STABILIZATION OF A UNICYCLE-TYPE MOBILE ROBOT USING HIGHER ORDER SLIDING MODE CONTROL

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Abstract

This paper deals with the problem of the practical stabilization of a unicycle-type mobile robot. The control strategy is divided into three steps and switches between different sliding mode controllers: a new third order sliding mode control with smooth manifolds that provides a practical stabilization and other sliding mode controls that perform finite time convergence (first order sliding mode and twisting algorithm). A simulation illustrates the results on the studied mobile robot.

1 Introduction

One of the motivations for tackling the stabilization (or tracking) of nonholonomic systems is the large number of applications, such as mobile robots. Obstacles to the stabilization of nonholonomic systems are the uncontrollability of their linear approximation and the fact that the Brockett's necessary condition to the existence of a smooth time-invariant state feedback is not satisfied [3]. To overcome those difficulties, various methods have been investigated: homogeneous and time-varying feedbacks [18, 19], sinusoidal and polynomial controls [15], piecewise controls [10, 14], flatness [8] or backstepping approaches [11]. In the present paper, it is aimed to design a control law for a unicycle-type mobile robot which:

- is a good compromise between performance and robustness,
- solves the disturbance rejection problem for some bounded matching perturbations,
- takes into account the actuator dynamics,
- leads to a practical stabilization: the system is stabilized in a ball containing the origin whose radius may be chosen as small as desired.

This objective will be achieved by switching between different sliding mode control laws. To this end, some smooth higher order sliding mode controllers will be introduced.

2 Problem statement

In this paper, we particularly focus on nonholonomic systems whose trajectories can be written as the solutions of the driftless system:

$$\dot{x} = g_1(x)u_1 + g_2(x)u_2 + p(x) \tag{1}$$

where $p(x)$ is a perturbation vector field (assumed to be smooth enough and thus bounded over some compact set). $u_1,\ u_2$ are the control inputs and the $g_j^{'}$ s are smooth vector fields on $\mathbb{R}^3$ that are linearly independent for all $x \in \mathbb{R}^3$. For instance, this is the case for the unicycle-type robot, which behavior can be described by the following system (see [4] for details):

$$\begin{cases} \dot{x} = \cos(\theta)\, u_1 + p_1(x) \\ \dot{y} = \sin(\theta)\, u_1 + p_2(x) \\ \dot{\theta} = u_2 + p_3(x) \end{cases}, \tag{2}$$

where x and y are the coordinates of the center gravity of the robot, θ is the orientation of the car with respect to the x-axis, $p_1(x)$, $p_2(x)$ and $p_3(x)$ are some additive perturbations and u_1 and u_2 refer respectively to the applied linear and the angular velocities (see Fig. 1).

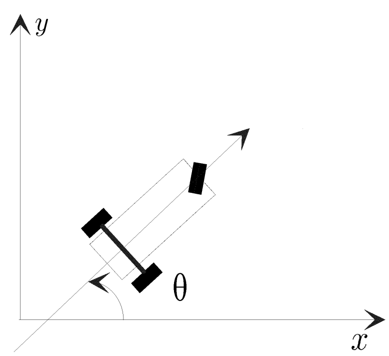

Figure 1: Unicycle robot kinematic

Using the smooth state change of coordinates and input transformations given in [16] (that allow to transform some classes of nonholonomic systems in the so-called one chained form), it has been shown in [9] that the system (1) can be written into the perturbed one-chained form

$$\begin{cases} \dot{z}_1 = v_1 + p_1(z) \\ \dot{z}_2 = v_2 + p_2(z) \\ \dot{z}_3 = z_2\,(v_1 + p_1(z)) \end{cases} \tag{3}$$

if and only if the perturbation vector field $p(x)$ belongs to the distribution spanned by the two vector fields $g_1(x)$ and $g_2(x)$. As it will be seen in the forthcoming developments, this form is convenient for designing stabilizing sliding mode control laws. For (2), one can use the following change of coordinates

$$\begin{cases} z_1 = \theta \\ z_2 = x\cos\theta + y\sin\theta \\ z_3 = x\sin\theta - y\cos\theta \end{cases}, \tag{4}$$

and the feedback control

$$\begin{cases} v_1 = u_1 \\ v_2 = u_2 - z_3 u_1 \end{cases}. \tag{5}$$

Discontinuous control laws have been developed in the literature in order to stabilize system (2). The main criticism when applying such strategies to a mobile robot would be the action of a discontinuous control directly on the mechanical part of the system (namely v_1). The purpose of the paper is to define a sliding mode control acting on the electrical parts of the system (which is more realistic since power converters are discontinuous actuators by nature). Taking into account the actuators dynamics remains to include some dynamical extensions (cascade integrators) in the system (3):

$$\begin{cases} \dot{z}_1 = v_1 + p_1(z) \\ \dot{v}_1 = w_1 \\ \dot{w}_1 = -aw_1 + \mu_1 = \tilde{\mu}_1 \\ \dot{z}_2 = v_2 + p_2(z) \\ \dot{z}_3 = z_2\left(v_1 + p_1(z)\right) \end{cases} \tag{6}$$

where v_1 is the linear velocity of the system, w_1 is the acceleration and μ_1 is the motor voltage of the electric actuator that will be considered as the control input. However, in order to preserve the properties of sliding mode, i.e robustness with respect to a class of perturbations and fast convergence, it is essential to use a higher order sliding mode since the relative degree of the system has been increased. In the present case, the stabilization of (6) requires at least a third order sliding mode strategy. One of the main contribution of this paper is to propose a real third order sliding mode leading to a practical stabilization of a triple integrator like system.

Assumption: The disturbances are supposed to be bounded as following:

$$|\ddot{p}_1(z)| \le \rho_1$$
$$|p_2(z)| \le \rho_2,\ z \in \mathbb{R}^3$$

where $\rho_1, \rho_2 > 0$.

3 Stabilization of a wheeled mobile robot

Sliding mode control, which consists in constraining the motion of the system along manifolds of reduced dimensionality in the state space, is quite popular in nonlinear control systems community. One can refer to [17, 20] for further details about this theory. Its robustness properties with respect to matching perturbations and its discontinuous character also motivated the authors to consider such an approach for the stabilization of the nonholonomic systems. Furthermore, as it will be seen in the following, the chosen chained form is quite appropriate for a sliding mode strategy.

The drawback of classical sliding modes is the well known chattering phenomenon, which may excite unmodeled high frequency modes which degrade the performance of the system and possibly lead to unstability. To get rid of this undesirable phenomenon, higher order sliding mode concept has been introduced by Emel'yanov et al. (see [7, 12]) which main objective is to obtain a finite time convergence onto the non empty manifold $\mathcal{S}^r = \left\{\sigma = \dot{\sigma} = ... = \sigma^{(r-1)} = 0\right\}$, where σ is the sliding variable. Higher order sliding modes not only avoid the chattering effects, but can also achieve a finite time convergence and a better accuracy than classical sliding modes. Taking into account the switching imperfections and the sampling period τ, the motion does not ideally take place on $\sigma = 0$, but stays in a small neighbourhood of the manifold, which is reached within an accuracy of $o(\tau^r)$ for a r-th order whereas it is only $o(\tau)$ for a first order.

In [13], the author designed ideal sliding mode algorithms for any order, i.e. control laws leading to the finite time convergence of the system trajectories exactly on the set $\mathcal{S}^r$ for all r. However, the implementation of those algorithms may present some difficulties since some singularities in the time derivatives of the sliding variable can appear. In order to overcome such difficulties, a higher sliding mode control strategy with smooth manifolds that was developed in [5] will be considered in this paper. Those algorithms are leading to a practical finite time stabilization, that is to say the finite time convergence into an ε-neighbourhood of the sliding manifold $\sigma = 0$.

The stabilization of the chained form (6) is made in three steps by switching between different types of sliding mode algorithms that are described in Appendices A and B.

The **first part** of the control algorithm is to constrain the subsystem

$$\begin{cases} \dot{z}_1 = v_1 + p_1(z) \\ \dot{v}_1 = w_1 \\ \dot{w}_1 = -aw_1 + \mu_1 = \tilde{\mu}_1 \end{cases} \tag{7}$$

to evolve on the manifold

$$\sigma_{11} = z_1 - at = 0. \tag{8}$$

One can note that the system (7), with input $\tilde{\mu}_1$ and output σ_{11}, has relative degree three

$$\sigma_{11}^{(3)} = \tilde{\mu}_1 + \ddot{p}_1(z).$$

Thus by applying a third order sliding mode (see Appendix A) of the form

$$\tilde{\mu}_1 = \Pi(\sigma_{11}, \dot{\sigma}_{11}, \ddot{\sigma}_{11}, k, m, A, \alpha),$$

a neighbourhood of the manifold (8) is reached in a finite time T_1. Since after T_1, $\dot{\sigma}_{11} = \nu(\varepsilon)$, where $\|\nu(\varepsilon)\| \ll 1$, one gets the following equivalent dynamics: $(v_1 + p_1(z))_{eq} = a + \nu(\varepsilon)$.

Second step: for $t \geq T_1$, the equivalent dynamics on the manifold (8) is given by

$$\left\{\begin{array}{l} \dot{z}_1 = a + \nu(\varepsilon) \\ \dot{z}_2 = v_2 + p_2(z) \\ \dot{z}_3 = (a + \nu(\varepsilon))\, z_2 \end{array}\right. .$$

The subsystem

$$\left\{\begin{array}{l} \dot{z}_2 = v_2 + p_2(z) \\ \dot{z}_3 = (a + \nu(\varepsilon))\, z_2 \end{array}\right.$$

has relative degree two with respect to the sliding variable $\sigma_{21} = z_3$:

$$\ddot{\sigma}_{21} = [(a + \nu(\varepsilon))\, p_2(z) + \dot{\nu}(\varepsilon) z_2] + (a + \nu(\varepsilon))\, v_2.$$

Thus, the second order sliding mode algorithm

$$v_2 = \Gamma(\sigma_{21}, \beta, \lambda_m, \lambda_M)$$

with a suitable choice of gains, implies the convergence of the state trajectories on the sliding set defined by $\{\sigma_{21} = \dot{\sigma}_{21} = 0\}$, i.e $z_3 = z_2 = 0$ in a finite time $t \leq T_2$ (see Appendix B).

Third step: after $t = T_1 + T_2$, the two control laws switch to $\tilde{\mu}_1 = \Pi(z_1, \dot{z}_1, \ddot{z}_1, \bar{k}, \bar{m}, \bar{A}, \bar{\alpha})$ and $v_2 = -k_2^{'} \operatorname{sgn}(z_2)$. Thus, z_2 and z_3 remains equal to zero and a neighbourhood of the manifold $z_1 = 0$ is reached. This ensures the finite time convergence of the whole state to a neighbourhood of the origin. This result is expressed in the following theorem

Theorem 1 *Under the variable structure control law*

$$\tilde{\mu}_1 = \left\{\begin{array}{ll} \tilde{\mu}_{11} = \Pi(\sigma_{11}, \dot{\sigma}_{11}, \ddot{\sigma}_{11}, k, m, A, \alpha), & t \leq T_1 + T_2 \\ \tilde{\mu}_{12} = \Pi(\sigma_{12}, \dot{\sigma}_{12}, \ddot{\sigma}_{12}, \bar{k}, \bar{m}, \bar{A}, \bar{\alpha}), & t > T_1 + T_2 \end{array}\right. \quad (9)$$

$$v_2 = \left\{\begin{array}{ll} v_{21} = \Gamma(\sigma_{21}, \beta, \lambda_m, \lambda_M), & T_1 \leq t \leq T_1 + T_2 \\ v_{22} = -k_2^{'} \operatorname{sgn}(\sigma_{22}), & t > T_1 + T_2 \end{array}\right. \quad (10)$$

where the sliding variables are defined by

$$\begin{array}{l} \sigma_{11} = z_1 - at, \quad a > 0 \\ \sigma_{12} = z_1 \\ \sigma_{21} = z_3 \\ \sigma_{22} = z_2 \end{array} ,$$

the solution of the closed-loop system (6-9-10) tends to a neighbourhood of the origin in finite time.

Note that the radius of this neighbourhood can be made as small as desired and that the convergence can be obtained in a prescribed time since T_1 and T_2 can be evaluated.

The application of a first order sliding mode in the first part of the algorithm would have resulted in discontinuous velocities and impulsive force and accelerations. This is naturally impossible in any real life application. The choice of a higher order sliding mode control strategy allows to get rid of this drawback since the discontinuous part of the variable structure control are henceforth embedded in the electrical part.

4 Simulation results

As a way of illustration, simulations based on the system (6), with the following controller parameters:

$$\begin{array}{rcl} k & = & \bar{k} = \dfrac{2}{3},\ m = \bar{m} = 100, \\ A & = & \bar{A} = 10,\ \alpha = \bar{\alpha} = 100, \\ \beta & = & 5,\ \lambda_m = 50,\ \lambda_M = 100,\ k_2^{'} = 10. \end{array}$$

Figure 2 shows the convergence of the state to zero while Figure 3 gives the behaviour of the actual system input v_1 which is continuous and of the motor voltage $\tilde{\mu}_1$ on which the sliding mode control is applied.

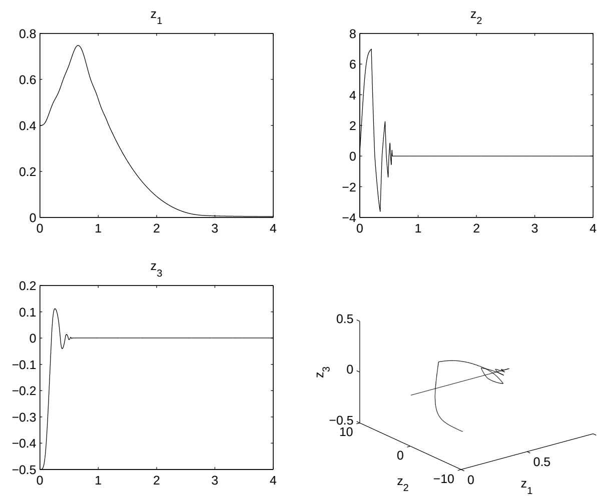

Figure 2: the z_1, z_2 and z_3 coordinates

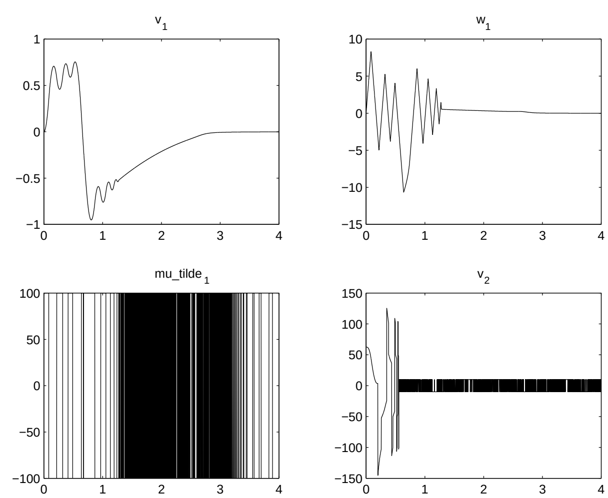

Figure 3: the control v_1 (and its derivatives) and v_2

5 Conclusion

The stabilization of a unicycle robot system has been studied. It is obtained by switching between several sliding mode controllers. So, a practical stabilization in finite time has been obtained (the origin is not attractive but the state can be made arbitrary small in a prescribed time). The main contribution of the paper is the design of a new kind of third order sliding mode control based on smooth manifolds. This allows to obtain continuous velocity and acceleration inputs for some practical applications on mechanical systems. Simulations on the example of a unicycle illustrated the performance of the controllers.

Appendix A: Third order sliding mode algorithm with time varying smooth manifolds

Here is described the third order sliding mode algorithm $\tilde{\mu}_1$. "Classical" sliding mode control theory provides several examples of systems that exhibit convergence to the equilibrium in finite time. A well-known example is the double integrator with bang-bang time feedback control. Some other types of finite time convergence are presented in [1] (this concept of stability will be used in the smooth manifold defined hereafter). Obviously, using a smooth manifold does not generate a stability in finite time, but a "practical " stability in finite time (convergence in finite time towards a ball of radius ε). On the other hand, using a variable structure control law enables to reject the disturbances if a kind of "*Matching Condition* " is satisfied. For a r-th order sliding mode, this condition can be expressed in the following way: the influence of the disturbance on the sliding variable σ and its derivatives $\sigma^{(i)}\ \forall\ i=1,...,r$ must be bounded.

Consider a system described by the differential equations

$$\begin{aligned}\dot{Z} &= f(Z)+g(Z)u+d(Z)\omega \\ y &= \sigma(Z)\end{aligned} \tag{11}$$

where u is the control input, y is an output whose vanishing fulfills the control objective and ω is a bounded perturbation satisfying the well-known matching condition given in [6]. Assume that the system has relative degree one with respect to y. Thus:

$$\dot{\sigma} = L_f\sigma(Z)+L_g\sigma(Z)u+L_d\sigma(Z)\omega$$

and $M_M > L_g\sigma > M_m > 0$. Taking into account that the perturbation is bounded, the control law defined by

$$\begin{aligned}u &= -\alpha sign(\sigma) \\ \alpha &> \frac{|L_d\sigma\omega|+|L_f\sigma|}{M_m}\end{aligned}$$

for any Z in the considered domain, implies the convergence on $\sigma(Z)=0$ in finite time. This elementary stability inequality may be generalized at any order, and more particularly for $r=3$ as it is shown here. For a sake of simplicity, it is assumed here that $\omega=0$. Let us describe the third-order sliding mode with a smooth manifold. For that, it is assumed that the Single Input Single Output system (11) has relative degree three with respect to y (that is to say $L_g\sigma = L_gL_f\sigma = 0$) and that there exist positive constants C_0, C_1, C_2, M_1, M_2 such that:

$$\begin{aligned}|L_f\sigma| &< C_0 \\ |L_f^2\sigma| &< C_1 \\ |L_f^3\sigma| &< C_2 \\ 0 < M_1 < L_gL_f^2\sigma &< M_2\end{aligned} \tag{12}$$

Two smooth manifolds S_{s1} and S_{s2} are designed:

$$S_{s1} = \dot{\sigma}+|\sigma|^{2/3}P_3(\sigma) \tag{13}$$

$$S_{s2} = \dot{S}_{s1}+A\,|S_{s1}|^{1/6}P_3(S_{s1})$$

in which the smooth function $P_3(\sigma)$ (which is a continuous approximation of the signum function) is defined by

$$P_3(\varsigma) = k\arctan(m\varsigma^3)$$

where k and m are two positive parameters.

The derivative of S_{s2} is given by

$$\dot{S}_{s2} = \ddot{S}_{s1}+A\frac{1}{6}\frac{|S_{s1}|^{\frac{1}{6}}}{S_{s1}}\dot{S}_{s1}P_3(S_{s1})+A\,|S_{s1}|^{\frac{1}{6}}\dot{P}_3(S_{s1}) \tag{14}$$

where $|S_{s1}|^{1/6}$ is determined as in [1] and where

$$\dot{S}_{s1} = \ddot{\sigma}+\frac{2}{3}\frac{|\sigma|^{2/3}}{\sigma}\dot{\sigma}P_3(\sigma)+\dot{P}_3(\sigma)\,|\sigma|^{2/3}$$

$$\begin{aligned}\ddot{S}_{s1} = {} & L_f^3\sigma-\frac{2}{9}\frac{|\sigma|^{2/3}}{\sigma^2}\dot{\sigma}^2P_3(\sigma) \\ & +\frac{2}{3}\frac{|\sigma|^{2/3}}{\sigma}\ddot{\sigma}P_3(\sigma)+\frac{2}{3}\frac{|\sigma|^{2/3}}{\sigma}\dot{\sigma}\dot{P}_3(\sigma) \\ & +\ddot{P}_3(\sigma)\,|\sigma|^{2/3}+\frac{2}{3}\frac{|\sigma|^{2/3}}{\sigma}\dot{\sigma}\dot{P}_3(\sigma)+L_gL_f^2\sigma u\end{aligned} \tag{15}$$

Note that the equation (14) is defined for all σ. Due to the assumptions (12) and thanks to the higher order sliding mode control law

$$u = -\alpha\left(L_gL_f^2\sigma\right)^{-1}sign(S_{s2}) = \Pi(\sigma,\dot{\sigma},\ddot{\sigma},k,m,A,\alpha) \tag{16}$$

one obtains first, for an appropriate choice of α, the finite time convergence of the system trajectories onto $S_{s2} = \dot{S}_{s1} + A\,|S_{s1}|^{1/6}P_3(S_{s1}) = 0$. Then, with a suitable choice of A, the manifold $S_{s1} = \dot{\sigma}+|\sigma|^{2/3}P_3(\sigma) = 0$ is reached. Finally, since on the manifold $S_{s1} = S_{s2} = 0$, $\dot{\sigma}+|\sigma|^{2/3}k\arctan\left(m\sigma^3\right) = 0$, with an appropriate choice of k and m , the trajectories evolve after a finite time in a neighbourhood of $\sigma=0$, whose layer is define by the parameters of $P_3(\sigma)$.

It is important to note that the convergence is only practical in this case and not in finite time. The robustness of the control law is ensured by taking into account, in the choice of α and in the inequalities (12), the bounds on the perturbations and some of its derivatives. To reject the perturbation, it is important to use the signum function (16). The details of the proof of the practical stability and of the case where $\omega \neq 0$ can be found in [5].

Appendix B: Second order sliding mode algorithm

Different kinds of second order algorithms have been given in the literature [2, 7, 12]. The algorithm used in this paper is a modified version of the twisting algorithm [7], which improves the convergence rate and which is useful to obtain a global convergence onto the chosen manifold. Consider the system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \zeta(t,y) + \chi(t,y)u \end{aligned} \tag{17}$$

where x_1, x_2, $u \in \mathbb{R}$. Let us denote $\sigma = x_1$ as the sliding variable. The system (17) has relative degree two with respect to σ. Assume that

$$\begin{gathered} 0 < K_m \leq \chi(t,y,u) \leq K_M, \\ |\zeta(t,y)| < C_0. \end{gathered}$$

then the control law

$$\begin{aligned} u &= \Gamma(\sigma,\beta,\lambda_m,\lambda_M) \\ &= -\beta^2\sigma - 2\beta\dot{\sigma} + \begin{cases} -\lambda_m \operatorname{sgn}(\sigma), & \text{if } \sigma\dot{\sigma} \leq 0 \\ -\lambda_M \operatorname{sgn}(\sigma), & \text{if } \sigma\dot{\sigma} > 0 \end{cases} \end{aligned}$$

with

$$\begin{aligned} &\lambda_m > \tfrac{C_0}{K_m} \\ &K_m\lambda_M - C_0 > K_M\lambda_m + C_0, \end{aligned} \tag{18}$$

generates a second order sliding mode with respect to the manifold $\sigma = 0$. Then the trajectories are describing an infinite number of rotations in the phase plane $(\sigma, \dot{\sigma})$ while converging in a finite time (as small as the value of β is high) to the set $\left\{(x_1,x_2) \in \mathbb{R}^2 : \sigma = \dot{\sigma} = 0\right\}$. It is also possible to define some upper-bounds for the convergence time.

References

[1] S.P. Bhat and D.S. Bernstein. "Finite time stability of continuous autonomous systems", *SIAM J. Control and Optimization*, Vol 38, pp. 751–766, (2000).

[2] G. Bartolini, A. Ferrara, and E. Usai. "Chattering Avoidance by Second-Order Sliding Mode Control", *IEEE Transactions on Automatic Control*, Vol. 43, No 2, pp. 241-246, (1998).

[3] R.W. Brockett. "Asymptotic stability and feedback stabilization", *Differential Geometric Control Theory*, R.W. Brockett, R.S. Millman and H.J. Sussmann, Eds., Birkhauser, (1983).

[4] C. Canudas de Wit and O. J. Sordalen. "Exponential stabilization of mobile robots with nonholonomic constraints", *IEEE Transactions on Automatic Control*, Vol. 37, No 11, pp. 1791 - 1797, (1992).

[5] M. Djemaï and J.P. Barbot. "Smooth manifolds and high order sliding mode control", *IEEE Conf. on Decision and Control*, (2002).

[6] B. Drazenovic. "The Invariance Conditions in Variable Structure Systems", *Automatica*, Vol. 5, No 3, pp. 287–295, (1969).

[7] S.V. Emel'yanov, S.V. Korovin and L.V. Levantovsky, "Higher Order Sliding Modes in the Binary Control System", *Soviet Physics*, Vol. 31, No 4, pp. 291–293, 1986).

[8] M. Fliess, J. Levine, P. Martin and P. Rouchon. "Flatness and defect of non-linear systems : Introductory theory and examples", *International Journal of Control*, Vol. 61, No 6, pp. 1327-1361, (1995).

[9] T. Floquet, J.P. Barbot, and W. Perruquetti. "One-chained form and sliding mode stabilization for a nonholonomic perturbed system", *Proceedings of the American Control Conference*, Chicago, USA, (2000).

[10] J. P. Hespanha and A. S. Morse. "Stabilization of nonholonomic integrators via logic-based switching", *Automatica*, 35, 385-393, (1999).

[11] Z.P. Jiang. "Robust exponential regulation of nonholonomic systems with uncertainties", *Automatica* 36, 189-209, (2000).

[12] A. Levant. "Sliding order and sliding accuracy in sliding mode control", *International Journal of Control*, Vol. 58 No. 6, pp. 1247–1263, (1993).

[13] Levant, A. (2001). Universal SISO sliding-mode controllers with finite-time convergence. *IEEE Transactions on Automatic Control*, Vol. 46, No 9, pp. 1447-1451.

[14] S. Monaco and D. Normand-Cyrot. "An introduction to motion planning under multirate digital control", *IEEE Conf. on Decision and Control*, Tucson, Arizona, pp. 1780-1785, (1992).

[15] R. Murray and S. Sastry. "Nonholonomic motion planning: steering using sinusoids", *IEEE Transactions on Automatic Control*, Vol. 38, pp. 700-716, (1993).

[16] R. Murray, Z. Li, and S. Sastry. *A Mathematical Introduction to Robotic Manipulation*. CRC Press, Inc., Florida, USA, (1994).

[17] W. Perruquetti and J.P. Barbot. *Sliding Mode Control in Engineering*. Ed. Marcel Dekker, (2002).

[18] J. Pomet. "Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift", *Systems and Control Letters*, Vol. 18, pp. 147-158, (1992).

[19] C. Samson. "Control of chained systems: Application to path following and time varying point-stabilization of mobile robots", *IEEE Transactions on Automatic Control*, Vol. 40, No 1, pp. 64-77, (1995).

[20] V.I. Utkin. *Sliding Modes in Control Optimization*. Communication and Control Engineering Series. Springer-Verlag, (1992).