MIXED H_2/H_∞-BASED PID CONTROL VIA GENETIC ALGORITHMS: AN EXPERIMENTAL EVALUATION

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Keywords: Genetic Algorithms, mixed H_2/H_∞ control, PID control, experimental evaluation.

Abstract

This paper concerns the experimental evaluation of a **PID** mixed H_2/H_∞ control methodology based on the application of genetic algorithms. The considered plant is an experimental DC servosystem affected by disturbances acting on its output, and the performance objective corresponds to the control of the load angular position.

1 Introduction

As is pointed out in [2], the so-called *mixed H_2/H_∞ control* designs are quite useful for robust performance design for systems under parameter perturbation and uncertain disturbance. However, as is also pointed out in [2], the conventional output feedback designs of mixed H_2/H_∞ optimal control are complicated and not easily implemented. By these reasons, it is quite common to fix the structure of the controller in order to express mixed H_2/H_∞ control in terms of a tractable numerical optimization problem in the parameter vector space. The real parameter vector obtained as a solution to the optimization problem corresponds then to a particular fixed-structure controller which satisfies the specified control problem. The Proportional Integral Derivative control (**PID** control) law is a very succesful industry-oriented fixed-structure controller. As far as numerical optimization techniques are concerned, evolutionary computing [5] offers some powerful tools. In particular, Genetic Algorithms, initially inspired from the processes of natural selection and evolutionary genetics, have been succesfuly applied in control and signal processing design (see for instance [11]). We are interested here in the experimental evaluation of a **PID** mixed H_2/H_∞ control methodology based on the application of a standard genetic algorithm. With this objective in mind, we follow the procedure described in [2] to obtain the gains of a **PID** controller which solves a positioning control problem. The concerned plant is an experimental DC servosystem affected by a disturbance acting on the output. The paper is organized as follows:

First of all, we discuss in Section 2 the problem statement, i.e., the **PID** mixed H_2/H_∞ control methodology applied to the positioning control problem. Section 3 is dedicated to a brief description of Genetic Algorithms. We also recall in Section 3 the **PID** mixed H_2/H_∞ control methodology based on the application of Genetic Algoritms. For our particular application we apply a comercial Genetic Algorithms software [3], which implements a general purpose algorithm.

We present in Section 4 the results obtained when applying the discussed methodology to a experimental DC servosystem. We conclude with some final remarks in Section 5.

2 Problem Statement

2.1 Mixed H_2/H_∞ control

Consider the feedback control scheme shown in Figure 1, where:

- r denotes the reference input signal;
- y_d denotes the output signal;
- d denotes the disturbance signal;
- e denotes the tracking error input signal;
- P denotes a Linear Time-Invariant (**LTI**) Single-Input Single-Output (**SISO**) plant;
- C denotes a **LTI SISO** controller, and:
- W denotes the so-called frequency profile of the disturbance signal d.

The control problem is then defined as follows:

Definition 1: Optimal Tracking Control Problem (**O-**

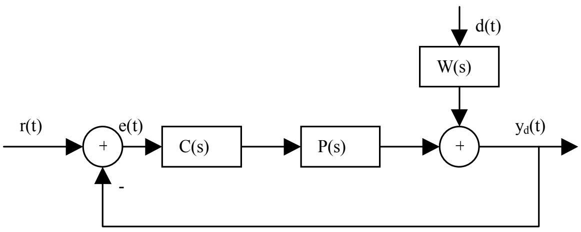

Fig. 1. Control system with disturbance.

TCP): taking into account the tracking control scheme shown in Figure 1, find a controller C which minimizes the tracking error signal e for a specific reference signal r, while insuring both disturbance attenuation and closed-loop internal stability.

Chosing the energy as the measure of a given signal, **OTCP** can be reformulated in formal terms as follows:

Definition 2: Let the **SISO** transfer functions $P(s)$ and $W(s)$ be given. Let also a real disturbance attenuation level $\gamma > 0$ be given. Find a controller $C(s)$ such that:

$$\min_{C} J, \text{ with } J := \int_0^{\infty} e^2(t)dt, \tag{1}$$

and:

$$\sup_{d(t)\epsilon L_2} \frac{\|y_d(t)\|_2}{\|d(t)\|_2} = \left\| \frac{W(s)}{1+P(s)C(s)} \right\|_{\infty} \leq \gamma, \tag{2}$$

while insuring closed-loop internal stability. L_2 stands for the space of all real valued Lebesgue integrable functions.

Please note that:

$$e(s) = \frac{r(s)}{1+P(s)C(s)}. \tag{3}$$

In the previous definition $\|\cdot\|_{\infty}$ stands for the H_{∞}-norm of the transfer function $\cdot$ (see for instance [4]).

Remark 3: The problem considered in Definition 2 is a typical mixed H_2/H_{∞} control one. In fact the problem corresponds to the minimization of the H_2-norm of a signal (1), with a H_{∞}-norm constraint (2).

2.2 PID mixed H_2/H_{∞} control

Let us now fix the structure of the controller to a **PID** one, i.e., $C(s) = k_1 + \frac{k_2}{s} + k_3 s$, where: k_1, k_2 and k_3 denote the **P**roportional, the **I**ntegral and the **D**erivative gains of the controller, respectively. If we suppose that the parameter domain of $\{k_1, k_2, k_3\}$ guarantees the stability of the closed-loop system (such parameter domain can be characterized by the Routh-Hurwitz criterion), and applying the Parseval's theorem (see for instance [7]), we have that (1) can be rewritten as follows:

$$\begin{aligned} J \quad : &= \int_0^{\infty} e^2(t)dt = \min_{k_1,k_2,k_3} \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} e(-s)\, e(s)\, ds \\ &= \min_{k_1,k_2,k_3} \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{B(s)B(-s)}{A(s)A(-s)} ds, \end{aligned} \tag{4}$$

where $A(s)$ and $B(s)$ are Hurwitz polynomials. Let us denote m the degree of $A(s)$, and let us assume that the degree of $B(s)$ is equal to $m-1$. Thus the polynomials $A(s)$ and $B(s)$ are given by $A(s) = \sum_{k=0}^{m} a_k s^k = a_m \prod_{i=0}^{m}(s - z_i)$ and $B(s) = \sum_{k=0}^{m-1} b_k s^k$, where $\{z_1, z_2, \ldots, z_m\}$ is the set of zeros of $A(s)$. It is also assumed that $a_0 \neq 0$ and $a_m > 0$.

Remark 4: In what follows we shall change J in (4) by $J_m(k_1, k_2, k_3)$ to indicate its dependence on m, k_1, k_2, and k_3.

As is pointed out in [2], the optimization problem (4) can be solved in this case by the residue theorem (see [12] and [10] for the details). Indeed $J_m(k_1,\, k_2,\, k_3) = J_m(k_1,\, k_2,\, k_3)_n \;/\; J_m(k_1,\, k_2,\, k_3)_d$, with:

$$\begin{aligned} & J_m(k_1,k_2,k_3)_n \\ : &= a_0 d_{m-1} Q_{m-1} \left[d_{m-2}Q_{m-2} - d_{m-3}Q_{m-3} \right. \\ & \left. + \cdots + (-1)^{m-1} d_1 Q_1 \right] + (-1)^{m-1} a_m d_0 Q_0 \end{aligned}$$

and:

$$\begin{aligned} & J_m(k_1,k_2,k_3)_d := (-1)^{m-1} 2 a_m a_0 \\ & \cdot \left[a_{m-1}Q_{m-1} - a_m \left(+a_{m-7}Q_{m-4} + \cdots \right) \right], \end{aligned} \tag{5}$$

where: $d_l = \sum_{i,j=0}^{m-1} (-1)^i b_i b_j$, with $i + j = 2l$. The Q_i, $i = 1, 2, , m-2$ are formed from $|\Omega|$, with:

$$\Omega := \begin{bmatrix} a_{m-1} & a_{m-3} & a_{m-5} & \cdots & \cdots & 0 & 0 \\ a_m & a_{m-2} & a_{m-4} & \cdots & \cdots & \vdots & \vdots \\ 0 & a_{m-1} & a_{m-3} & a_{m-5} & \cdots & \vdots & \vdots \\ \vdots & a_m & a_{m-2} & a_{m-4} & \cdots & \vdots & \vdots \\ \vdots & 0 & a_{m-1} & a_{m-3} & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & a_1 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & a_2 & a_0 \end{bmatrix},$$

by deleting the first, $(m-1)$-th and m-th columns and the first $(i+1)$-th and m-th rows, and Q_0 and Q_{m-1} are given by:

$$\begin{aligned} Q_0 &= a_2Q_1 - a_4Q_2 + a_6Q_3 - a_8Q_4 + \cdots \\ Q_1 &= a_{m-2}Q_{m-2} - a_{m-4}Q_{m-3} + a_{m-6}Q_{m-4} - \cdots \end{aligned}$$

As far as the attenuation disturbance constraint (2) is concerned, we have that:

$$\left\| \frac{W(s)}{1+P(s)C(s)} \right\|_\infty = \sqrt{\sup_{\omega\in[0,\infty)} \frac{\beta(\omega)}{\alpha(\omega)}} \leq \gamma,$$

where $\beta(\omega)$ and $\alpha(\omega)$ are some appropiate polynomials in ω. Since the peaks of $\beta(\omega)/\alpha(\omega)$ occur at the point which satisfy:

$$\frac{d}{d\omega}\frac{\beta(\omega)}{\alpha(\omega)} = \frac{\alpha(\omega)\frac{d\beta(\omega)}{d\omega} - \beta(\omega)\frac{d\alpha(\omega)}{d\omega}}{\alpha^2(\omega)} = 0,$$

only the real roots of:

$$\begin{aligned} & num_{\frac{d}{d\omega}\frac{\beta(\omega)}{\alpha(\omega)}}(\omega) \\ : \ & = \alpha(\omega)\frac{d\beta(\omega)}{d\omega} - \beta(\omega)\frac{d\alpha(\omega)}{d\omega} = \prod_{i=1}^{n}(\omega-\lambda_i) = 0 \end{aligned} \tag{6}$$

need be found.

Remark 5: In what follows we shall denote $\{\lambda_{r_1},\ \lambda_{r_2},\ \ldots,\ \lambda_{r_l}\}$ the set of real roots of $num_{\frac{d}{d\omega}\frac{\beta(\omega)}{\alpha(\omega)}}(\omega)$.

Thus, the attenuation disturbance constraint (2) is equivalent to:

$$\sqrt{\sup_{\lambda_{r_i}\in\{\lambda_{r_1},\ \lambda_{r_2},\ldots,\ \lambda_{r_l}\}} \frac{\beta(\lambda_{r_i})}{\alpha(\lambda_{r_i})}} \leq \gamma. \tag{7}$$

Summarizing, the **PID** mixed H_2/H_∞ Control Problem ($\mathbf{PID}_{H_2/H_\infty}\mathbf{CP}$) is defined as follows:

Definition 6: $\mathbf{PID}_{H_2/H_\infty}\mathbf{CP}$: let the transfer functions $P(s)$ and $W(s)$ be given. Consider a disturbance attenuation level γ also be given. Find a **PID** controller $C(s) = k_1 + \frac{k_2}{s} + k_3 s$, assuming that the parameter domain of $\{k_1,\ k_2,\ k_3\}$ guarantees the stability of the closed-loop system, such that $J_m(k_1,k_2,k_3)$ is minimized and $\sqrt{\sup_{\lambda_{r_i}}(\beta(\lambda_{r_i})/\alpha(\lambda_{r_i}))} \leq \gamma$ is satisfied.

2.3 An illustrative example

In order to illustrate the **PID** mixed H_2/H_∞ control methodology we have identified the model of an experimental DC Servosystem. This servosystem is included in a Client-Server experimental setup, mainly oriented to the evaluation of control laws and model identification algorithms. The Client is allocated in a Celeron^{TM} based computer running at 333 MHz. The sampling rate was set to 1 KHz. The DC motor is from Clifton Precision^{TM}, model JDTH-2250-BQ-DC. Angular position is measured using an optical encoder with 2500 pulses per turn. A Copley^{TM} Controls Power amplifier, model 413, is employed for driving the motor. An inertial load is attached to the motor through a belt. The data acquisition card is the MultiQ^{TM} model from Quanser Consulting with optical encoder inputs, which multiply by 4 their count, then, the number of pulses per turn is 10000. This output was further scaled down by a factor of 10000, which correspond, to 1 turn. The card also has 12 bits digital to analog converters with an output range [-5 volts, +5 volts]. The model identification algorithm, in fact a well known least-squares algorithm (see for instance [1]), was implemented using the MatLab^{TM}/Simulink^{TM} software running under the WINCON^{TM} environment. The Server is installed in a Pentium^{TM} based computer running at 200 MHz. The model obtained from the identification process is given by $P(s) = \frac{317}{s(s+5{,}57)} = \frac{y(s)}{u(s)}$, where $u(s)$ denotes the control signal and $y(s)$ denote the output signal. The output (angular position) is measured in number of turns, while the input signal is given in volts. We take $r(s) = \frac{1}{s}$ as the reference signal. We take as the frequency profile of the disturbance the transfer function $W(s) = \frac{1}{s+1}$.

2.3.1 The internal stability constraint

First of all, we have $e(s) = r(s) - y_d(s) = T_e(s)\,r(s)$, where:

$$\begin{aligned} T_e(s) &:= \left(\frac{1}{1+P(s)C(s)}\right) \\ &= \frac{s^2(s+5{,}57)}{s^3+(5{,}57+317k_3)s^2+317k_1 s+317k_2}. \end{aligned}$$

Consequently, the characteristic polynomial, say $p_c(s)$, is given by:

$$p_c(s) = s^3+(5{,}57+317k_3)s^2+317k_1 s+317k_2. \tag{8}$$

Now, the Routh-Hurtwitz criterium let us to conclude that the parameter domain of $\{k_1,\ k_2,\ k_3\}$ insuring internal stability (of the closed-loop system) is characterized by:

$$\begin{gathered} k_2 > 0, \\ k_3 > -5{,}57/317, \\ k_1 > k_2/(317k_3+5{,}57). \end{gathered} \tag{9}$$

2.3.2 The H_2 optimization problem

Taking into account (4) and (5) we have:

$$\begin{aligned} & \int_{-j\infty}^{j\infty} e(-s)\,e(s)\,ds \\ = & \int_{-j\infty}^{j\infty} \frac{r(-s)r(s)}{(1+P(-s)C(-s))\,(1+P(s)C(s))}ds \\ = & \int_{-j\infty}^{j\infty} \frac{B(s)B(-s)}{A(s)A(-s)}ds, \end{aligned}$$

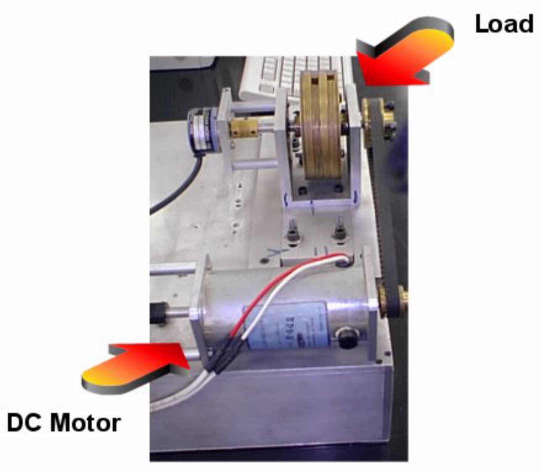

Fig. 2. DC Servosystem.

where $B(s) := b_0 + b_1 s + b_2 s^2$ and $A(s) := a_0 + a_1 s + a_2 s^2 + a_3 s^3$, with: $a_0 = 317k_2$, $a_1 = 317k_1$, $a_2 = (5{,}57 + 317k_3)$, $a_3 = 1$, $b_0 = 0$, $b_1 = 5{,}57$, and $b_2 = 1$. Remark that for our current example $m = 3$. Consequently:

$$\begin{aligned} & J_3(k_1,k_2,k_3) \\ = \; & \frac{b_2^2 a_0 a_1 + (b_1^2 - 2b_0 b_2) a_0 a_3 + b_0^2 a_2 a_3}{2a_0 a_3(-a_0 a_3 + a_1 a_2)} \\ = \; & 0{,}00001577287066 \left(\frac{31700k_1 + 3102{,}49}{-k_2 + 5{,}57k_1 + 317k_1 k_3} \right). \end{aligned}$$

2.3.3 The H_∞ optimization problem

As far as the disturbance attenuation constraint is concerned, we have $\frac{W(s)}{1+P(s)C(s)} = \frac{s^2(s+5{,}57)}{(s+1)(s^3+cs^2+317k_1 s+317k_2)}$, and:

$$\begin{aligned} \beta(\omega) &= \omega^6 + 31{,}0249\omega^4, \\ \alpha(\omega) &= \omega^8 \\ & +(3531{,}38k_3 + 100489k_3^2 - 634k_1 + 32{,}0249)\omega^6 \\ & + \left(-3531{,}38k_2 + 100489k_3^2 - 634k_1 \right. \\ & +100489k_1^2 + 3531{,}38 * k_3 - 200978k_2k_3 \\ & \left. +31{,}0249\right)\omega^4 \\ & + \left(100489k_2^2 + 100489k_1^2 - 3531{,}38k_2 \right. \\ & \left. -200978k_2k_3\right)\omega^2 \\ & +100489k_2^2, \end{aligned}$$

with $c := 5{,}57 + 317k_3$ and:

$$\begin{aligned} & num_{\frac{d}{d\omega}\frac{\beta(\omega)}{\alpha(\omega)}}(\omega) \\ = 2\omega^{13} \quad & +124{,}09960\omega^{11} \\ & + (1925{,}08884 + 212058{,}6628k_3 \\ & +6034344{,}36k_3^2 - 38071{,}57320k_1 \\ & +7062{,}76k_2 + 401956k_2k_3 - 200978k_1^2)\,\omega^9 \\ & (+803912k_2k_3 - 401956k_2^2 + 14125{,}52k_2 \\ & -401956k_1^2)\,\omega^7 \\ & + (12470644{,}71k_2k_3 - 6235322{,}348k_1^2 \\ & +219121{,}4227k_2 \\ & -6838256{,}348k_2^2)\,\omega^5 \\ & -12470644{,}7k_2^2\omega^3. \end{aligned}$$

2.3.4 PID$_{H_2/H_\infty}$CP

Summarizing, for our current example **PID$_{H_2/H_\infty}$CP** is defined as follows (we fix $\gamma = 0{,}1$):

Problem 7: Let $P(s) = \frac{317}{s(s+5{,}57)}$, $W(s) = \frac{1}{s+1}$, $\lambda = 0{,}1$, and $r(s) = \frac{1}{s}$, be given. Consider the parameter stability domain of $\{k_1, k_2, k_3\}$ (9), i.e., $k_2 > 0$, $k_3 > -5{,}57/317$, $k_1 > k_2/(317k_3 + 5{,}57)$, find a set $\{k_1, k_2, k_3\}$ satisfying the previous inequality constraints and such that:

$$\begin{aligned} & \min_{\{k_1,k_2,k_3\}} J_3(k_1,k_2,k_3) \\ = \; & \min_{\{k_1,k_2,k_3\}} \left(\frac{1577287066 \times 10^{-5}(31700k_1 + 3102{,}49)}{-k_2 + 5{,}57k_1 + 317k_1k_3} \right) \end{aligned}$$

and:

$$\sqrt{\sup_{\lambda_{r_i} \in \{\lambda_{r_{1i}}, \lambda_{r_2}, \ldots, \lambda_{r_l}\}} \left(\frac{\lambda_{r_i}^6 + 31{,}0249\lambda_{r_i}^4}{\alpha(\lambda_{r_i})} \right)} < 0{,}1,$$

where λ_{r_i}, for $i = 1, ..., l$, denote the r_i-th real root of :

$$\begin{aligned} & num_{\frac{d}{d\omega}\frac{\beta(\omega)}{\alpha(\omega)}}(\omega) \\ = 2\omega^{13} \quad & \\ & +124{,}09960\omega^{11} \\ & + (1925{,}08884 + 212058{,}6628k_3 \\ & +6034344{,}36k_3^2 \\ & -38071{,}57320k_1 + 7062{,}76k_2 \\ & +401956k_2k_3 - 200978k_1^2)\,\omega^9 \\ & + (803912k_2k_3 - 401956k_2^2 \\ & +14125{,}52k_2 - 401956k_1^2)\,\omega^7 \\ & + (12470644{,}71k_2k_3 - 6235322{,}348k_1^2 \\ & +219121{,}4227k_2 \\ & -6838256{,}348k_2^2\omega^5 - 12470644{,}7k_2^2)\,\omega^3. \end{aligned}$$

Remark 8: In what follows we shall denote $\{k_1^*, k_2^*, k_3^*\}$ the solution to Problem 7.

We can at this level proceed to solve the proposed problem.

3 The Genetic Algorithms Approach

There exists a huge quantity of publications concerning Genetic Algorithms and its applications in Automatic Control and Signal Processing (see for instance [3], [11], [8], and [6]). As is pointed out in [6], Genetic Algorithms (as a class of stochastic optimization techniques) can be interpreted as one particular implementation of a Monte Carlo optimization technique and can be applied to arbitrary optimization problems (like the one specified by Definition 6). Motivated by the mechanisms of natural selection and evolutionary genetics, a typical Genetic Algorithm behaves as specified by the following genetic dialect (see [6]): "*We start out with a randomely chosen qualitative genetic pool. We evaluate the quality of the entire genetic pool. We rank the genetic strings according to their quality. We define the fitness of a genetic string as* $\text{fitness} = \frac{1{,}0}{\text{total error}}$. *We then add up the fitnesses of all genetic strings in the genetic pool and define the relative fitness of a genetic string as* $\text{relative fitness} = \frac{\text{fitness}}{\text{sum over all fitnesses}}$. *We then replace the entire genetic pool by a new pool in which each genetic string is represented never, once, or multiple times proportional to its relative fitness. Poor genetic strings are removed, while excellent genetic strings are duplicated many times. We then pair the genetic strings up arbitrarely. Each pair produces exactly two offspring, one consisting of the head of the first string concatenated with the tail of the second and the other consisting of the head of the second string concatenated with the tail of the first. We then let the old generation die and replace the entire genetic pool by the new generation. The algorithm is repeated until convergence*".

There exist several implementations of the genetic computing strategy schematized above. For our purposes we chose a MATLAB^{TM}-based tool called FlexTools^{TM} [3]. This MATLAB^{TM} toolbox implements a general-purpose Genetic Algorithm which constraint each genetic string to be coded as a binary pattern. Both the fitness function (i.e., $J_m(k_1, k_2, k_3)$ in our case) and the disturbance attenuation constraint function are coded as MATLAB^{TM} procedures, each functions receiving as their argument the vector $\{k_1, k_2, k_3\}$. It is quite obvious that the initial genetic pool belongs to the parameter domain characterized by (9). As far as the set of Genetic Algorithm descriptors, we must choose the following one:

- Number of generations.
- Population size.
- Crossover probability.
- Mutation probability.
- Selection operator.

3.1 The computing procedure

In order to implement the solution of $\mathbf{PID}_{H_2/H_\infty}\mathbf{CP}$ through the Genetic Algorithm Approach we follow the following sequential procedure:

1. We fix the Genetic Algorithms descriptors mentioned above.
2. We randomely choose a genetic pool of **PID** gains $\{k_1, k_2, k_3\}$ (coded in binary terms) constrained to belong to the stability parameter domain. In fact, we fix range values for k_1, k_2, and k_3.
3. We compute the fitnesses of all genetic strings, taking directly $J_m(k_1, k_2, k_3)$ as the fitness function.
4. We apply the roulette wheel selection technique to choose the best subset of the population of **PID** gains.
5. We proceed to pair the genetic strings (and to apply mutation) in order to obtain a new population.
6. We verify the disturbance attenuation constraint for each member of the chosen population. We decrease the fitness of the members which do not satisfy the disturbance attenuation constraint.
7. We repeat the procedure step 3 to step 6 until the fixed number of generations is attained. The final binary result is finally decoded to obtain the PID gains.

Remark 9: Both the crossover probability and the mutation probability are chosen following what is indicated in [5].

4 Experimental Results

Let us now continue with our illustrative example (see Subsection 2.3):

Consider Problem 7. The set of Genetic Algorithms descriptors is fixed as follows:

- Number of generations = 20.
- Population size = 77.
- Crossover probability = 0.77.
- Mutation probability = 0.077.
- Roulette wheel selection.

In order to compute $\{k_1^*, k_2^*, k_3^*\}$ we take a subset of the parameter stability domain charcaterized by $k_1 \in [0, 8]$, $k_2 \in [0, 8]$, and $k_3 \in [0, 8]$. We apply the specified procedure to our current example, and we obtain the following

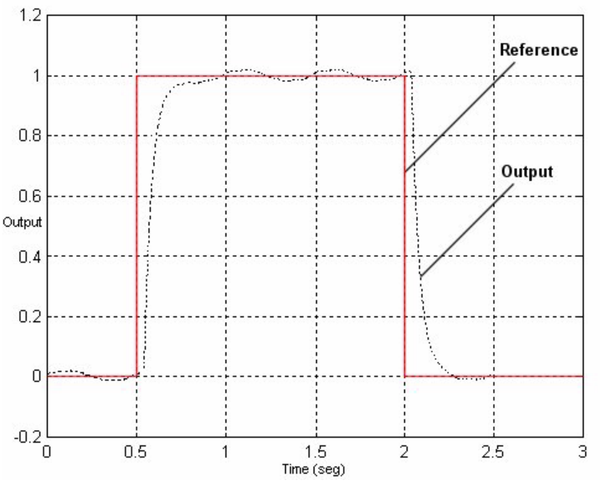

Fig. 3. DC servosystem output behaviour.

results:

$$k_1^* = 8, \quad k_2^* = 0{,}0073, \quad k_3^* = 8$$

$$\sqrt{\sup_{\lambda_{r_i} \in \{\lambda_{r_{1i}}, \lambda_{r_2}, \ldots, \lambda_{r_l}\}} \left(\frac{\lambda_{r_i}^6 + 31{,}0249 \lambda_{r_i}^4}{\alpha(\lambda_{r_i})} \right)} = 0{,}0669 < 0{,}1.$$

The computed solution shows some particularities: k_1^* and k_3^* take the extreme values of theirs corresponding ranges, while k_2^* tends to be equal to zero. Because of the nature of $J_3(k_1, k_2, k_3)$, k_1^* and k_3^* must be small to guarantee a small value of $J_3(k_1, k_2, k_3)$. As far as k_2^* is concerned, the obtained result confirms that integral control is not necessary, since the model of the DC servosystem includes an integral action. It is necessary to say that a big value of the proportional gain gives rise to response overshoots. This undesired behaviour can be corrected injecting damping, i.e., increasing the derivative gain. However, high noise level impose low derivative gains. By these reasons, the **PID** gains were adjusted to the following values:

$$k_1^* = 8, \quad k_2^* = 0{,}0073, \quad k_3^* = 0{,}4. \tag{10}$$

Figure 3 shows the behaviour of the real system corresponding to the **PID** gains. Remark that the disturbance is attenuated, but it is still present on the output.

5 Concluding remarks

Concerning our experimental evaluation, we compare the behaviour obtained when applying the computed controller to the real system with the behaviour obtained when applying the computed controller in a simulated control scheme. Both behaviours are very close, but the simulated control scheme does not present an appreciable effect of the disturbance. It is quite obvious that the remarked difference is due to the nonlinear nature of the real system.

In general, we can conclude that the Genetic Algorithms Approach to solve mixed H_2/H_∞ control problems (considering **PID** controllers) is a good choice when the dynamics of the real plant are close to the dynamics of a linear system. In our study we apply a standard Genetic Algorithm which does not profit from both the nature of the plant and the controller. The nature of the control scheme must be coded in the optimization procedure, e.g., the constraint limiting the value of the derivative gain in our illustrative example (becausF pressence of undesired vibrations acting on the DC motor when the derivative gain takes big values) can be coded in terms of the specified range for the gain. As far as the Genetic Algorithm descriptors are concerned, it seems suitable to proceed to an intensive experimental study concerning the development of evolutionary computing techniques adapted to the intimate knowledge of the nature of the systems to be controlled; descriptors (such like crossover probability and mutation probability) must be obtained from real knowledge.

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