ON μ-ANALYSIS AND SYNTHESIS FOR SYSTEMS SUBJECT TO REAL UNCERTAINTY

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Abstract

The paper introduces a new approach for the computation of a lower bound on the *structured singular value* (SSV), μ in the presence of purely real and mixed/complex uncertainties. The approach utilises a frequency sweeping technique based on a linear fractional transformation representation of structured uncertainty. The technique is applied to a well-known civil transport aircraft example. A fixed structure controller synthesis strategy is developed, which addresses potential stability problems that can occur using standard design methodologies.

Notation

$\mu(M)$	structured singular value (SSV) of M
$\overline{\sigma}(M)$	largest singular value of M
$\mathcal{F}_l(M,\Delta)$	lower Linear Fractional Transformation of M, Δ
$\Re$	real part of a complex number
$\Im$	imaginary part of a complex number
$\mathcal{R}$	field of real numbers
$\mathcal{C}$	field of complex numbers
$\lambda(M)$	eigenvalues of M

1 Introduction

The concept of robustness analysis for systems with structured uncertainties first appeared in 1980 when the so called *excess stability margin*, which later became known as the *multiloop stability margin* was introduced, [14]:

$$k_m = \min_{\Delta\in\mathcal{D}}\{k \in [0,\infty) \ : \ \det(I - k\Delta M) = 0\} \qquad (1)$$

where M represents the value of the transfer function matrix $M(s)$ at $s = \jmath\omega$, $\mathcal{D}$ is the set of all admissible perturbations and Δ is a structured perturbation defined as

$$\begin{aligned}\Delta \ = \ & diag(\delta_1^r I_1, \ldots, \delta_{m_r}^r I_{m_r}, \delta_1^c I_{m_r+1}, \ldots, \delta_{m_c}^c I_{m_r+m_c}, \\ & \Delta_1^C, \ldots, \Delta_{m_C}^C), \ \ \delta_i^r \in \mathcal{R}, \ \delta_j^c \in \mathcal{C}, \ \Delta_k^C \in \mathcal{C}^{p_k \times q_k} \\ & i = 1,\ldots,m_r; \ j = 1,\ldots,m_c; \ k = 1,\ldots,m_C \qquad (2)\end{aligned}$$

In [5], the term *structured singular value* for the reciprocal of k_m was coined and denoted by μ:

$$\mu(M) = \left\{\min_{\Delta\in\mathcal{D}} \overline{\sigma}(\Delta) \ : \ \det(I - M\Delta) = 0\right\}^{-1} \qquad (3)$$

The exact computation of μ is an NP-hard problem, therefore lower and upper bounds are considered in the literature. The upper bounds are defined as convex optimisation problems [11]. MATLAB software, which uses a suitably defined frequency grid to determine an upper bound on μ [1, 8] is known to work quite well, with the possible exception of when the uncertainty consists of purely real blocks, [13]. This is a particular problem for the practical interacting systems where the μ plot exhibits multiple narrow peaks. These peaks can necessitate a prohibitively narrow grid using standard μ-techniques. An upper bound solution to the frequency gridding problem has been presented by Feron [6], whereby stability guarantees, (albeit with fairly mild restrictions), are possible within a prespecified frequency interval. However, the algorithm can potentially yield a conservative upper bound if the frequency interval is quite wide.

All available lower bound computation techniques consist of finding a perturbation which corresponds to the limit of stability. A fixed point *power algorithm* is presented in [11]. Unfortunately, when uncertainties are modelled as real parameter variations this approach does not converge well enough. An improvement can be achieved by adding a small amount of complex uncertainty [12] but this amount needs to be fixed by trial and induces approximation in the results that is difficult to evaluate. Moreover, the solution is suboptimal in the real parameters. An approach for purely real uncertainties is presented in [3], but the method is of exponential time and its practical use is only for small uncertainty sets. An optimisation based approach presented in [9] provides a satisfactory lower bound for large Δ's, but the algorithm is quite sensitive to the choice of initial starting point and recalculations are required at some frequencies.

A different lower bound approach, which features a frequency independent μ computation is considered here. It is has been found that this approach works quite well on a wide variety of practically motivated problems, reducing the gap between the lower and upper bounds on μ to a very small level. A key feature of the approach is that an accurate combination of parameters that results in a destabilising perturbation is returned at each iteration. This "unwrapping" procedure may be used in conjunction with upper bound real μ-solvers like e.g., [6], to inform the frequency interval selection process and therefore also offer an easy way of reducing the potential conservatism on the upper bound for real μ.

The paper is organised as follows: Section 2 is dedicated to the proposed new approach for the computation of a lower bound on real μ. Section 3 demonstrates the use of the new approach as a tool for robustness analysis and controller design. An easily reproducible civil transport aircraft model, presented in [7], is taken as a representative example. Finally, some conclusions and future research directions are given in section 4.

2 A Pole Placement Approach for the Computation of a μ Lower Bound

2.1 Description

Typical strategies for the computation of the structured singular value involve the evaluation of a nominal system $M(s)$ at different frequencies, i.e., a so called *frequency gridding* approach. However for the lightly damped systems that are now appearing in the literature it is quite possible to miss a frequency where one of possibly multiple resonant peaks occur. Inaccurate μ bounds can be observed with this type of problem. Moreover, the number of frequency points and indeed the frequency range necessary to obtain good quality μ bounds can also be difficult to select in advance.

More sophisticated methods that address this problem have been considered in the literature. The migration of a set of closed loop nominal poles through the imaginary axis due to a suitably scaled uncertainty set is considered in [7, 10]. Denoted as *frequency sweeping*, the idea is to find the smallest perturbation $\Delta \in \mathcal{D}$ that will move a pole (or a complex conjugate pair) onto the imaginary axis while all other poles remain in the LHP. In [10], the idea is formulated as a quadratic optimisation under linear constraints. A different formulation of the frequency sweeping technique is presented in this paper.

Let (A, B, C, D) be the nominal state space representation of an uncertain system $M(s)$. Note that B, C and D are sized compatibly with the size of Δ. If the quadruple (A, B, C, D) is perturbed by Δ, the state matrix of the closed loop $\mathcal{F}_l(M(s), \Delta)$ is

$$A_0 = A + B\Delta(I - D\Delta)^{-1}C \qquad (4)$$

Denote the eigenvalues of A_0 by $\lambda_i,\ i = 1, \ldots, n$. The matrix Δ_0 will be called a *destabilising perturbation* if and only if $\lambda_{\max}(A_0) \geq 0$, where

$$\lambda_{\max}(A_0) = \arg\left(\max_i \Re\left\{\lambda_i(A_0)\right\}\right) \qquad (5)$$

The "minimum" perturbation Δ is defined in terms of the largest singular value of Δ, i.e., $\overline{\sigma}(\Delta)$. It can be shown that for a Δ of the structure defined in eqn. (2) with $m_C = 0$ the following equality holds

$$\overline{\sigma}(\Delta) = \|\Delta\|_1 = \|\Delta\|_2 = \|\Delta\|_\infty \qquad (6)$$

For the case of scalar perturbations, μ corresponds to the smallest Δ_0 of appropriate structure that will move an eigenvalue of A onto the imaginary axis. The singularity condition in the μ definition $\det(M(\jmath\omega)\Delta - I) = 0$ corresponds to $\lambda_{\max}(A_0) = 0$. Hence,

$$\mu(M(s)) = \left\{\max_{\omega} \min_{\Delta\in\mathcal{D}} (\overline{\sigma}(\Delta)) : \det(M(\jmath\omega)\Delta - I) = 0\right\}^{-1} \qquad (7)$$

is identical to

$$\mu(M(s)) = \left\{\min_{\Delta\in\mathcal{D}} \|\Delta\|_\infty : \lambda_{\max}(A + B\Delta(I - D\Delta)^{-1}C) = 0\right\}^{-1} \qquad (8)$$

Eqn. (8) is a minimisation with a nonlinear constraint which replaces the minimax problem of eqn. (7). Also eqn. (8) determines a (suboptimal) destabilising Δ and essentially gives the basis of the proposed *pole placement approach* (PPA).

2.2 On the Practical Implementation of the Approach

The μ lower bound algorithm motivated by eqn. (8) can be addressed in *MATLAB* using constraint optimisation software provided by the *Optimisation toolbox*, [2]. To locate the minimisation vector x, it is common for the optimisation algorithms to consider the first two terms of the Taylor approximation of $f(x)$ at a candidate x. This recasts the minimisation to a *sequential quadratic programming* problem:

$$f(x) = \frac{1}{2}x^T H(x)x + x^T g(x)$$

where $H(x)$ is the Hessian and $g(x)$ is the gradient.

It should be emphasised that in order to allow complex parameters to enter the optimisation procedure in this scheme, each complex entry has to be factored into two real optimisation variables. It is clear that for a Δ with $m_c = 0$ and $m_C = 0$, the number of optimisation variables involved in the problem (8) is m_r. Suppose that $m_c \neq 0$. Then each δ_i^c is separated into two real variables, i.e., $\delta_i^c = a_i + \jmath b_i$. Hence δ_i^c will appear in the optimisation code as a_i and b_i. The number of optimisation variables m would then be

$$m = m_r + 2m_c \qquad (9)$$

Eqn. (9) indicates that the algorithm is likely to be inefficient for mixed or complex uncertainty sets.

A major problem in the non-convex optimisation algorithms is the choice of initial conditions. An interesting approach of choosing the starting point for optimisation is employed in this algorithm. For simplicity, the case when Δ is a real perturbation (i.e., $m_c = 0$ and $m_C = 0$) is presented, but this can easily be recasted for mixed and complex Δ's. Consider the partial derivatives

$$\nabla = \left[\frac{\partial \lambda_{max}(A)}{\partial \delta_1^r}, \ldots, \frac{\partial \lambda_{max}(A)}{\partial \delta_{m_r}^r} \right] \tag{10}$$

which are expected to give a rough estimate of how the parameter perturbations δ_i^r impact on the migration of the dominant eigenvalue of A. Hence, an initial starting point can be provided by simply taking $x_0 := \nabla$.

Although potential discontinuities in the optimisation problem of eqn. (8) are known to exist, numerical experience suggests that (10) is a good way to locate an initial starting point. Initialisation and local minima recovery can be implemented using a so called "Tree" test which is motivated by the exponential time Tree Structured Decomposition approach of Degaston & Safonov, [4]. The procedure generates 2^{m_r} sets of Δ's, where each entry δ_i^r takes a value of either -1 or 1. Hence, the Δ that causes largest $\Re\{\lambda_{max}(A_0)\}$ is chosen as initial condition.

It should be noted that in addition to the destabilising Δ_0 unwrapped from the optimisation variables, the critical peak frequency is also extracted:

$$\omega_p = |\Im\{\lambda_{max}(A_0)\}|$$

3 Application: Analysis and Synthesis for a Civil Aircraft

3.1 Description of the Civil Aircraft

The civil aircraft that is considered here is described in detail, (including full non-linear equations of motion) in [7]. The system has 2 inputs and 4 outputs and a linear aerodynamic model is obtained using standard MATLAB trimming techniques. The lateral-axis model has 4 states that depend on 14 stability derivatives. For this example, uncertainty is introduced to all of the stability derivatives at an arbitrarily designated level of 30% of the nominal values. The number of uncertain parameters means that this represents a class of problem that is beyond the point where conventional exact μ-solvers like [3] can be used.

The system has an unstable mode in open loop. A nominally stabilising static output feedback controller K_1 has been synthesised using an approach presented in [15]. Four of 8 entries are fixed at zero thus allowing the remaining 4 to determine the closed loop shape. The static output feedback design is:

$$K_1 = \begin{bmatrix} 2.995 & 0 & 0 & 0 \\ 0 & -2.935 & -80.894 & 30 \end{bmatrix} \tag{11}$$

Standard Linear Fractional Transformation (LFT) tools are used to implement the closed loop design with attached actuators (A_{c1} and A_{c2}) as shown in Figure 1.

$$A_{c1} = \frac{-1.77s + 399}{s^2 + 48.2s + 399}, \quad A_{c2} = \frac{2.6s^2 - 1185s + 27350}{s^3 + 77.7s^2 + 3331s + 27350}$$

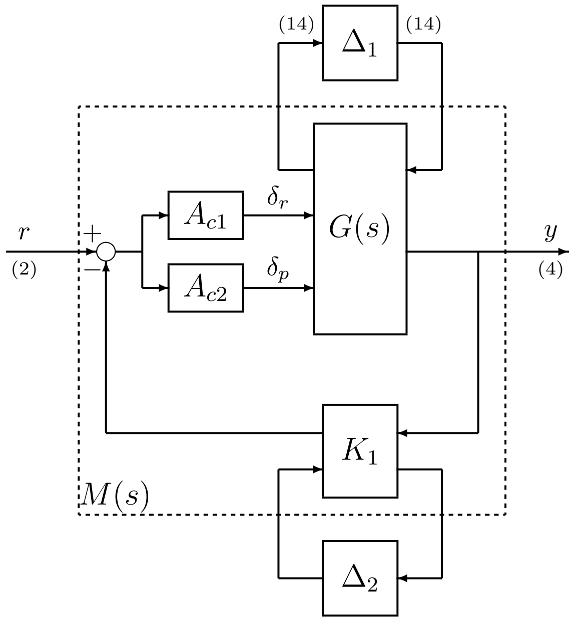

Figure 1: Lateral flight control system for a civil aircraft

Some comments are appropriate about this block diagram. The LFT is set up in such a way that the uncertainty in the stability derivatives is captured by Δ_1. The Δ_2 block acts on the non-zero elements of the controller K_1. Δ_2 will be a zero matrix for the initial analysis problem. Later, Δ_2 will be non-zero when the new algorithm is used for controller synthesis.

3.2 Analysis of Stability Robustness

Robustness analysis for the civil transport aircraft is initially performed on $M(s)$ without the actuators in the problem formulation (i.e., $A_{c1} = 1$ and $A_{c2} = 1$). The following μ algorithms were used:

1. μ_{boa} - a basic optimisation algorithm (BOA) proposed in [9], which computes good lower bounds on real μ.
2. μ_{ppa} - the pole placement approach (PPA) presented in this paper.
3. μ_l, μ_u/μ_u^* - standard μ-toolbox code [1], which normally provides poor lower bounds and potentially inaccurate upper bounds for purely real uncertainty sets. Upper bounds on μ are computed to both default accuracy (option `u`, and denoted by μ_u) and greatest accuracy (option `uC9`, and denoted by μ_u^*), while lower bounds are computed with maximal accuracy (option `ltR9`, denoted by μ_l).

Approaches μ_{boa}, μ_l and μ_u/μ_u^* are used initially for a frequency grid of 300 points in the range $[10^{-2},\ 10^{2}]\ rad/s$. The corresponding μ plots are shown in Figure 2. Table 1 illustrates μ bounds, critical frequencies and computation time for each of the considered approaches.

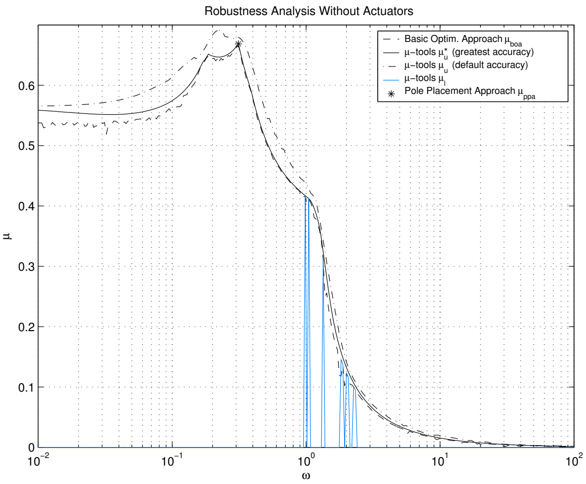

Figure 2: μ-analysis for 30% level of uncertainty

	$\mu(M(\jmath\omega_p))$	ω_p $[rad/s]$	CPU time $[s]$
μ_{boa}	0.6629	0.3055	6360
μ_{ppa}	0.6686	0.3121	20
μ_l	0.4170	0.9847	1437
μ_u	0.6910	0.2245	64
μ_u^*	0.6662	0.3055	549

Table 1: Robustness Analysis for the Civil Transport Aircraft

The following observations can be outlined:

1. The peak lower bound achieved by μ_{ppa} is slightly larger than the one obtained by μ_{boa}. Note that the computation time required for μ_{ppa} is about 20 sec while approach μ_{boa} needs more than an hour and half to evaluate μ for 300 frequency points[1]. However, both approaches found the critical peak at approximately the same frequency ($\omega_p \approx 0.31\ rad/s$). It should be emphasised that the lower bound μ_l is zero for almost all frequency points.

2. Note the conservatism of μ_u and the computational cost associated with μ_u^*. It can be noted that the peak value of μ determined by μ_u^* is in fact smaller than the peak lower bound achieved by μ_{ppa}. Computation of μ_u^* at a frequency of $\omega = 0.3121\ rad/s$ returns a value which is virtually the "same" (to within an accuracy of 4 significant digits) as the one computed by the new algorithm, i.e., $\mu_u^* = \mu_{ppa} = 0.6686$.

3. Consider the returned worst-case destabilising Δ_1's, that are presented in Table 2. It can be noted that the uncertainty entries $\delta_4, \delta_5, \delta_9, \delta_{10}, \delta_{14}$ have no effect on the system stability as indicated by μ_{ppa} and μ_l. Moreover, the nonzero Δ_1 entries obtained by μ_{ppa} have values of either k_m or $-k_m$. This is an intuitively pleasing result bearing in mind the nature of the worst case perturbation that is determined by an exact μ algorithm like for example, that given in [3].

[1]Analysis is carried out on a PC with MATLAB 6.1 benchmark 6.8

Δ_1 entries	μ_{boa}	μ_{ppa}	μ_l
δ_1^r	-1.5042	-1.4956	-2.3982
δ_2^r	-0.9736	-1.4956	-2.3982
δ_3^r	0.9015	1.4955	2.3982
δ_4^r	-1.3747	0	0
δ_5^r	0.1999	0	0
δ_6^r	-1.5085	-1.4956	1.7238
δ_7^r	-1.4949	-1.4956	-2.3982
δ_8^r	1.5075	1.4956	2.3982
δ_9^r	0.6040	0	0
δ_{10}^r	0.7679	0	0
δ_{11}^r	1.5081	1.4956	-2.3982
δ_{12}^r	-1.5048	-1.4955	-2.3982
δ_{13}^r	-1.5060	-1.4956	2.3982
δ_{14}^r	0.5531	0	0

Table 2: Destabilising Δ_1's obtained from the μ lower bounds

3.3 Output Feedback Controller Design using the PPA

For this design example the actuators are now introduced into the problem formulation. Note that the output feedback controller design presented here is significantly different from the standard D-K iteration approach to μ-synthesis. The proposed approach is based on nonlinear unconstrained optimisation[2]. The objective is to perturb the coefficients in the nominal controller K_1 so that the robustness indicator $\mu(M(s))$ is minimised. The objective for controller design is

$$\min_{\Delta_2 \in \mathcal{K}} \left\{ \max_{\omega} \mu(M(\jmath\omega)) \right\} \tag{12}$$

where $\mathcal{K}$ is the set of all suitably structured stabilising controllers for this design. The objective $\max \mu(M(\jmath\omega))$ can be readily addressed using the new μ lower bound algorithm. It should be noted that the objective function in eqn. (12) is nonlinear and therefore the solution K_2 is suboptimal. The nominally stabilising K_1 is determined when $\Delta_2 \equiv 0$, while K_2 is determined by the perturbation Δ_2 that satisfies eqn. (12):

$$\Delta_2 = \arg \left\{ \min_{\Delta_2 \in \mathcal{K}} \mu_{ppa}(M(s)) \right\} \tag{13}$$

Robustness analysis is carried out for the closed loop system with controller K_1 using the same μ approaches as in the previous subsection. Results from the analysis are presented in Table 3. Figures 3 and 4 depict the bounds on μ for 200 and 600 frequency points, respectively. As the μ-tools lower bound (μ_l) has shown to be very poor it is omitted from the figures. The computed lower bound $\mu_{ppa} = 1.4256$ was found at frequency of $\omega_p = 11.0632\ rad/s$. In order to confirm the reliability of

[2]The Simplex method of Nelder-Mead, [2] is used by the authors

this indicator, a comparison with analysis results achieved from frequency-grid based approaches is necessary.

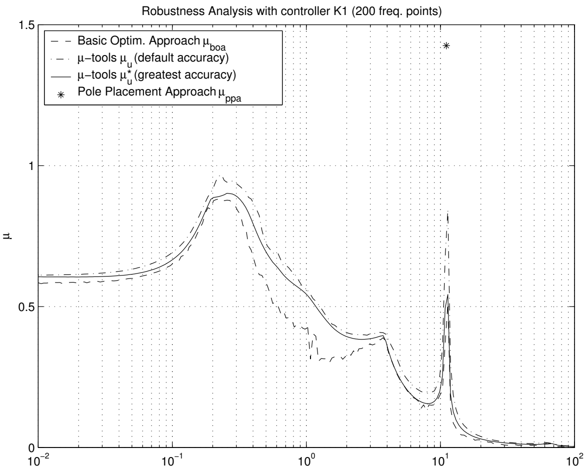

Figure 3: Robustness analysis with K_1 (200 pts.)

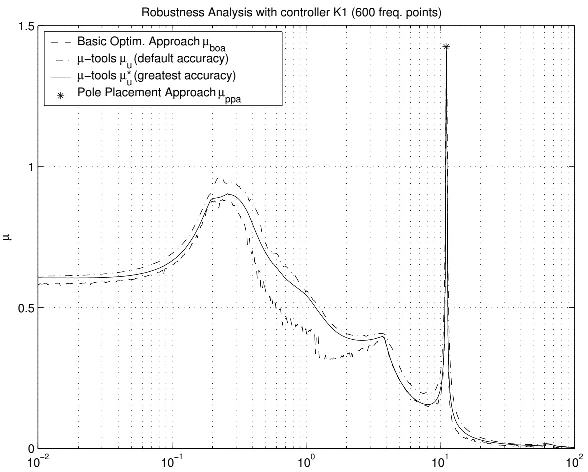

Figure 4: Robustness analysis with K_1 (600 pts.)

	200/**600** frequency points		
	$\mu(M(\jmath\omega_p))$	$\omega_p\ [rad/s]$	CPU time $[s]$
μ_{boa}	0.8846/**1.4079**	0.2327/**11.0937**	4698/**11972**
μ_{ppa}	1.4256	11.0632	28
μ_u	0.9675/**1.4306**	0.2327/**11.0937**	45/**138**
μ_u^*	0.9018/**1.4108**	0.2552/**11.0937**	465/**1291**

Table 3: Robustness indicators with K_1

First, consider the frequency grid of 200 points. Note that two resonant peaks can now be seen in the μ-plots. The second peak at approximately $10\ rad/s$ is particularly narrow and is the focus of our observations. Both μ upper bound plots (representing μ_u and μ_u^*) reach their maximum peaks around the first resonant peak. Note that these upper bounds are significantly *smaller* than $\mu_{ppa}(M(s))$.

The lower bound determined by the pole placement algorithm clearly indicates the need for a much finer frequency grid. Increasing the number of frequency points from 200 to 600 allows the $\mu_u(M(s))$ algorithms to catch the more significant second resonant peak, as shown in Figure 4. Note that the peak value for both μ_u and μ_u^* is now $\omega_p = 11.0937\ rad/s$, which is much closer to the critical frequency determined by μ_{ppa} ($11.0632\ rad/s$). Nevertheless, the accurate upper bound solution μ_u^* is still smaller than the lower bound μ_{ppa}. This suggests that an even more narrow frequency band has to be considered. Computing μ at the critical frequency associated with μ_{ppa} now returns larger values for both μ_u and μ_u^*: 1.4416 and 1.4272, respectively. The latter is larger than μ_{ppa} and the gap between them is negligible: $1.4272 - 1.4256 = 0.0016$. The fact that the closed-loop system with K_1 is obviously not robustly stable suggests that a new controller needs to be designed.

Applying the proposed synthesis algorithm formulated by eqn. (12) determines a new output feedback controller:

$$K_2 = \begin{bmatrix} 2.494 & 0 & 0 & 0 \\ 0 & -31.637 & -70.352 & 19.632 \end{bmatrix} \qquad (14)$$

K_2 achieves a robustness indicator of $\mu_{ppa} = 0.7468$ ($\omega_p = 0.1505\ rad/s$). To verify this design, μ analysis is carried out using a narrow frequency grid of 600 points. Analysis results are illustrated in Figure 5 and Table 4.

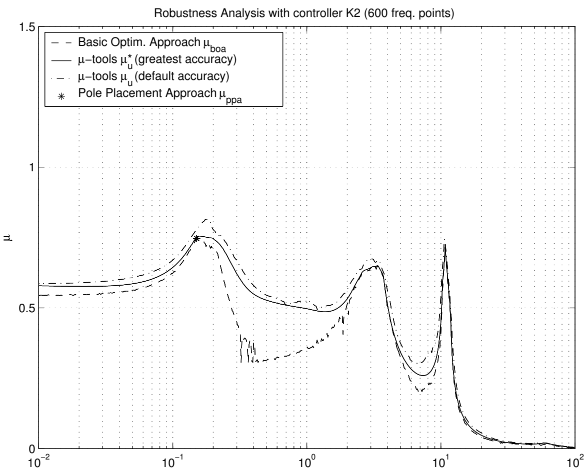

Figure 5: Robustness analysis with K_2

	$\mu(M(\jmath\omega_p))$	$\omega_p\ [rad/s]$	CPU time $[s]$
μ_{boa}	0.7446	0.1520	9287
μ_{ppa}	0.7468	0.1506	24
μ_u	0.8153	0.1801	141
μ_u^*	0.7550	0.1592	1298

Table 4: Robustness indicators with K_2

Note that the gap between the peak upper μ_u^* and lower μ_{ppa} bounds is insignificant: $0.75497 - 0.7468 = 0.0082$. The synthesised controller K_2 ensures a closed loop with multivariable stability margin of $k_m \approx 1.33$. This is a significant increase in robustness when compared with the performance of K_1. Typical step responses are presented in Figure 6. This figure illustrates roll rate in response to a step demand on aileron deflection for this aircraft. Note that the combination of stability derivatives that will cause μ peak to occur for design K_1 will be different to that for design K_2. Both combinations are easily unwrapped using the new algorithm and the δ_i's are constrained to be within $[-1,\ 1]$. It is interesting to note that controller K_2 offers better performance for both Δ_1 combinations.

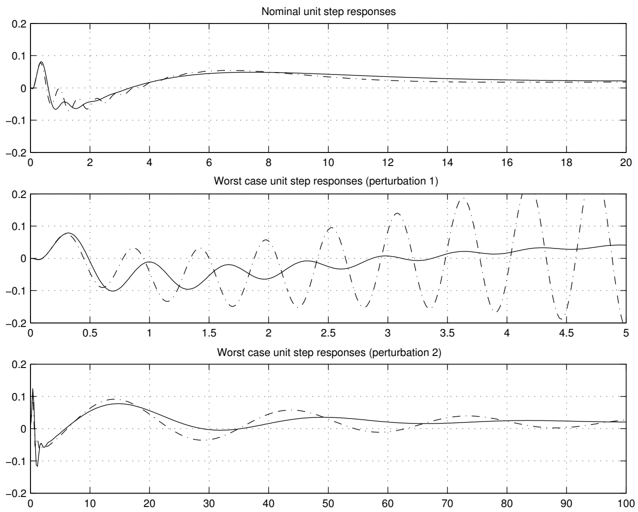

Figure 6: Step responses with K_1 (dashed), K_2 (solid)

4 Conclusions

This paper has considered the computation of a good lower bound on μ for systems that are subject to strictly real parameter uncertainty. A new algorithm that improves on crude lower bound optimisation approaches has been presented. The algorithm works very well, pointing out potential difficulties in frequency grid selection for conventional upper bound based μ-analysis. The new algorithm has been shown to be superior to any of the currently available tools that compute strictly real destabilising perturbations. A fixed structure synthesis algorithm has also been introduced. The algorithm is easy to use and works well on a representative practical example. Research is ongoing on the development of *a priori* tests that can indicate whether lower bound μ algorithms are likely to exhibit convergence difficulties.

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