GLOBAL AND LOCAL ANALYSIS OF COPRIME FACTOR-BASED ANTI-WINDUP FOR STABLE AND UNSTABLE PLANTS

S. Crawshaw[†]

[†] AMS Integrated Systems Division, Portsmouth PO3 5PQ, UK
email: stuart.crawshaw@amsjv.com

Keywords: Anti-windup, Saturation, Local stability

Abstract

This paper considers analysis of anti-windup compensators based on coprime factorizations, for linear time-invariant plants with an input saturation nonlinearity. An explicit local $\mathcal{L}_2$ gain bound is presented which relates the size of the external disturbances to the difference between signals in the saturated system and the corresponding signals in the nominal (unsaturated) system. This result shows how stability and performance degrade gracefully as the external disturbances grow larger, and also gives some intuition into how the local properties are determined by the $\mathcal{H}_\infty$- and $\mathcal{H}_2$-norms of the system components.

1 Introduction

In this paper we consider the coprime factor-based anti-windup synthesis method of [1], which was in turn built on the framework of [2] and [3]. The plant is assumed to be linear and time-invariant, apart from a saturation at the input, and a linear time-invariant controller is also assumed to be given. Together these define the required closed-loop behaviour in the unsaturated regime. Compensation is then added to the controller. This compensation only acts when the plant input saturates, and its purpose is to lessen the effects of this saturation.

The framework is an extension of that of Kothare *et al* [3] which employed a parametrization of all coprime factors of the linear controller which have the same McMillan degree as that controller. This was extended in [2] to a parametrization of *all* coprime factors of the controller and, crucially, the problem of choosing a suitable coprime factorization of the controller was then seen to be equivalent to that of choosing a suitable coprime factorization of the plant.

A synthesis method for stable plants was proposed in [2], which used an $\mathcal{H}_\infty$ optimisation to guarantee stability and performance, measured in terms of the gain from the difference between the controller output & plant input in the saturated system, to the plant output and/or controller output. An alternative synthesis technique, which is also guaranteed to always lead to a globally stabilizing solution, was presented by Teel and Kapoor [4]. Their scheme can also be interpreted as a choice of plant coprime factorization, although nonlinear factorizations of a linear plant are also considered. Their paper introduced a performance criterion relating the difference between the real behaviour and the idealized behaviour without saturation to the amount the idealized plant input exceeds the saturation level.

This paper has three purposes: firstly, to bring together the local and global analysis results of [5] and [1] in the context of coprime factor anti-windup, secondly, to re-express one local result in a more intuitively useful form, and lastly to motivate the search for extensions to the local results for unstable plants which would more closely resemble the stable plant results.

2 Problem Definition

Figure 1 shows a nominal tracking problem where the signals $\boldsymbol{y}_{\text{ref}}$, $\boldsymbol{u}_{\text{lin}}$, $\boldsymbol{y}_{\text{lin}}$ and $\boldsymbol{e}_{\text{lin}}$ are the reference signal, controller output, plant output measurement and error signal respectively, and $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ are disturbances. The plant and stabilizing controller are assumed to be finite dimensional, linear, time-invariant systems with transfer functions $P(s)$ and $C(s)$ respectively, and the plant is furthermore assumed to be strictly proper. This interconnection has the following closed-loop relation:

$$\begin{bmatrix} \boldsymbol{e}_{\text{lin}} \\ \boldsymbol{y}_{\text{lin}} \\ \boldsymbol{u}_{\text{lin}} \end{bmatrix} = \begin{bmatrix} S & -SP & -S \\ I-S & SP & S \\ CS & -CSP & -CS \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_{\text{ref}} \\ \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \end{bmatrix} \tag{1}$$

where $S := (I + PC)^{-1}$ is the sensitivity function.

The same basic system with input saturation $\mathbf{Sat}$ and a linear anti-windup compensator is shown in Figure 2, where $\hat{\boldsymbol{u}}$ and $\boldsymbol{u}$ are the input and output of the saturation nonlinearity:

$$u_i(t) := \begin{cases} 1 & \text{if } \hat{u}_i(t) > 1 \\ \hat{u}_i(t) & \text{if } |\hat{u}_i(t)| \le 1 \\ -1 & \text{if } \hat{u}_i(t) < -1 \end{cases} \tag{2}$$

Identical external signals $\boldsymbol{y}_{\text{ref}}, \boldsymbol{d}_1, \boldsymbol{d}_2$ are assumed.

Remark 1 *Note that this is an idealized picture for the purpose of analysis. In practice, the saturation element shown would be implemented as part of the controller – thus guaranteeing that the saturation at the plant input itself never activates. As a result, the signal $\boldsymbol{d}_1$ can only represent disturbances at the plant input which enter independently of the saturating actuators; it cannot be used to represent noise on the controller output.*

By choosing initial left- and right-coprime factorizations of the controller $C = \tilde{V}_0^{-1}\tilde{U}_0$ and the plant $P = N_0 M_0^{-1}$ such that

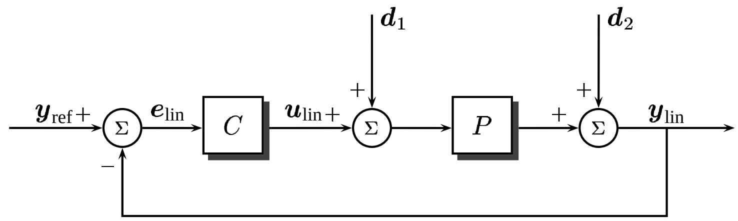

Figure 1: Nominal linear system

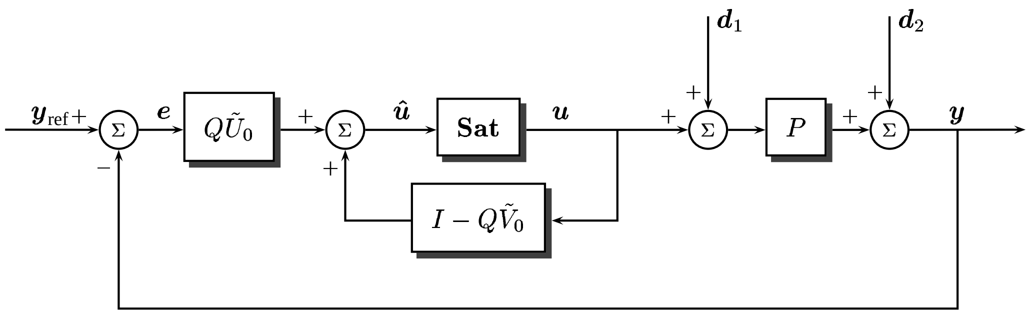

Figure 2: System with saturation and anti-windup compensator

they satisfy the Bezout identity $\tilde{V}_0 M_0 + \tilde{U}_0 N_0 = I$, and by letting $\mathcal{Q}$ represent the set of units in $\mathcal{RH}_\infty$, i.e.

$$\mathcal{Q} := \left\{ Q : Q, Q^{-1} \in \mathcal{RH}_\infty \right\} \tag{3}$$

then we can express the family of all coprime factor anti-windup compensators ([2]) as

$$\hat{\boldsymbol{u}} = Q\tilde{U}_0 \boldsymbol{e} + (I - Q\tilde{V}_0)\boldsymbol{u} \tag{4}$$

over all $Q \in \mathcal{Q}$; the synthesis problem is to choose a suitable coprime factorization of C, or of P, or to choose Q.

3 Stability & Performance Analysis

By considering the interconnection of Figure 2 as a LFT on the deadzone $\mathbf{Dzn}$ ([6], [7], [1]), it may be seen that stability of Figure 2 is equivalent to stability of Figure 3, where we assume that $M_0 Q^{-1} - I$ is strictly proper[1].

Furthermore, $\boldsymbol{e}, \hat{\boldsymbol{u}}, \boldsymbol{u}$ and $\boldsymbol{y}$ in Figure 2 can be given in terms of the nominal linear responses of Figure 1 and the output of the deadzone nonlinearity in Figure 3:

$$\boldsymbol{e} - \boldsymbol{e}_{\text{lin}} = N_0 Q^{-1}(\hat{\boldsymbol{u}} - \boldsymbol{u}) \tag{5}$$

$$\hat{\boldsymbol{u}} - \boldsymbol{u}_{\text{lin}} = (I - M_0 Q^{-1})(\hat{\boldsymbol{u}} - \boldsymbol{u}) \tag{6}$$

$$\boldsymbol{u} - \boldsymbol{u}_{\text{lin}} = -M_0 Q^{-1}(\hat{\boldsymbol{u}} - \boldsymbol{u}) \tag{7}$$

$$\boldsymbol{y} - \boldsymbol{y}_{\text{lin}} = -N_0 Q^{-1}(\hat{\boldsymbol{u}} - \boldsymbol{u}) \tag{8}$$

[1]For well-posedness of the interconnection in Figure 3, it is desirable that $M_0 Q^{-1}(\infty) = I$; by a happy coincidence, and under the mild assumption that P is strictly proper, this is equivalent to $Q\tilde{V}_0(\infty) = I$, which guarantees that there is no algebraic loop in the interconnection of Figure 2.

We denote the external signals by $\boldsymbol{z}$, and the transfer function from $\boldsymbol{z}$ to $\boldsymbol{u}_{\text{lin}}$ by F

$$\boldsymbol{z} := \begin{bmatrix} \boldsymbol{y}_{\text{ref}} \\ \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \end{bmatrix} \tag{9}$$

$$F := \begin{bmatrix} CS & -CSP & -CS \end{bmatrix} \tag{10}$$

so that $F\boldsymbol{z} = \boldsymbol{u}_{\text{lin}}$. When we wish to consider only an isolated disturbance, say at $\boldsymbol{d}_1$, this is simply achieved by ignoring the other rows of $\boldsymbol{z}$ and the corresponding columns of F.

We would like to address, both locally and globally, two $\mathcal{L}_2$ performance criteria which have been proposed for the interconnection in Figure 2:

- In [2] the suggestion is to minimize the $\mathcal{L}_2$ gain from the difference between the saturated & unsaturated inputs $(\hat{\boldsymbol{u}} - \boldsymbol{u})$ to the difference between the actual & nominal outputs $(\boldsymbol{y} - \boldsymbol{y}_{\text{lin}})$, weighted by some suitable W.

  This gain can be seen easily to be given by $\left\| W N_0 Q^{-1} \right\|_\infty$.

- In [4] the suggestion is to minimize the $\mathcal{L}_2$ gain from $\mathbf{Dzn}(\boldsymbol{u}_{\text{lin}})$ to the difference between any actual signal and its corresponding nominal signal, such as $(\boldsymbol{u} - \boldsymbol{u}_{\text{lin}})$ or $(\boldsymbol{y} - \boldsymbol{y}_{\text{lin}})$.

  Bearing in mind the first criterion and Equations (5) to (8), we see that we can address this second criterion indirectly by seeking to bound the $\mathcal{L}_2$ gain from $\mathbf{Dzn}(\boldsymbol{u}_{\text{lin}})$ to $(\hat{\boldsymbol{u}} - \boldsymbol{u})$.

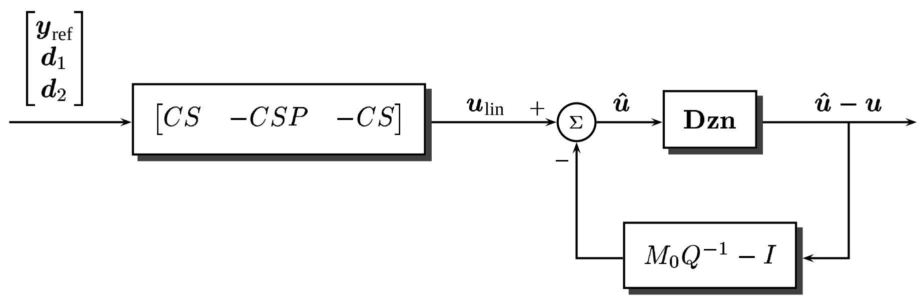

Figure 3: Equivalent system for stability & performance analysis

3.1 Global stability & performance

In the case that the plant P is stable, the system can be stabilized while addressing both performance criteria:

Proposition 1 ([1]) *The interconnection in Figure 2 is stable if $Q \in \mathcal{Q}$ is chosen such that $\left\| M_0Q^{-1} - I \right\|_\infty < 1$.*

Furthermore, provided this condition is satisfied, $\|\hat{\boldsymbol{u}} - \boldsymbol{u}\|_2$ is bounded by

$$\|\hat{\boldsymbol{u}} - \boldsymbol{u}\|_2 \leq \frac{1}{1 - \|M_0Q^{-1} - I\|_\infty} \|\mathbf{Dzn}(\boldsymbol{u}_{lin})\|_2 \qquad (11)$$

Proposition 2 ([2]) *For an asymptotically stable plant P and any weight W such that $W, W^{-1} \in \mathcal{RH}_\infty$*

$$\inf_{Q \,\in\, \mathcal{Q} \text{ such that } M_0Q^{-1}(\infty) = I} \left\| \begin{bmatrix} WN_0Q^{-1} \\ M_0Q^{-1} - I \end{bmatrix} \right\|_\infty = \frac{\|WP\|_\infty}{\sqrt{1 + \|WP\|_\infty^2}} < 1 \qquad (12)$$

Remark 2 *From Propositions 1 and 2 it is simple to deduce that we can always choose Q so as to achieve closed-loop stability while addressing both of the performance criteria.*

See [8] and references therein for further discussion of this approach to the anti-windup problem for stable plants, including a full state-space characterisation of the solutions to finding Q such that

$$\left\| \begin{bmatrix} WN_0Q^{-1} \\ M_0Q^{-1} - I \end{bmatrix} \right\|_\infty < \gamma$$

for achievable values of γ.

3.2 Local stability & performance

In the case that the plant P is unstable, it is well known that the system in Figure 2 **cannot** be made globally stable by any choice of Q, or indeed by any other linear or nonlinear anti-windup scheme. However, if we can show that $(\hat{\boldsymbol{u}} - \boldsymbol{u}) \in \mathcal{L}_2$ in Figure 3, for some restricted class of disturbances, then all of the signals in Figure 2 are also bounded, by Equations (5) to (8).

We define two scalar quantities ρ and μ, which will be seen to have an important influence on the local properties of the interconnection.

$$\mu := \left\| M_0Q^{-1} - I \right\|_\infty \qquad (13)$$

$$\rho := \tfrac{\|F\|_\infty}{\|F\|_2} \left\| M_0Q^{-1} - I \right\|_2 \qquad (14)$$

Proposition 3, which is an alternative statement of some local stability results previously presented in [5] and [9], applies to Figure 3 provided that both F and $M_0Q^{-1} - I$ are stable and strictly proper:

Proposition 3 ([5], [9]) *For the interconnection in Figure 3 with F, $\boldsymbol{z}$, μ and ρ as defined in Equations (10), (9), (13) and (14) respectively:*

(a) If $\mu < 1$, then for any $\lambda \in (0, \frac{1}{1-\mu})$

$$\begin{aligned} \textit{if} \quad & \|F\|_2 \|\boldsymbol{z}\|_2 \leq \frac{1 + \mu\lambda}{(1 + \rho\lambda)(1 - (1 - \mu)\lambda)} \\ \textit{then} \quad & \|\hat{\boldsymbol{u}} - \boldsymbol{u}\|_2 \leq \lambda \|F\|_\infty \|\boldsymbol{z}\|_2 \end{aligned} \qquad (15)$$

(b) If $\mu = 1$ and $\rho < 1$, then for any $\lambda \in (0, \infty)$

$$\begin{aligned} \textit{if} \quad & \|F\|_2 \|\boldsymbol{z}\|_2 \leq \frac{1 + \lambda}{(1 + \rho\lambda)} \\ \textit{then} \quad & \|\hat{\boldsymbol{u}} - \boldsymbol{u}\|_2 \leq \lambda \|F\|_\infty \|\boldsymbol{z}\|_2 \end{aligned} \qquad (16)$$

(c) If $\mu > 1$ and $\rho < 1$, then for any $\lambda \in (0, \infty)$

$$\begin{aligned} \textit{if} \quad & \|F\|_2 \|\boldsymbol{z}\|_2 \leq \frac{1 + \mu\lambda}{(1 + \rho\lambda)(1 + (\mu - 1)\lambda)} \\ \textit{then} \quad & \|\hat{\boldsymbol{u}} - \boldsymbol{u}\|_2 \leq \lambda \|F\|_\infty \|\boldsymbol{z}\|_2 \end{aligned} \qquad (17)$$

(d) If $\mu \geq 1$ and $\rho \geq 1$

$$\begin{aligned} \textit{if} \quad & \|F\|_2 \|\boldsymbol{z}\|_2 \leq 1 \\ \textit{then} \quad & \|\hat{\boldsymbol{u}} - \boldsymbol{u}\|_2 = 0 \end{aligned} \qquad (18)$$

Proposition 3 admits a graphical interpretation, which we will illustrate assuming that $\mu > 1$ and $\rho < 1$ (so that case (c) applies):

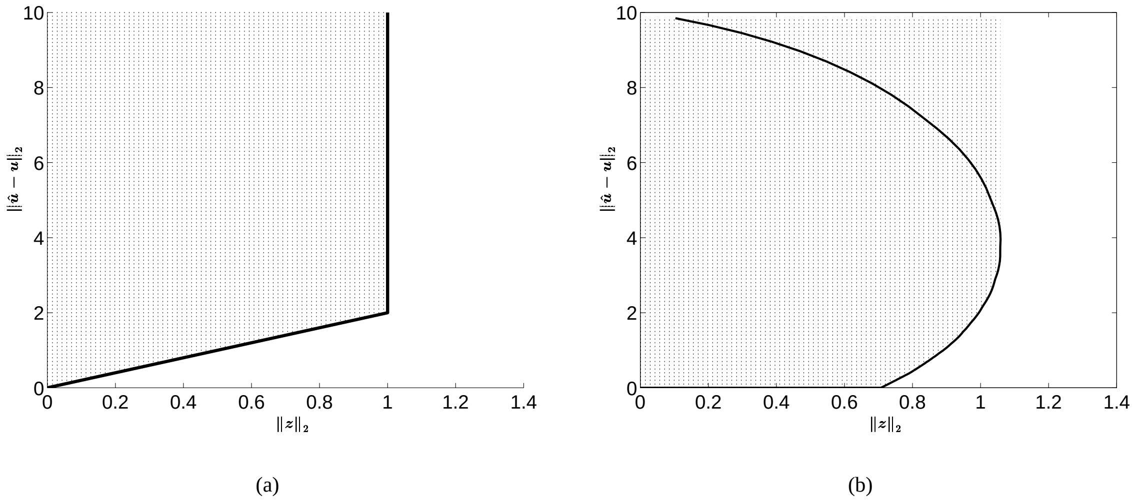

Figure 4: Graphical interpretation of Proposition 3

For any particular value of λ, and for any $\boldsymbol{z}$, the point defined by $\left[\|\boldsymbol{z}\|_2 \quad \|\hat{\boldsymbol{u}}-\boldsymbol{u}\|_2\right]$ will not be within the shaded region of Figure 4 (a), ie that area to the left of $\frac{1}{\|F\|_2}\frac{1+\mu\lambda}{\left(1+\rho\lambda\right)\left(1+(\mu-1)\lambda\right)}$ and above the line with gradient $\lambda\,\|F\|_\infty$.

By showing the equivalent shaded region for each possible value of λ on the same plot, the curved region shown in Figure 4 (b) is obtained. Again, the interpretation is that for any $\boldsymbol{z}$, the point $\left[\|\boldsymbol{z}\|_2 \quad \|\hat{\boldsymbol{u}}-\boldsymbol{u}\|_2\right]$ will not be within the shaded region.

The results given in Proposition 3 are analytically invertible, so we can obtain an explicit "local $\mathcal{L}_2$ gain" from $\boldsymbol{z}$ to $\hat{\boldsymbol{u}}-\boldsymbol{u}$. This gain is expressed in Theorem 1:

Theorem 1 *Define a gain function $f_{\mu,\rho}(*) : \mathbb{R} \to \mathbb{R}$ depending on $\mu, \rho \in \mathbb{R}$ as follows:*

(a) If $\mu < 1$, then

$$f_{\mu,\rho}(\psi) := \begin{cases} 0 & \text{if } 0 \le \psi \le 1 \\ \frac{\Theta-\sqrt{\Theta^2-4(\mu-1)(\psi-1)\rho\psi}}{2(\mu-1)\rho\psi} & \text{if } 1 \le \psi \end{cases} \tag{19}$$

where $\Theta := (1-\rho\psi) - (\mu-1)(\psi-1)$.

(b) If $\mu = 1$ and $\rho < 1$, then

$$f_{\mu,\rho}(\psi) := \begin{cases} 0 & \text{if } 0 \le \psi \le 1 \\ \frac{\psi-1}{1-\rho\psi} & \text{if } 1 \le \psi < \frac{1}{\rho} \\ \textit{undefined} & \text{if } \frac{1}{\rho} \le \psi \end{cases} \tag{20}$$

(c) If $\mu > 1$ and $\rho < 1$, then

$$f_{\mu,\rho}(\psi) := \begin{cases} 0 & \text{if } 0 \le \psi \le 1 \\ \frac{-\Sigma+\sqrt{\Sigma^2+4(1-\mu)(\psi-1)\rho\psi}}{2(1-\mu)\rho\psi} & \text{if } 1 \le \psi \le X \\ \textit{undefined} & \text{if } X < \psi \end{cases} \tag{21}$$

where[2] $\Sigma := (1-\rho\psi) + (1-\mu)(\psi-1)$ and

$$X := \left(\frac{\sqrt{\rho(\mu-1)}+\sqrt{\mu-\rho}}{\sqrt{\rho(\mu-\rho)}+\sqrt{\mu-1}}\right)^2 \tag{22}$$

(d) If $\mu \ge 1$ and $\rho \ge 1$, then

$$f_{\mu,\rho}(\psi) := \begin{cases} 0 & \text{if } 0 \le \psi \le 1 \\ \textit{undefined} & \text{if } 1 < \psi \end{cases} \tag{23}$$

Then for the interconnection in Figure 2 with F, $\boldsymbol{z}$, μ and ρ as defined in Equations (10), (9), (13) and (14) respectively, and provided that $\|\boldsymbol{z}\|_2$ is such that $f_{\mu,\rho}\left(\|F\|_2\,\|\boldsymbol{z}\|_2\right)$ is defined:

$$\|\hat{\boldsymbol{u}}-\boldsymbol{u}\|_2 \le f_{\mu,\rho}\Big(\|F\|_2\,\|\boldsymbol{z}\|_2\Big)\,\|F\|_\infty\,\|\boldsymbol{z}\|_2 \tag{24}$$

PROOF OF THEOREM 1: As shown Figure 4, Proposition 3 clearly expresses a relation of the form shown in Equation (24); it remains only to determine $f_{\mu,\rho}(*)$:

The value of $f_{\mu,\rho}(\psi)$, over the range $0 \le \psi \le 1$, can be obtained from (a) to (c) in Proposition 3 by taking $\lambda = 0$, and trivially from (d).

For each of (a) to (c) in Proposition 3, it is then relatively simple algebra to solve for λ in terms of $\|F\|_2\,\|\boldsymbol{z}\|_2$, and also to determine the valid range of $\|F\|_2\,\|\boldsymbol{z}\|_2$ over which each such expression holds. This gives the second part of $f_{\mu,\rho}(\psi)$.

Finally, in (b), (c) and (d) there is a range of $\|F\|_2\,\|\boldsymbol{z}\|_2$ for which Proposition 3 gives no information. ∎

Remark 3 *Synthesis of anti-windup compensators based on this type of local stability analysis is described in [9] and [10], along with discussion on the usage of such compensators.*

[2] Note that Σ and Θ denote the same value, but written such that each term in parentheses is positive.

Theorem 1 has some interesting properties:

- It uses simple $\mathcal{H}_\infty$- and $\mathcal{H}_2$-norms of the nominal system to give an indication of local properties.
- In (a), where $\left\|M_0Q^{-1}-I\right\|_\infty < 1$ and the interconnection is stable, we can see that as $\|z\|_2$ increases there is a graceful degradation in $\|\hat{u}-u\|_2$ with limiting $\mathcal{L}_2$ gain bound given by

$$\|\hat{u}-u\|_2 \le \frac{1}{1-\|M_0Q^{-1}-I\|_\infty}\|F\|_\infty\|z\|_2 \quad (25)$$

  which applies for all $z \in \mathcal{L}_2$.
- The same sort of graceful degradation is seen in (b) and (c), ie even when the interconnection is unstable. (Note, of course, that $\mu \ge 1$ does not imply instability, but that instability *does* imply $\mu \ge 1$.) In each of these cases there is a limiting value of $\|z\|_2$ beyond which we cannot guarantee boundedness of $\|\hat{u}-u\|_2$.
- The graceful degradation in cases (a), (b) and (c) gives us some confidence that "small" excursions into the saturated regime lead to "small" differences between the nominal and saturated systems.
- If $\mu \ge 1$ and $\rho \ge 1$, as in (d), then this local stability analysis gives no interesting results; the method is not usefully applicable if $\left\|M_0Q^{-1}-I\right\|_2$ is too large.

Remark 4 *The local stability analysis of [5] and [9] actually gave a stronger version of Proposition 3, which in any specific situation could be used to obtain a stronger gain bound than $f_{\mu,\rho}(*)$. However, in that case the equivalent gain bound cannot be expressed explicitly, and hence the intuition is partially lost.*

It should also be noted at this point that there are numerical methods (using, for example, Linear Matrix Inequalities), which can be applied to specific problems. However, these techniques have the disadvantage of being non-intuitive: you simply "turn the handle" and use the result which pops out. If a small change is made to the system, the calculations must be performed again, with (in general) no guarantees about how the result will change.

4 Conclusions

We have considered stability and performance, both locally and globally, for saturated systems with coprime factor anti-windup compensation.

When the closed-loop can be stabilized, we have have a global gain bound (Proposition 1) which explicitly shows that the difference between the nominal and saturated systems is directly bounded by the amount by which the nominal plant input u_{lin} exceeds the saturation limits:

$$\|\hat{u}-u\|_2 \le \frac{1}{1-\|M_0Q^{-1}-I\|_\infty}\|\mathbf{Dzn}(u_{\text{lin}})\|_2 \quad (26)$$

When the closed-loop cannot be stabilized, we have a somewhat different expression (Theorem 1):

$$\|\hat{u}-u\|_2 \le f_{\mu,\rho}\Big(\|F\|_2\|z\|_2\Big)\|F\|_\infty\|z\|_2 \quad (27)$$

which only applies over a certain class of inputs z. As a means of getting some intuition into the local properties of the system this is a useful result, however it is limited in that the external disturbances are invoked directly.

Remark 5 *As a closing remark, note that the component parts of the expression in Equations (27) and (24) are related to known quantities: for example, $\|F\|_2\|z\|_2$ is an upper bound on $\|u_{lin}\|_\infty$, and $\|F\|_\infty\|z\|_2$ is an upper bound on $\|u_{lin}\|_2$. This hints that a more direct relationship between u_{lin} and ($\hat{u}-u$) might exist, although this remains an open question.*

References

[1] S. Crawshaw and G. Vinnicombe, "Anti-windup for guaranteed $\mathcal{L}_2$ stability," in *Proceedings of the 39th IEEE Conference on Decision & Control*, 2000.

[2] S. Miyamoto and G. Vinnicombe, "Robust control of plants with saturation nonlinearity based on coprime factor representations," in *Proceedings of the 35th IEEE Conference on Decision & Control*, 1996.

[3] M. V. Kothare, P. J. Campo, M. Morari, and C. N. Nett, "A unified framework for the study of anti-windup designs," *Automatica*, vol. 30, no. 12, pp. 1869–1883, 1994.

[4] A. R. Teel and N. Kapoor, "The $\mathcal{L}_2$ anti-windup problem: its definition and solution," in *Proceedings of the European Control Conference*, 1997.

[5] S. Crawshaw and G. Vinnicombe, "Local stability of systems with saturation and deadzone nonlinearities," in *Proceedings of the 37th IEEE Conference on Decision and Control*, 1998.

[6] S. Miyamoto and G. Vinnicombe, "Robust control of plants with saturation nonlinearity based on coprime factor representations," Tech. Rep. CUED/F-INFENG/TR 264, Cambridge University Engineering Dept, 1996.

[7] P. Weston and I. Postlethwaite, "Linear conditioning schemes for systems containing saturating actuators," in *Proceedings of the IFAC Nonlinear Control System Design Symposium*, 1998.

[8] S. Crawshaw, "Anti-windup synthesis for guaranteed $\mathcal{L}_2$ performance," in *Proceedings of the 2002 American Control Conference*, 2002.

[9] S. Crawshaw, *Control of systems with actuator nonlinearities*. PhD thesis, Cambridge University Engineering Dept, 2000.

[10] S. Crawshaw and G. Vinnicombe, "Anti-windup for local stability of unstable plants," in *Proceedings of the 2002 American Control Conference*, 2002.