag{35b}$$

Taking differentiation with respect to k_p on both sides of (35a), we obtain

$$(\omega\frac{\partial h}{\partial k_p} + h\frac{\partial \omega}{\partial k_p})\cos(h\omega) = \frac{(k_i\omega - k_p\omega^3)(2k_p\omega^2 + 2kp^2\omega\frac{\partial\omega}{\partial k_p})}{(k_i^2 + k_p^2\omega^2)^2} + \frac{\omega^3 - k_i\frac{\partial\omega}{\partial k_p} + 3k_p\omega^2\frac{\partial\omega}{\partial k_p}}{k_i^2 + k_p^2\omega^2} \tag{36}$$

Substituting (35b) into (36), we have the partial derivative

$$\frac{\partial h}{\partial k_p} = \frac{2k_pk_i\omega^3 + k_i^2\omega^3 - k_p^2\omega^5 + (k_p^2k_i\omega^2 - k_i^3 + 3k_pk_i^2\omega^2 - hk_pk_i^2\omega^2 - hk_i^3\omega^2 + k_p^3\omega^4 - hk_p^3\omega^4 - hk_p^2k_i\omega^4)\frac{\partial\omega}{\partial k_p}}{(k_i + k_p)\omega^3(k_i^2 + k_p^2\omega^2)} \tag{37}$$

where

$$h = \frac{1}{\omega}\tan^{-1}\left(\frac{k_p\omega^2 - k_i}{k_i\omega + k_p\omega}\right) \tag{38a}$$

$$\frac{\partial\omega}{\partial k_p} = \frac{k_p^3 - k_p + k_p\sqrt{4k_i^2 + (k_p^2-1)^2}}{\sqrt{2}\sqrt{4k_i^2 + (k_p^2-1)^2}\sqrt{k_p^2 - 1 + \sqrt{4k_i^2 + (k_p^2-1)^2}}} \tag{38b}$$

The dash curves shown in Fig. 6 is the curves defined by $\partial h/\partial k_p = 0$.

3.4 Stabilization of FOPDT Unstable Systems Using PID Controllers

As given in (10), the closed-loop characteristic equation of the PID-controlled system shown in Fig. 1 is

$$F(s; h, k_p, k_i, k_d) = s(s-1) + (k_ds^2 + k_ps + k_i)e^{-hs} = 0 \tag{39}$$

The $\Delta(s)$ polynomial corresponding to (39) is given by

$$\Delta(s) = (1 - k_d^2)s^4 + (k_p^2 - 2k_dk_i - 1)s^2 - k_i^2 \tag{40}$$

It follows from (39) that

$$\partial D_0: \quad k_i = 0 \tag{41}$$

$$\partial D_\infty: \quad k_d = 1 \tag{42}$$

Solving the equation $\Delta(s) = 0$ for $x = s^2$ gives

$$x_\pm = \frac{-(k_p^2 - 2k_dk_i - 1) \pm \sqrt{(k_p^2 - 2k_ik_d - 1)^2 + 4k_i^2(1 - k_d^2)}}{2(1 - k_d^2)} \tag{43}$$

Under the condition $1 - k_d^2 > 0$, the polynomial $\Delta(s)$ has only the following pair of complex-conjugate pure-imaginary roots:

$$s = \pm j\omega, \quad \omega = \sqrt{-x_-} \tag{44}$$

Substituting $s = j\omega$ into (39), we obtain

$$F_r = k_i\cos(h\omega) - \omega^2 + \omega k_p\sin(h\omega) - \omega^2k_d\cos(h\omega) = 0 \tag{45a}$$

$$F_i = -k_i\sin(h\omega) - \omega + k_p\omega\cos(h\omega) + k_d\omega^2\sin(h\omega) = 0 \tag{45b}$$

Hence, the D-partition boundary ∂D_ω is described by

$$\partial D_\omega: \quad f(h, k_p, k_i, k_d) = F_r^2 + F_i^2 \tag{46}$$

For illustration, we construct the stability domains in the k_p–k_i plane for $h = 0.5$ and various values of the derivative gain k_d. These domains are shown in Fig. 7. To investigate the effect of deadtime h on the stabilizability of FOPDT unstable systems, the stability domains in k_p–h plane for various values of the integral gain k_i are constructed and shown in Fig. 8. This figure shows that the maximum time delay h_{max} of the FOPDT unstable systems that can be stabilized by a PID controller decreases as the value of the integral gain k_i increase. In the following we derive expressions for establishing the relation between h_{max} and k_i.

First, we have from (45) the following relations:

$$\sin(h\omega) = \frac{k_p\omega^3 - k_i\omega + k_d\omega^3}{k_i^2 + k_p^2\omega^2 - 2k_ik_d\omega^2 + k_d\omega^4} \tag{47a}$$

$$\cos(h\omega) = \frac{k_p\omega^2 + k_i\omega^2 - k_d\omega^4}{(k_i^2 + k_p^2\omega^2 - 2k_ik_d\omega^2 + k_d\omega^4)^2} \tag{47b}$$

Then, differentiating both sides of (47a) with respect to k_p, we obtain

$$\begin{aligned}(\omega\frac{\partial h}{\partial k_p} + h\frac{\partial \omega}{\partial k_p})\cos(h\omega) =& \frac{(k_i\omega - k_p\omega^3 - k_d\omega^3)[2k_p\omega^2 + (2k_p^2\omega - 4k_id_d\omega + 4k_d^2\omega^3)\frac{\partial\omega}{\partial k_p}]}{(k_i^2 + k_p^2\omega^2 - 2k_ik_d\omega^2 + k_d\omega^4)^2} \\ &+ \frac{\omega^3 + (3k_p\omega^2 - k_i + 3k_d\omega^2)\frac{\partial\omega}{\partial k_p}}{k_i^2 + k_p^2\omega^2 - 2k_ik_d\omega^2 + k_d\omega^4}\end{aligned} \tag{48}$$

By using (47b) for $\cos(h\omega)$ and making rearrangement, the derivative $\partial h/\partial k_p$ is given by

$$\frac{\partial h}{\partial k_p} = \frac{c_0 + c_1\frac{\partial\omega}{\partial k_p}}{\omega^3(k_d\omega^2 - k_p - k_i)(k_i^2 + k_p^2\omega^2 - 2k_ik_d\omega^2 + k_d\omega^4)} \tag{49}$$

where

$$\frac{\partial\omega}{\partial k_p} = \frac{k_p - k_p^3 + 2k_pk_ik_d - k_p\sqrt{4k_i^2(1-k_d^2)^2 + (k_p^2 - 2k_ik_d - 1)^2}}{2\omega(k_d^2 - 1)\sqrt{4k_i^2(1-k_d^2)^2 + (k_p^2 - 2k_ik_d - 1)^2}} \tag{50a}$$

$$c_0 = k_p^2\omega^5 + 2k_pk_i\omega^5 + 2k_ik_d\omega^5 - 2k_pk_i\omega^3 - k_i\omega^3 - k_d^2\omega^7 \tag{50b}$$

$$\begin{aligned}c_1 =& k_i^3 - k_p^2k_i\omega^2 + 3k_pk_i^2\omega^2 + hk_pk_i^2\omega^2 - k_i^2k_d\omega^2 + hk_i^3\omega^2 - k_p^3\omega^4 \\ &+ hk_p^3\omega^4 - k_p^2k_d\omega^4 + hk_p^2k_i\omega^4 + 2k_pk_ik_d\omega^4 - 2hk_pk_ik_d\omega^4 - k_ik_d^2\omega^4 - 3hk_i^2k_d\omega^4 \\ &- hk_p^2k_d\omega^6 + k_pk_d^2\omega^6 + hk_pk_d^2\omega^6 + k_d^3\omega^6 + 3hk_ik_d^2\omega^6 - hk_d^3\omega^6\end{aligned} \tag{50c}$$

and

$$h = \frac{1}{\omega}\tan^{-1}\left(\frac{k_p\omega^2 - k_i + k_d\omega^2}{k_p\omega + k_i\omega - k_d\omega^3}\right) \tag{50d}$$

By tracing the curves defined by $\partial h/\partial k_p = 0$, the diagram of h_{max} versus k_i for various values of the derivative gain k_d are shown in Fig 9. As it can be seen, the larger the derivative gain used the larger the maximum delay time h_{max}.

5. Conclusions

The D-partition technique has been applied to the problem of stabilizing FOPDT unstable systems using PID controllers. The main contribution of the paper lies in deriving analytical expressions for the D-partition boundaries. Based on these expressions, we have investigated the stabilizability of P-, PI-, PD-, and PID-controlled FOPDT unstable systems. These explicit D-partition equations greatly facilitate the construction of the entire set of stabilizing controller parameters. The presented approach to the stabilizability analysis of PID-controlled FOPDT unstable systems is notably simpler than the classical D-partition technique [15] and the method of using Hermite-Biehler theorem [19-22].

Acknowledgement

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References

[1] Bialkowski, W. L., *The Control Handbook,* (Editor: W. S. Levine), pp. 1219-1242, 1996.

[2] Åström, K. J., and Haägglund, T. *PID Controllers: Theory, Design, and Tuning,* Research Triangle Park, NC, USA, 2nd Ed, 1995.

[3] Koivo, H. N., and Tanttu, J. T., *Proc. IFAC Intelligent Tuning and Adaptive Control Symposium,* Singapore, pp. 75-81, 1991.

[4] Gorez, R., *Journal A,* vol. 38, no. 1, pp. 3-10, 1997.

[5] O'Dwyer, A., *Proc. IFAC Workshop on Digital Control: Past, Present and Future of PID Control,* pp. 175-180, Terrassa, Spain, 5-7 April, 2000.

[6] O'Dwyer, A., *ibid,* pp. 242-247, Terrassa, Spain, 5-7 April, 2000.

[7] Zhuang, M., and Atherton, D. P., *IEE Proc.–Control Theory Applic.,* vol. 140, no. 3, pp. 216-224, 1993.

[8] Calistru, C.-N., *Proc. IEEE Conf. on Control Applications,* pp. 174-175, 1995.

[9] Chen, W. Z., *Proc. 1992 American Control Conf.,* pp. 3124-3129, 1992.

[10] Hwang, C., Chen, J. J., and Shieh, L. S., *J. Chinese Inst. Chem. Engrs,* vol. 32, no. 2, pp. 163-175, 2001.

[11] Cheng, S. L., and Hwang, C., *Chem. Eng. Communications,* vol. 170, pp. 83-115, 1998.

[12] Åström, K. J., Panagopoulos, H., and Hägglund, T., *Automatica,* vol. 34, no. 5, pp. 585-601, 1998.

[13] Panagopoulos, H., Åström, K. J., and Hägglund, T., *IEE Proc.–Control Theory Appl.,* vol. 149, no. 1, pp. 32-40, 2002.

[14] Besson, V., and Shenton, A. T., *IEEE Trans. Automat. Control,* vol. 42, no. 7, pp. 946-955, 1997.

[15] Cheng, S. L., and Hwang, C., *J. of Chinese Inst. Chem. Engrs,* vol. 30, no. 2, pp. 123-140, 1998.

[16] McKay, J., *Measurement and Control,* vol. 3, no. 10, pp. 293-294, 1970.

[17] Neimark, Ju. I., American Mathematical Society Translations, Series 2. Vol. 102: Ten papers in analysis. American Mathematical Society, Providence, R.I., pp. 95-131, 1973.

[18] Porter, B., and Bradshaw, A., *Int. J. of Systems Sci.,* vol. 5, no. 9, pp. 807-815, 1974.

[19] Silva, G. J., Datta, A., and Bhattacharyya, S. P., *Proc. 2000 American Control Conf.,* vol. 2, pp. 963-970, 2000.

[20] Silva, G. J., Datta, A., and Bhattacharrya, S. P., *Automatica,* vol. 37, no. 12, pp. 2025-2031, 2001.

[21] Silva, G. J., Datta, A., and Bhattacharrya, S. P., *Proceedings 2001 American Control Conf.,* vol. 6, pp. 4658-4663, 2001.

[22] Silva, G. J., Datta, A., and Bhattacharyya, S. P., *IEEE Trans. on Automat. Contr.,* vol. 47, no. 2, pp. 241-252, 2002.

[23] Krall, Allan M., *Bulletin of the American Mathematical Society,* vol. 70, pp. 291-292, 1964.

[24] Walton, K, and Marshall, J. E., *IEE Proc.–Control Theory and Applic.,* vol. 134, pp. 101-107, 1987.

[25] Huang, H. P., and Chen, C. C., *J. Chinese Inst. Chem. Engrs,* vol. 28, no. 4, pp. 295-299, 1997.

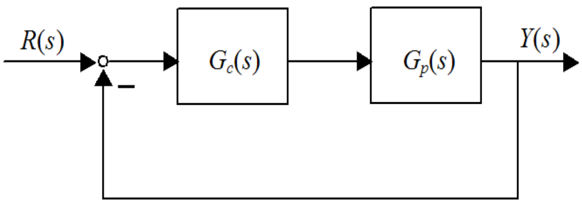

Fig. 1. Feedback control system.

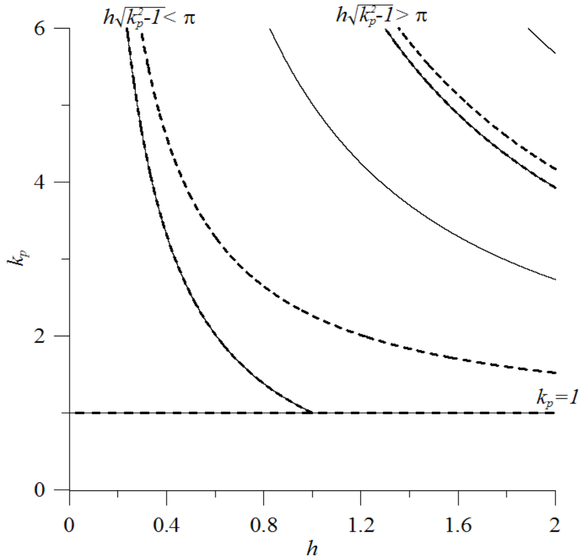

Fig. 2. Stability domain in k_p-h plane for the stabilization of FOPDT unstable systems using a P controller, Eq. (15a) gives solid curves, Eq. (15b) gives dash curves, and Eq. (17) gives the curves represented by the overlap of solid and dash curves.

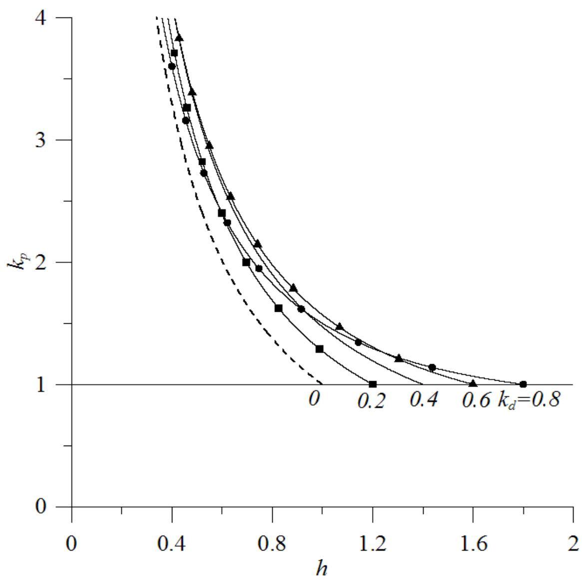

Fig. 3. Stability boundaries in k_p-h plane for using $k_d = 0, 0.2, 0.4, 0.6, 0.8$ in the PD stabilization of FOPDT unstable processes.

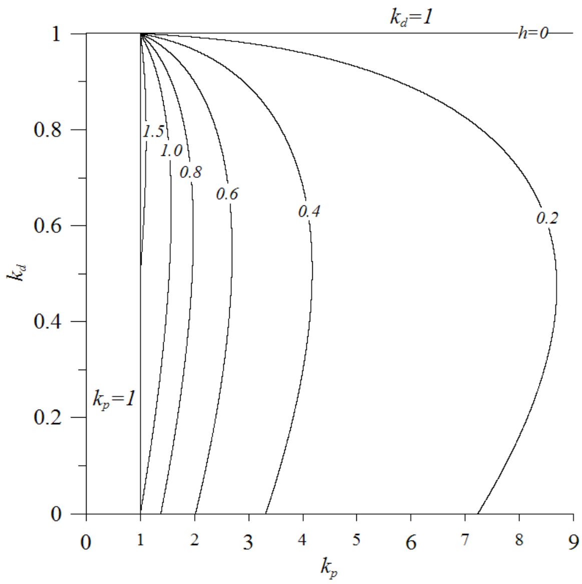

Fig. 4. Stability domains in k_p-k_d plane for the PD stabilization FOPDT unstable processes with various values of delay time h.

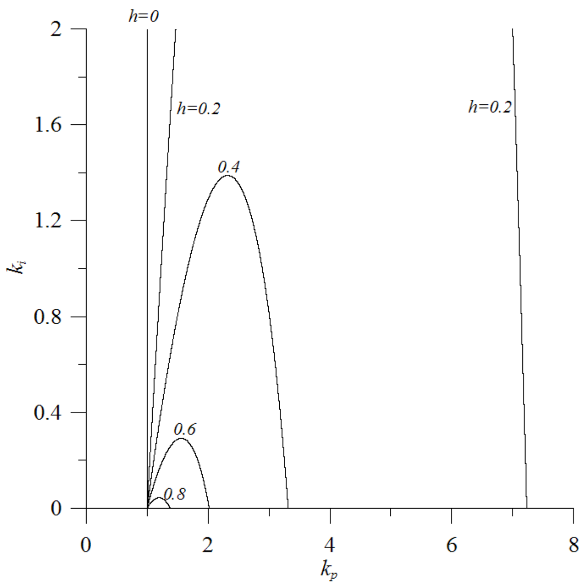

Fig. 5. Stability domains in k_p-k_i plane for the PI stabilization of FOPDT unstable processes with various values of delay time h.

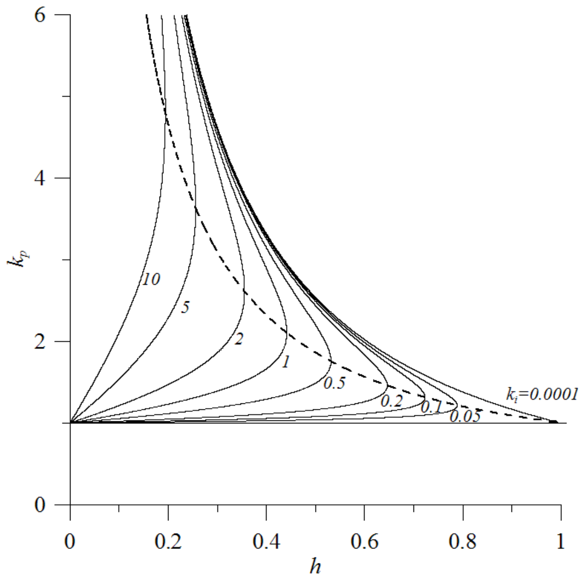

Fig. 6. Stability domains in k_p-h plane for the PI stabilization of FOPDT unstable processes with various values of delay time k_i.

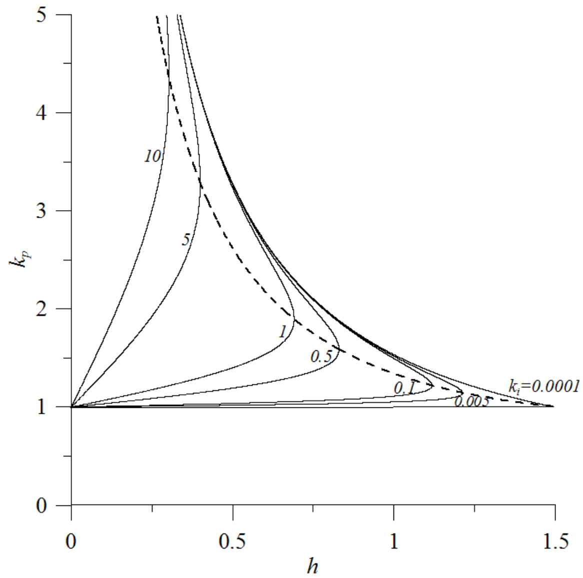

Fig. 8. Stability domains in k_p-h plane for using $k_d = 0.5$ and various values of k_i in the PID stabilization of FOPDT unstable processes.

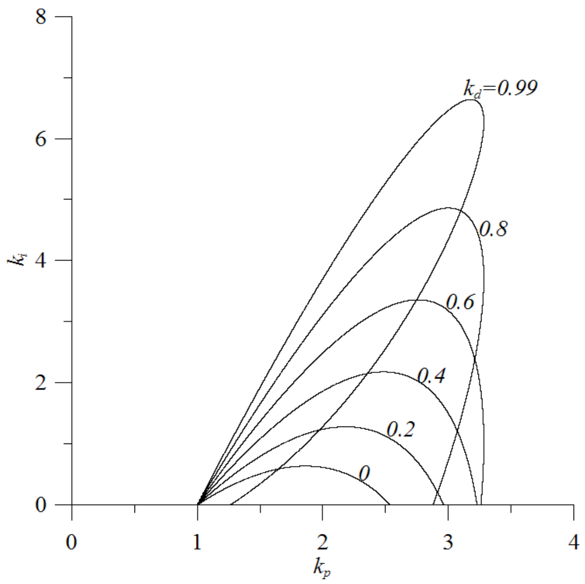

Fig. 7. Stability domains in k_p-k_i plane for using various values of k_d in the PID stabilization of an FOPDT unstable process having delay time $h = 0.5$.

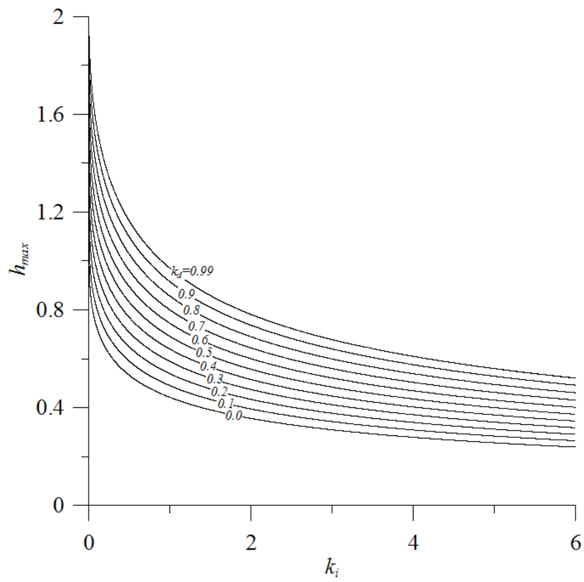

Fig. 9. The plot of the maximum stabilizable delay time versus k_i for using different values of k_d in the PID stabilization of FOPDT unstable processes.