ADVANCED SLIDING MODE STABILIZATION OF A LEVITATION SYSTEM

O. Bethoux *, T. Floquet #, J-P. Barbot *

* Control of System Research Team (E.C.S.)
E.N.S.E.A. , 6, Avenue du Ponceau - 95 014 Cergy-Pontoise – France
e-mail: bethoux@ensea.fr, barbot@ensea.fr

LAIL, UMR 8021 CNRS
Ecole Centrale de LiIle , BP48 Cité Scientifique, 59651 Villeneuve d'Ascq – France
e-mail: floquet@ec-lille.fr

Keywords: magnetic levitation system, nonlinear system, higher order sliding mode control.

Abstract

Levitation bearings are intrinsically unstable, nonlinear and highly uncertain systems. In this paper, we focus our attention on sliding mode controllers which allow robust design and more particularly on second order sliding mode control which appears very relevant with respect to the process structure.

1 Introduction

Magnetic levitation systems have received much attention as a mean of eliminating Coulomb friction due to mechanical contact. They are becoming popular in two different kinds of realization: high-speed motion and precision engineering industry [6].

Levitation bearing has been used from the beginning in rotating machinery to support rotors without friction providing low energy consumption, high rotational speed, no lubrication and greater reliability. It also allows a simpler and safer mechanical design as in the case of pumps used in nuclear installations where fluid leakage avoidance is of primary importance. The most famous application is high-speed ground transportation systems: Japanese "Maglev" and German "Transrapid", shown Figure 1, are very fast trains with linear motor.

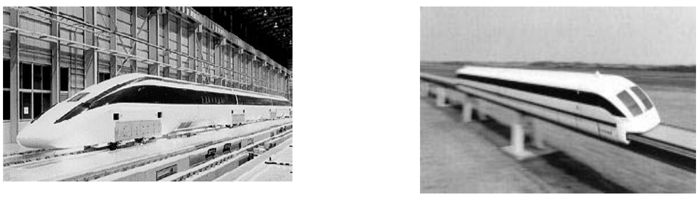

Figure 1. Japanese "Maglev" and German "Transrapid"

On an other side, magnetic bearings are becoming increasingly popular in the precision industry, which places significant demands on accurate positioning. One can quote nanometric servo-position actuator in micro-lithography industry as well as vibration isolation in precision scientific instruments.

Magnetic levitation highlights phenomenon like nonlinearities, fast dynamics and actuator saturation. Many control techniques have been quite successfully implemented on levitation systems. Within the control methodologies, we can, for instance, distinguish feedback linearization control [2,3,15], flatness based control [11], passivity based control [14] or backstepping design approach [13]. Many of these implentations are limited by the model relevance as well as its parameters accuracy.

Among robust control methods, one can quote nonlinear output regulation [8], adaptive control laws [18] and sliding mode control laws [2,3,9]. The latter works highlighted some robustness properties of sliding mode to unmodeled dynamics and some classes of disturbances. However, it suffers from the chattering phenomenon leading to high frequency switching currents. In this paper, we intend to optimize the use of the sliding mode approach in order to design a new control law robust to a larger class of disturbances and well suited to the levitator structure. Process physical properties lead us to a second order sliding mode control law.

2 Magnetic levitation

2.1 Process presentation

The system we consider in this paper is a gravity-biased one degree-of-freedom magnetic levitation system.

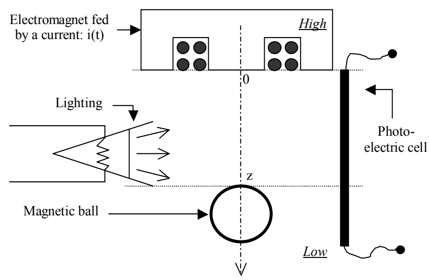

Figure 2. The electromagnetic suspension system.

The main goal is to keep a magnetic ball in levitation at a reference distance of an electromagnet. The only control variable is the current feeding the coil located above the ball. The distance between the ball and the electromagnet is indirectly measured: a photoelectric cell delivers a voltage proportional to the light flux coming from a lamp situated on the other side of the ball. Figure 2 gives a front view of the main elements.

2.2 Model of the magnetic levitation system

The system dynamics describing the behaviour of the moving ball is derived from the Newton's laws:

$$m\ddot{z} = F_{mag}(i,z) + mg + d \tag{1}$$

where z denotes the position of the ball (as indicated figure 2), m its mass, g the acceleration of gravity, i the coil current and $F_{mag}(i,z)$ the electromagnetic force applied to the ball. d denotes a bounded perturbation.

Drawing up the energy balance of the whole system and under the assumption that the magnetic core is non saturated (which occurs because of the air gap), the electromagnetic force can be expressed as following:

$$F_{mag}(i,z) = -\frac{i^2}{2}\frac{dL}{dz}(z) \tag{2}$$

where L is the coil inductance.

From equation (2), it can be seen that the electromagnetic force is always a negative term since the inductance parameter decreases while the ball moves away (z increases). Hence the magnetic ball is always pulled over to the electromagnet whatever the sign of the coil current is. Moreover magnetic force is all the greater as L(z) decreases steeply from L(0) to L(∞). As the value of L(∞) is always the same (coil inductance value without influence), L(0) has to be as great as possible, which is the case with high ball permeability.

Denoting

$$k(z) = -\frac{1}{2m}\frac{dL}{dz}(z) \qquad \text{and} \qquad \delta = \frac{d}{m}, \tag{3}$$

the model of the considered system can be expressed as:

$$\ddot{z} = g - k(z)i^2 + \delta \tag{4}$$

In the case of a good magnetic linkage between the electromagnet and the magnetic ball, the term k(z) is generally given by the relation:

$$k(z) = \frac{k_0}{\left(1 + \frac{z}{l_0}\right)^2} \tag{5}$$

where k_0 and l_0 are some physical parameters depending on the electromagnet, the mass and the permeability of the free space [17].

Since the objective is to find a control law i(t) such that the mass position is stabilized at a desired constant position, the system is rewritten in terms of the state tracking error variables i.e. $e_1 = (z - z_{ref})$ and $e_2 = \dot{e}_1$:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = g - k(z)i^2 + \delta \end{cases} \tag{6}$$

This magnetic levitation system is strongly nonlinear, open-loop unstable and uncontrollable if $g + \delta < 0$. It is also of importance to note that this system is subject to parametric uncertainties and external perturbations. Indeed, the accurate value of parameter k(z) is difficult to evaluate, except by a finite-element analysis. Furthermore, the term k(z) depends on the magnetic environment and can be perturbed by magnetic object and low frequency magnetic fields. In this paper, we focus our attention on sliding mode control so as to design a robust algorithm.

Assumption 1: It will be assumed throughout the paper that δ is a bounded perturbation and that the lower and upper bounds of k(z) are known:

$$|\delta| < \Delta_1$$

$$k_{min} < k(z) < k_{max}$$

2.3 Linear levitation controller

The relationship between the position and the current is non linear which is the most common case in general modelling. Nevertheless, a stabilisation of this unstable process can be achieved by the use of a linear controller such as a lead-phase compensator. This linear feedback design remains on a small-signal model.

Denoting $(i_0;z_0)$ the steady state and $(i_0 + i\,;\, z_0 + z)$ the operating point, we write a first order approximation of equation (4). We hence deduce a linear second order model:

$$H(s) = \frac{z(s)}{i(s)} = \frac{h_0(i_0,z_0)}{\left(\frac{s}{\omega_0(i_0,z_0)}\right)^2 - 1} \tag{7}$$

The harmonic design of such a feedback is based on Nyquist stability criterion (Cauchy theorem). As shown in Figure 3, a lead compensator provides a stable closed loop behaviour. But its three parameters (C_0, a, τ) depend deeply on local model values (h_0, ω_0). Hence the performance is degraded as the system moves away from the domain of validity of the modelling approximation.

These poor stability margins on large travel motivate the application of nonlinear controls to achieve satisfactory performance. Among them, nonlinear control strategies based on feedback linearization are limited in the sense that the parameters of the suspension must be well known. To achieve robustness of the control system, strategies such adaptive nonlinear control or sliding mode techniques have been applied. In this paper, it is shown that second order sliding modes are of particular interest for the robust control of magnetic levitation systems. For this, the basic notions of higher order sliding modes are given hereafter.

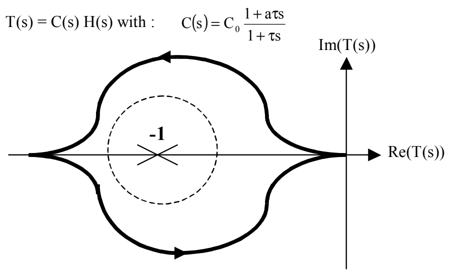

Figure 3: Open loop Nyquist diagram of levitation bearing H(s) with stabilizing compensator C(s).

3 Sliding mode background

In this section are presented the general notions of first and higher order sliding modes. The definitions are given in the framework of the magnetic levitation and thus in the case of a single input and nonlinear system whose dynamics is defined by the differential equations:

$$\begin{cases} \dot{x}(t) = f(t, x, u) \\ s = s(t, x) \end{cases} \quad (8)$$

where $x \in X \subset \Re^n$ is the state vector, $u \in U \subset \Re$ is the bounded input, $f : \Re^+ \times X \times U \rightarrow \Re^n$ is a sufficiently smooth uncertain vector field and $s : \Re^+ \times X \rightarrow \Re$ is the *sliding variable*. Assume that the control task is fulfilled by constraining the state trajectory on a proper *sliding manifold* S in the state space defined by the vanishing of the sliding variable i.e $S = \{x \in X : s(t, x) = 0\}$. The resulting behaviour of the system is called *sliding mode*. In classical sliding mode control (see [12,16]), this is achieved by means of a discontinuous control acting on the first time derivative of the sliding variable. In sliding mode, the system can be shown to be unsensitive to perturbations and parametric uncertainties which are satisfying the well known matching conditions (see [5]). However the major drawback is the so-called chattering which consists in large oscillation in the neighbourhood of the sliding manifold (due to the fact that the control is switching between high amplitude opposite values with theoretically infinite frequency). To overcome this undesirable phenomenon, higher order sliding modes have been introduced [7]. Assume that the function s and its (r-1) first total time derivatives, along the system trajectories, exist and are single valued functions of the state system (then discontinuity appears only in the r^{th} total time derivative $s^{(r)}$).

Definition (see [10]) If the state trajectory of the system (8) lies, after a finite time, in the following manifold

$$S^r = \left\{ x \in X : s = \dot{s} = \ldots = s^{(r-1)} = 0 \right\}$$

then it is said that this system evolves featuring a r^{th} order ideal sliding mode with respect to s (or a r^{th} order ideal sliding mode on the sliding manifold S).

A control law u leading to such a behaviour is called a r^{th} order ideal sliding mode algorithm with respect to s. In this definition, it is supposed that the prescribed constraint s = 0 is ideally kept. That is not the case in practice since it would imply that the control commutes at an infinite frequency. Because of the technological limitations of the actuators, such as switching time delays and/or small time constants in the actuators, this frequency is finite. Thus, the motion only takes place in a neighbourhood of the sliding manifold and is said to be a *r^{th} order real sliding mode* if the following relations are satisfied:

$$\begin{aligned} |s| &= O(\tau^r) \\ |\dot{s}| &= O(\tau^{r-1}) \\ &\vdots \\ |s^{(r-1)}| &= O(\tau) \end{aligned}$$

where τ is the sampling period.

From this definition, it can be seen that the higher the order of the sliding mode is, the more accurate the convergence on S is. Here, we are more particularly interested in second order algorithms, whom several examples have been develop in the literature [10]. The following one is called the real twisting algorithm [7]. Assume that f and s are respectively C^1 and C^2 functions and that the relative degree of the system with respect to s is r = 1. By differentiating the sliding variable s twice, the following expression is derived:

$$\ddot{s} = \frac{\partial \dot{s}(t,x,u)}{\partial t} + \frac{\partial \dot{s}(t,x,u)}{\partial x} f(t,x,u) + \frac{\partial \dot{s}(t,x,u)}{\partial u} \dot{u}$$

where it is assumed that there exists some positive constants s_0, K_m, K_M and C_0 such that in a neighbourhood $|s(t, x)| < s_0$, the following inequalities hold

$$\begin{cases} 0 < K_m \leq \dfrac{\partial \dot{s}(t,x,u)}{\partial u} \leq K_M \\ \left| \dfrac{\partial \dot{s}(t,x,u)}{\partial t} + \dfrac{\partial \dot{s}(t,x,u)}{\partial x} f(t,x,u) \right| < C_0 \end{cases} \quad (9)$$

Then under additive assumptions (see [7]), it can be shown that with the control law

$$\dot{u} = \begin{cases} -u & \text{if } |u| > 1 \\ -\lambda_m \operatorname{sgn}(s) & \text{if } s\Delta_S \leq 0, |u| \leq 1 \\ -\lambda_M \operatorname{sgn}(s) & \text{if } s\Delta_S > 0, |u| \leq 1 \end{cases} \quad (10)$$

$$\Delta_S \overset{\Delta}{=} \begin{cases} 0 & , k = 0 \\ s(k\tau) - s((k-1)\tau) & , k \geq 1 \end{cases},$$

where λ_m and λ_M are satisfying the conditions:

$$\lambda_m > \frac{C_0}{K_m}$$
$$K_m \lambda_M - C_0 > K_M \lambda_m + C_0$$

the system trajectories evolve, after a finite time, in the sliding set

$$\left\{ x \in X : |s| = O(\tau^2),\ |\dot{s}| = O(\tau) \right\} \quad .$$

The interest of the real twisting algorithm with respect to other algorithms is that it does not require the knowledge of the derivative of the sliding variable and takes into account some practical constraints such as the sampling of the measures and the control law.

4 First order sliding mode levitation stabilization

4.1 Control law

It is proposed in this section to stabilize the ball at its desired position with a first order sliding mode control law.

For this, let us consider the nonlinear system (6) and define the sliding surface:

$$L_s = \{(e_1, e_2) : s(e_1, e_2) = e_1 + T e_2 = 0\} \tag{11}$$

If a control law can constrain the levitation system to remain on the line L_s, then the ball motion satisfies a linear first order differential equation: the tracking error reaches exponentially the origin. This property occurs if the control action i(t) implies the fulfillment of the reachability condition:

$$s\dot{s} \leq -\mu|s| \tag{12}$$

The time derivative of the sliding variable is given by:

$$\dot{s}(e_1, e_2) = -T i^2 k(z) + \{e_2 + T(g + \delta)\} \tag{13}$$

If the relation

$$-\frac{e_2}{T} < g + \delta \tag{14}$$

is satisfied, the following discontinuous control law

$$i = \begin{cases} 0 & \text{if } s < 0 \\ I_{MAX} & \text{if } s \geq 0 \end{cases} \tag{15}$$

where
$$(I_{MAX}) \geq \sqrt{\frac{g + \frac{\mu}{T} + \Delta_1 + \frac{|e_2|}{T}}{k_{MIN}}} \tag{16}$$

guarantees the L_s reachability condition (12). This ensures that, in finite time, the levitator motion intercepts the sliding surface L_s and is forced to remain on it whatever the disturbances or mismatches between the rough model and the real system are.

4.2 Simulations

Some simulation results (Figure 4) have been obtained using the control law (15-16) with I_{MAX} = 20A and τ = 0.1s. The values of the system parameters have been chosen as g=9.81m/s^2, l_0 = 0.01m and k_0 = 1 m/s^2/A^2. The initial conditions are z = 4 mm and $\dot{z} = 0$ whereas the reference value is set at z_{ref} = 5 mm. A sinusoidal disturbance with amplitude D=0.5m/s^2 and frequency f=10Hz occurs at time t=0.8s.

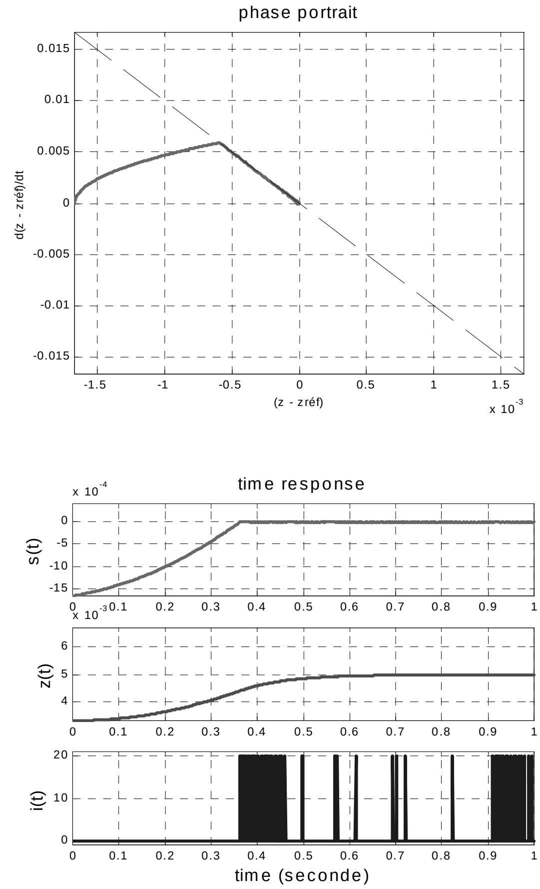

Figure 4 : Levitation with a 1^{srt} order sliding mode controller

It is important to note that this control strategy faces several drawbacks. The action required to bring about such a sliding mode is discontinuous and results in the chattering phenomenon, large control efforts (particularly in order to reject the parametric variations and the perturbations satisfying the matching condition, see (16)) and heat dissipation since the control force F_{mag} is driven by the current variable i(t). Furthermore, this variable is a continuous variable and its maximum slope rate is limited by the voltage value of the power supply. Therefore, the real control signal applied to the system will be an approximation of the desired robust and stabilizing control required by the switching function.

Another difficulty to apply a first order sliding mode for that kind of system is linked to the fact that the control input enters square in the model. As it was shown in [1], the equivalent control may in that case not be uniquely defined (or not exist) and some unstability may appear [4]. Furthemore, a control law that switches between high amplitude opposite values can not be designed. Thus, the inequality (14) ensuring the reachability condition defines a sliding domain that can be only enlarged by increasing the time constant T. However the greater T is, the slower the convergence of the tracking errors is.

In order to deal with these problems while keeping the key property of robustness due to the switching function, we intend to design a control feedback based on second order sliding mode. In this approach, the coil current i(t) will be a continuous variable.

5 Second order sliding mode levitation stabilization

5.1 Control law

If we consider that the control input is the coil voltage and if we neglect the coil resistance, the magnetic levitation state equations are now described by the following third-order nonlinear system:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = g - k(z)u^2 + \delta \\ \dot{u} = \dfrac{V}{L(z)} \end{cases} \quad (17)$$

where u denotes the coil current i(t) and V the new control input. One can note that the uncertainties does not enter the state equation at the same point as the control input.

Assumption 2:

- The coil inductance is bounded:

$$L_{min} < L(z) < L_{max}$$

- The disturbance δ is now assumed to be a bounded and derivable disturbance (which is the case of low frequency magnetic fields) such that:

$$\left|\dot{\delta}\right| \le \Delta_2$$

- The time derivative of k(z) is bounded:

$$\left|\dot{k}(z)\right| \le \Gamma$$

The discontinuous control input is the voltage V(t) and the current is in that case a smooth variable. This is well suited here since the actuator is in practice a power converter which is itself of discontinuous kind.

The sliding variable s whose vanishing implies the asymptotic stabilization of the tracking error remains $s(e_1, e_2) = e_1 + T\, e_2$. The second time derivative of s is given by:

$$\ddot{s} = g - k(z)u^2 + \delta + T\left[-\frac{dk(z)}{dt}u^2 + \dot{\delta} - 2k(z)u\,\dot{u}\right]$$

$$\ddot{s} = \left[g + \delta + T\dot{s} - u^2\left[k(z) + T\frac{dk(z)}{dt}\right]\right] - \frac{k(z)u}{L(z)}V$$

Note that thanks to a suitable choice of the control gains, the coil current i(t) (that is to say u, the integral of the discontinuous control) can be maintained at a strictly positive value. Thus, under assumptions 1 and 2, there exist K_m, K_M, C_0 such that:

$$\left|g + \delta + T\dot{\delta} - u^2\left(k + T\frac{dk}{dt}\right)\right| < C_0$$

$$0 < K_m < \frac{2K(z)u}{L(z)} < K_M$$

Applying the real twisting algorithm

$$V = \begin{cases} \lambda_m \operatorname{sgn}(s) & \text{if } s\Delta_S \le 0 \\ \lambda_M \operatorname{sgn}(s) & \text{if } s\Delta_S > 0 \end{cases}$$

where the gains are defined by (10), the system evolves after a finite time in the domain defined by:

$$P = \left\{(e_1, e_2, e_3) : s(e_1, e_2) = \frac{ds}{dt}(e_1, e_2, e_3) = 0\right\}$$

Then, in sliding mode, z is asymptotically stabilized at its desired position z_{ref} since s=0 defines a differential equation whose dynamics asymptotically converges to zero.

Remark: Although it is not essential to know the nominal model (5) to design the sliding mode control laws, one could take advantage of its use, particularly in order to reduce the control gains.

This second order sliding mode exhibits some good properties. In sliding mode, since

$$\frac{ds}{dt}(e_1, e_2, e_3) = e_2 + T\left[g - k\left(zi^2 + \delta\right)\right] = 0$$

the value of the coil current is stabilized at $i = \sqrt{\dfrac{g + \delta}{k(z_{ref})}}$.

In steady state, first order sliding mode also leads to the mean control value: $i^2 = \dfrac{g + \delta}{k(z_{ref})}$. Hence, the electrical coil power losses are the same in both cases. But 2nd order sliding mode law yields a continuous control value which means no magnetic core losses, whereas 1st order sliding mode algorithm yields a varying control value at high frequency which means a varying induction field generating core heating. On an other hand, the accuracy of the convergence onto the sliding manifold is improved since it is of the order of the square of the sampling period (whereas it is of the order of the sampling period for a first order sliding mode). That can be an important point of interest for applications in the precision industry. It has also been seen that the second order sliding mode has some robustness properties, even in the case of disturbances and parametric uncertainties that are not satisfying the matching condition, provided that they are derivable. This is essential for the design of a sliding mode controller for the system (17). Indeed, in [2], Charara et al. developed a first order sliding mode control law for a magnetic levitation system whose control input was the coil voltage. However, some disturbances are not matching so that the sliding condition (12) can not be satisfied in all cases. Furthermore, the control has the drawback to require the knowledge of the acceleration, which is not the case with a real second order sliding mode strategy.

5.2 Simulations

The Figure 5 illustrates simulation results with the second order sliding mode control strategy, obtained in the same condition as in the paragraph 4.2. It can be seen that the magnetic ball is stabilized at the desired position following the dynamics imposed by the sliding manifold and that the coil current is smooth. This simulation also highlights the robustness properties of the control law since its design only relies on the bounds of some uncertain model. Furthermore, it is shown that the system is not affected by the sinusoidal disturbance appearing at t=0.8s.

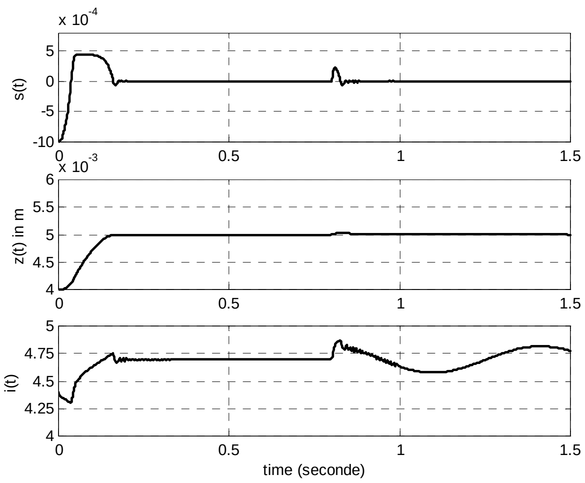

Figure 5: 2nd order sliding mode control

6 Conclusion

In this paper, we proposed a robust nonlinear controller based on second order sliding mode for the stabilization of a one degree-of-freedom magnetic levitation system. This strategy, which is of a relative simplicity of design, particularly allows to take into account the nature of the process: the discontinuous control acts on the power converter and not on the coil current. This strategy results in low energy consumption and even in a high-resolution position accuracy, which is of crucial importance in precision industry.

References

[1] G. Bartolini, T. Zolezzi, "Control of nonlinear variable structure systems", *J. Math. Anal. Appl.*, **Vol. 118**, pp. 42-62, (1986).

[2] A. Charara, J. De Miras and B. Caron, "Nonlinear Control of a Magnetic Levitation System without premagnetization", *IEEE Transactions on Control Systems Technology*, **vol 4 - n°5**, pp. 513-523, (1996).

[3] D. Cho, Y. Kato and D. Spilman, "Sliding Mode and Classical Controller – Magnetic Levitation Systems", *IEEE Control Systems Magazine*, pp. 42-48, (1993).

[4] M. Djemaï, J.P. Barbot et T. Boukhobza, "Some comments on higher order sliding modes", *European Control Conference*, Karlshruhe, Germany, (1999).

[5] B. Drazenovic, "The Invariance Conditions in Variable Structure System", *Automatica*, **vol. 5 – n°3**, pp. 287-295, (1969).

[6] M. Dussaux, "The Industrial Applications of the Active Magnetic Bearings Technology", *American Control Conference*, pp. 33-38, (1990).

[7] S.V. Emel'yanov, S.V. Korovin et L.V. Levantovsky, "Higher Order Sliding Modes in the Binary Control System", *Soviet Physics*, **vol. 31 – n°4**, pp. 291-293, (1986).

[8] L. Gentili and L. Marconi, "Robust Disturbance Suppression of a Magnetic Levitation System with Input Constraint", IFAC, 15th Triennial World Congress, (2002).

[9] H.M. Gutierrez and P.I. Ro, "Sliding-Mode Control of a Nonlinear-Input System: Application to a Magnetically Levitated Fast-Tool Servo", *IEEE Transactions on Industrial Electronics*, **vol 45 – n°6**, pp. 921-927, (1998).

[10] A. Levant, "Sliding order and sliding accuracy in sliding mode control", *International Journal of Control*, **vol. 58 – n°6**, pp. 1247-1263, (1993).

[11] J. Levine, J. Lotin and J. Ponsart, "A Nonlinear Approach to the Control of Magnetic Bearings", *IEEE Transactions on Control Systems Technology*, **vol 4**, pp. 524-544, (1996).

[12] W. Perruquetti and J.P. Barbot, "Sliding Mode Control in Engineering", **Marcel Dekker**, 2002.

[13] M. Queiroz, and D. Dawson, "Nonlinear Control of Magnetic Bearing: a Backstepping Approach", *IEEE Transactions on Control Systems Technology*, **vol 4**, pp. 545-552, (1996).

[14] M. Torres and R. Ortega "Feedback Linearization, Integrator Backstepping and Passivity Based Controller Design: a Comparison Example", *Perspective in Control: Theory and Application*, **Springer-Verlag**, pp. 97-115, (1998).

[15] D.L. Trumper, S.M Olson and P.K. Subrahmanyan, "Linearizing Control of Magnetic Suspension Systems", *IEEE Transactions on Control Systems Technology*, **vol 5 - n°4**, pp. 427-437, (1997).

[16] V.I. Utkin, "Sliding Modes in Control and Optimization", **Springer-Verlag**, (1992).

[17] H.H. Woodson, J.R. Melcher, "Electromechanical Dynamics-Part I", **John Wiley – New York**, (1968).

[18] Z.J Yang and M. Tateishi, "Adaptative Robust Nonlinear Control of a Magnetic Levitation System", *Automatica*, **vol 37**, pp. 1125 - 1131, (2001).