DOES DIRECT ADAPTIVE CONTROL HAVE A FUTURE?

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Abstract

In the context of discrete-time adaptive control of plants subject to parameter variations, direct adaptive control (despite various attempts: model reference control, minimum variance self-tuning) cannot effectively be used in practice because the discrete-time plant models feature generically unstable zeros.

There is however another extremely important area for application of adaptive control: rejection of unknown disturbances. The paper shows that in this case it is possible to develop direct adaptive control schemes, which are very effective in practice. These schemes perform better and they are simpler than the indirect adaptive control schemes. This is illustrated by their application to adaptive rejection of unknown narrow band disturbances in an active suspension system.

1 Introduction

Most of the literature dedicated to adaptive control is focused toward the problem of high performance control of plants subject to (significant) plant parameters variations (or uncertainty).

Discrete time solutions have been proposed for this problem (for a state of the art see [1, 4]). Unfortunately the discrete time models of the plant feature generically unstable zeros. For this reason, direct adaptive control schemes (model reference adaptive control, minimum variance stochastic self-tuning controllers), which assume that the plant zeros are stable, have a very limited use in practice (in the reported applications a careful selection of the sampling period was necessary in order to avoid unstable zeros in the plant model). The standard solution for adaptive control in practice is to use an indirect adaptive control scheme (combining real-time estimation of the plant model with on-line redesign of the controller).

The combination system identification and robust control allows now to tune a robust controller which in many cases assures a satisfactory performance in practical situations where the parametric variations are limited.

However, even in this very realistic practical context, another problem can become very important, namely the rejection of unknown disturbances.

It appears (based on an extensive review of the literature) that solutions for this important practical problem have been proposed by the signal processing community and many applications are reported. However these solutions (inspired by Widrow's technique for adaptive noise cancellation [10]) ignore the "internal model principle" and require an additional transducer. The principle of this "signal processing solution " for adaptive rejection of unknown disturbances is illustrated in figure 1. The basic idea is that a "well located " transducer can provide a measurement, highly correlated with the unknown disturbance. This information is applied to the control input of the plant through an adaptive filter (in general a FIR) whose parameters are adapted such that the effect of the disturbance upon the output is minimized. The disadvantages of this approach are:(1)It requires the use of an additional transducer. (2)Difficult choice for the location of this transducer (it is probably the main disadvantage.(3)It requires adaptation of many parameters.

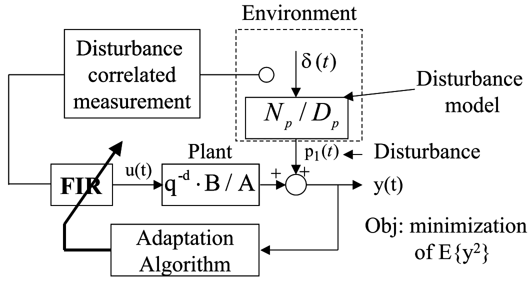

Figure 1: Signal processing approach to rejection of unknown disturbances

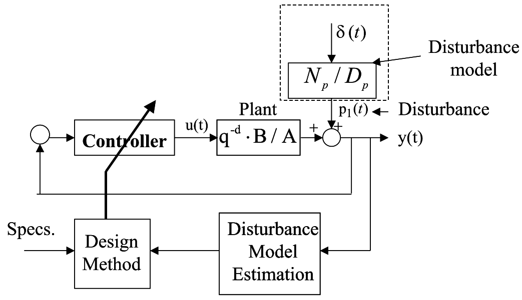

Figure 2: Indirect adaptive control scheme for rejection of unknown disturbances

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To achieve the rejection of the disturbances (at least asymptotically) without measuring it, the controller should incorporate the model of the disturbance (the internal model principle [3]). Therefore the rejection of unknown disturbances raises the problem of adapting the internal model of the controller and its re-design in real-time. The natural way for solving this problem is to try to estimate in real time the model of the disturbance and re-compute the controller, which will incorporate the estimated model of the disturbance (as a pre-specified element of the controller). This will lead to an indirect adaptive control scheme. The principle of such a scheme is illustrated in figure 2. The estimation of the disturbance model can be done by using standard parameter estimation algorithms (see for example [6, 7]. The time consuming part is the redesign of the controller at each sampling time. The reason is that in many applications the plant model can be of very high dimension and despite that this model is constant, one has to re-compute the controller because a new internal model should be considered. For details on an indirect adaptive control scheme for disturbance rejection see [2].

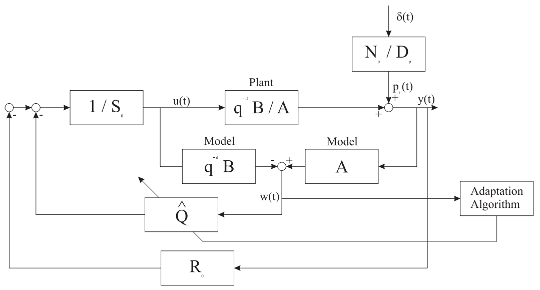

Figure 3: Direct adaptive control scheme for rejection of unknown disturbances

However, by considering the Youla-Kucera parameterization of the controller (known also as the Q-parameterization) (see figure 3) it is possible to insert and adjust the internal model in the controller by adjusting the parameters of the Q polynomial. It comes out that in the presence of unknown disturbances it is possible to build a direct adaptive control scheme where directly the parameters of Q are adapted in order to have the desired internal model without recomputing the controller (polynomials R_0 and S_0 in figure 3 remain unchanged). The number of the controller parameters to be directly adapted is roughly equal to the number of parameters of the disturbance model. In other words the size of the adaptation algorithm will depend upon the complexity of the disturbance model.

The objective of the paper is to show that indeed a direct adaptive control scheme can be developed and implemented and this scheme is simpler and provides better performance than an indirect adaptive control scheme.

The paper is organized as follows. The formulation of the problem and basic notations are given in Section 2. In Section 3, using the Youla-Kucera parameterization and the output sensitivity function a direct adaptive control scheme will be developed. Section 4 will present experimental results obtained on an active suspension using the direct adaptive control scheme introduced in Section 3 and which will be compared with those obtained with an indirect adaptive control scheme. The results presented in this paper go beyond those given in [2] where only the case of auto-tuning of the controller has been considered. Section 5 will present some conclusions.

2 Problem Formulation and Basic Notations

The structure of a linear time invariant discrete time model of the plant (on which is based the design of the controller) is

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})},$$

where:

$$\begin{aligned} d &= \text{the plant pure time delay in number of sampling periods;} \\ A &= 1 + a_1 z^{-1} + \ldots + a_{n_A} z^{-n_A}; \\ B &= b_1 z^{-1} + \ldots + b_{n_B} z^{-n_B}. \end{aligned}$$

$A(z^{-1})$, $B(z^{-1})$ are polynomials in the complex variable z^{-1} and n_A, n_B represent their orders [1].

The controller to be designed is a RS-type polynomial controller (see fig. 4).

The output of the plant $y(t)$ and the input $u(t)$ may be written as:

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p_1(t); \quad (1)$$

$$S(q^{-1}) \cdot u(t) = -R(q^{-1}) \cdot y(t), \quad (2)$$

where q^{-1} is the delay (shift) operator, $p_1(t)$ is the resulting additive disturbance on the output of the system and $R(z^{-1})$, $S(z^{-1})$ are polynomials in z^{-1} having the orders n_R, n_S, with the following expressions:

$$\begin{aligned} R(z^{-1}) &= r_0 + r_1 z^{-1} + \ldots + r_{n_R} z^{-n_R}; \\ S(z^{-1}) &= 1 + s_1 z^{-1} + \ldots + s_{n_S} z^{-n_S}. \end{aligned}$$

Using the equations (1) and (2), we can write the output of the

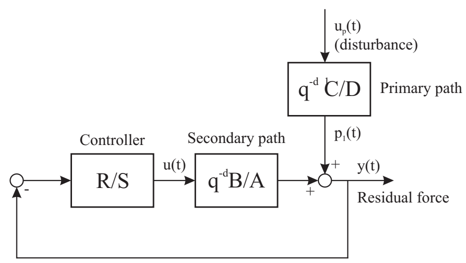

Figure 4: Block diagram of the active suspension system

system as:

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p_1(t); \quad (3)$$

[1]The complex variable z^{-1} will be used for characterizing the system behaviour in the frequency domain and the delay operator q^{-1} will be used for describing the system behaviour in the time domain.

where

$$P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \quad (4)$$

represent the poles of the closed loop.

We define the output sensitivity function (the transfer function between the disturbance $p_1(t)$ and the output of the system $y(t)$):

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{P(z^{-1})}.$$

This sensitivity function allows to describe the performances of the system with respect to disturbances and to specify certain parts of $S(z^{-1})$ in order to obtain a satisfactory disturbance rejection.

The polynomials $R(z^{-1})$ and $S(z^{-1})$ are expressed as:

$$\begin{aligned} R(z^{-1}) &= R'(z^{-1}) \cdot H_R(z^{-1}); \\ S(z^{-1}) &= S'(z^{-1}) \cdot H_S(z^{-1}), \end{aligned} \quad (5)$$

where H_R and H_S are fixed parts of the controller [5].

Suppose that $p_1(t)$ is a deterministic disturbance, so it can be written as

$$p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t), \quad (6)$$

where $\delta(t)$ is a Dirac impulsion. The effect of the disturbance $p_1(t)$ on the output $y(t)$ is given by (3):

$$y(t) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t). \quad (7)$$

We are interested in the rejection of narrow band disturbances. In this case the energy of the disturbance is essentially represented by D_p which will be characterized by a second order polynomial with roots on the unit circle (or very close); the contribution of the terms of N_p is weak compared to the effect of D_p, so we can consider that the effect of N_p is negligible.

In order to apply the Internal Model Principle (to introduce the disturbance model into the controller) we shall consider $H_S(z^{-1}) = D_p(z^{-1})$. For details on the Internal Model Principle see [3].

3 Direct Adaptive Control

Consider a nominal controller (without the internal model of the disturbance) who verifies the imposed robustness constraints and let use the Q-parameterization to define the family of all stabilizing controllers who explicitly take into account the disturbance acting on the system (see [8, 9]). The use of the Q-parameterization introduces a supplementary degree of freedom into the controller, who allows to treat separately the problem of disturbance rejection. Afterwards we shall use a RS-type controller with Q-parameterization.

In the RS structure, the placement of the closed loop poles is obtained by solving a diophantine equation. If the plant and the disturbance models are known with a good precision, the choice of Q for the disturbance rejection may be done using a second diophantine equation (see [9]).

3.1 Q-parameterization

The Q-parameterization for disturbance rejection has been explicitely proposed by Y. Z. Tsypkin in [9]. Let $[R_0(z^{-1}), S_0(z^{-1})]$ be the nominal controller (without internal model of the disturbance), verifying the diophantine equation (4) and satisfying the imposed robustness constraints. The control law is

$$S_0(q^{-1}) \cdot u(t) = -R_0(q^{-1}) \cdot y(t).$$

Using the Q-parameterization (also known as Youla-Kucera parameterization) the RS polynomial controller is defined as:

$$R(z^{-1}) = R_0(z^{-1}) + A(z^{-1})Q(z^{-1}); \quad (8)$$

$$S(z^{-1}) = S_0(z^{-1}) - z^{-d}B(z^{-1})Q(z^{-1}). \quad (9)$$

We obtain the family of all stabilizing controllers $[R(z^{-1}), S(z^{-1})]$, where $Q(z^{-1})$ is a polynomial of degree n_Q. $[R(z^{-1}), S(z^{-1})]$ corresponds to the general solution of the diophantine equation (4). The polynomial $Q(z^{-1})$ will be computed such as the equivalent controller $R(z^{-1})/S(z^{-1})$ contains the internal model of the disturbance.

The controller equation becomes

$$S_0(q^{-1}) \cdot u(t) = -R_0(q^{-1}) \cdot y(t) - Q(q^{-1}) \cdot w(t), \quad (10)$$

where

$$w(t) = A(q^{-1}) \cdot y(t) - q^{-d} \cdot B(q^{-1}) \cdot u(t). \quad (11)$$

The closed loop, with the Q-parameterized controller, is presented in the figure 3.

If we take into account the equations (8) and (9), the equation (4) defining the closed loop poles remains unchanged.

Consider the disturbance $p_1(t)$ of the form (6). With the controller parameterized as in (8) and (9), the disturbance effect on the output of the system, (7), becomes:

$$y(t) = \frac{S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})}{P(q^{-1})} \cdot w(t), \quad (12)$$

where $w(t) = \dfrac{A(q^{-1})N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t)$ is given in (11).

The output sensitivity function is, in this case:

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})[S_0(z^{-1}) - z^{-d}B(z^{-1})Q(z^{-1})]}{P(z^{-1})}.$$

In order to reject asymptotically the effect of the disturbance $p_1(t)$ on the output of the system, $Q(z^{-1})$ must be chosen such as $S_0(z^{-1}) - z^{-d}B(z^{-1})Q(z^{-1}) = M(z^{-1})D_p(z^{-1})$, which is equivalent to solve the diophantine equation

$$M(z^{-1})D_p(z^{-1}) + z^{-d}B(z^{-1})Q(z^{-1}) = S_0(z^{-1}), \quad (13)$$

where

- $D_p(z^{-1})$, d, $B(z^{-1})$ et $S_0(z^{-1})$ are known;
- $M(z^{-1})$ et $Q(z^{-1})$ are unknown.

Hence we compensate the poles of the disturbance model, $D_p(z^{-1})$.

Equation (13) has a unique solution for $M(z^{-1})$ and $Q(z^{-1})$ if

$$\begin{aligned} n_{S_0} &\leq n_{D_p} + n_B + d + 1; \\ n_M &= n_B + d - 1; \\ n_Q &= n_{D_p} - 1, \end{aligned}$$

in which n_{S_0}, n_{D_p}, n_M and n_Q are the orders of the polynomials S_0, D_p, M and Q.

3.2 Known Parameters Case

In the case when the parameters of $D_p(z^{-1})$ are known, we compute $Q(z^{-1})$ by solving the diophantine equation (13), the controller being obtained using the relations (8) and (9).

3.3 Unknown Parameters Case

The objective is to minimize $y(t)$ in the sense of a certain criterion.

By defining $\varepsilon(t) = y(t)$ we obtain from (12):

$$\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d}B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t). \quad (14)$$

Suppose that the structure of $Q(z^{-1})$ is known, defined as a function of the nature of the disturbance. Define $\hat{Q}(t, z^{-1})$ as the estimation of the unknown $Q(z^{-1})$ at instant t.

From equation (14) we obtain the *a priori* and *a posteriori* errors:

$$\varepsilon^0(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \hat{Q}(t, q^{-1}) \cdot w(t) \quad (15)$$

and

$$\varepsilon(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \hat{Q}(t+1, q^{-1}) \cdot w(t). \quad (16)$$

Replacing $S_0(q^{-1})$ from the last equation by (13) we obtain

$$\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1, q^{-1})] \cdot \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t+1), \quad (17)$$

where

$$v(t) = \frac{M(q^{-1})D_p(q^{-1})}{P(q^{-1})} \cdot w(t) = \frac{M(q^{-1})A(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \delta(t)$$

is a disturbance who tends asymptotically towards zero.

Consider $\hat{Q}(t, q^{-1}) = \hat{q}_0(t) + \hat{q}_1(t)q^{-1} + \ldots + \hat{q}_{n_Q}(t)q^{-n_Q}$ and let $\hat{\theta}(t) = [\hat{q}_0(t)\, \hat{q}_1(t) \ldots \hat{q}_{n_Q}(t)]^T$ be the parameters vector; note $w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t)$ and consider $\phi^T(t) = [w_2(t) \quad w_2(t-1) \ldots w_2(t-n_Q)]$. Equation (17) becomes

$$\varepsilon(t+1) = [\theta^T - \hat{\theta}^T(t+1)] \cdot \phi(t) + v(t+1). \quad (18)$$

We remark that $\varepsilon(t)$ has the structure of an adaptation error [4].

From equation (15) we obtain the *a priori* adaptation error:

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t),$$

with

$$\begin{aligned} w_1(t+1) &= \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1)\,; && (19) \\ w_2(t) &= \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t)\,; && (20) \\ w(t+1) &= A(q^{-1}) \cdot y(t+1) - q^{-d}B^*(q^{-1}) \cdot u(t), \end{aligned}$$

where $B(q^{-1})u(t+1) = B^*(q^{-1})u(t)$.

The *a posteriori* adaptation error is obtained from (16):

$$\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t).$$

For the estimation of the parameters of $\hat{Q}(t, q^{-1})$ we use the following parametric adaptation algorithm [4]:

$$\begin{aligned} \hat{\theta}(t+1) &= \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1)\,; && (21) \\ \varepsilon(t+1) &= \frac{\varepsilon^0(t+1)}{1 + \phi^T(t)F(t)\phi(t)}\,; && (22) \\ \varepsilon^0(t+1) &= w_1(t+1) - \hat{\theta}^T(t)\phi(t)\,; && (23) \\ F(t+1) &= \frac{1}{\lambda_1(t)} \left[F(t) - \frac{F(t)\phi(t)\phi^T(t)F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \phi^T(t)F(t)\phi(t)} \right] && (24) \end{aligned}$$

$\lambda_1(t)$ and $\lambda_2(t)$ allow to adjust the adaptation speed (for details see [4]).

In order to implement this methodology (for the $p_1(t)$ disturbance rejection (see figure **??**)), we suppose that the plant model $\frac{z^{-d}B(z^{-1})}{A(z^{-1})}$ is known (identified) and that it exists a controller $[R_0(z^{-1}), S_0(z^{-1})]$ who verifies the desired specifications in absence of the disturbance. We also suppose that the degree n_Q of the polynomial $Q(z^{-1})$ is fixed, $n_Q = n_{D_p} - 1$, when the structure of the disturbance is known.

4 Experimental Results

4.1 The Active Suspension

The structure of the system (the active suspension) that we use in this paper is presented in figure 5. The controller will act upon the piston (through a power amplifier) in order to reduce

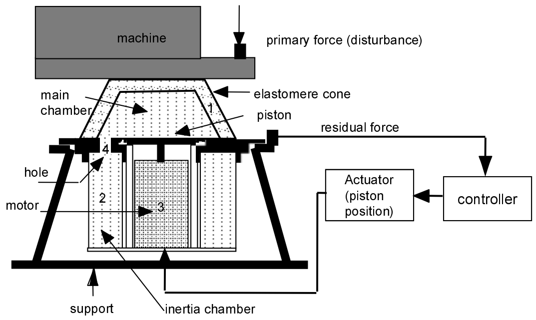

Figure 5: Active suspension system (scheme)

the residual force. The sampling frequency is $800Hz$. The equivalent scheme has been presented in figure 4.

The system input, $u(t)$ is the position of the piston, the output $y(t)$ being the residual force measured by a force sensor.In our case (for testing purposes), the primary force is generated by a shaker controlled by a signal u_p given by the computer.

The transfer function ($q^{-d_1}\dfrac{C}{D}$), between the signal sent to the shaker, u_p, and the residual force $y(t)$ is called primary path. The transfer function ($q^{-d}\dfrac{B}{A}$) between the input of the system, $u(t)$, and the residual force is called secondary path.

4.2 Real-Time Results

The narrow band disturbance rejection procedure using the direct adaptive methodology proposed in section 3 is illustrated in real time for the case of the control of the active suspension presented previously. The results will be compared with those obtained using an indirect adaptive method, presented in [2]. In our case the disturbance will be a time-varying frequency sinusoid. The goal of the control is to reject the effect of this disturbance on the output of the system, $y(t)$, by adapting the controller parameters as a function of the disturbance frequency.

The frequency characteristic of the identified primary path model, between the signal u_p sent to the shaker and the residual force $y(t)$, is presented in figure 6. The first vibration mode of the primary path model is near $32Hz$.

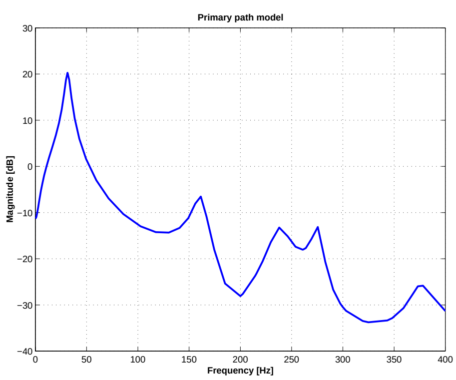

Figure 6: Frequency characteristic of the primary path model

The identified secondary path model (closed loop identification) has the following complexity: $n_B = 14$, $n_A = 16$, $d = 0$. There exist several vibration modes on the secondary path, the first one being at $31.8Hz$ with a damping factor 0.07. The system contains also a double differentiator.

The nominal controller (without the internal model of the disturbance) has been designed using the pole placement method and the secondary path identified model. A pair of dominant poles has been fixed at the frequency of the first vibration mode ($31.8Hz$), with a damping $\xi = 0.8$, and we considered as fixed auxiliary poles the other poles of the model. In addition a fixed part $H_R = 1 + q^{-1}$ ($R = H_R R'$) which assures the opening of the loop at $0.5f_s$ and 10 auxiliary poles at 0.7 have been introduced into the controller. The resulting nominal controller has the following complexity: $n_R = 14$, $n_S = 16$ and it satisfies the imposed robustness constraints in low frequencies.

In order to evaluate the performances of the direct and indirect adaptive methods in real time, we use time-varying frequency sinusoidal disturbances. For the implementation of the adaptive loop we use the parameters of the secondary path identified model. We consider sinusoidal disturbances having the frequencies between 25 and $47Hz$, the first vibration mode of the primary path being near $32Hz$.

The tests done with the two methods (indirect and direct adaptive control) have been as follows:

1. Start up: The system is started in open-loop. After 5 seconds (4000 samples) a sinusoidal disturbance of $32Hz$ is applied on the shaker. The model of the disturbance is estimated and an initial controller is computed (the same initial controller for both direct and indirect adaptive control).

2. Adaptive operation: Once a first controller is applied, the adaptation algorithms work permanently and the controller is recomputed (in the indirect approach) or updated (in the direct approach) at each sampling instant. A sequence of sinusoidal disturbances is then applied starting at 12000 samples (15 seconds).

The measured residual force obtained is presented in figures 7 (direct adaptation method) respectively 8 (indirect adaptation method). In the case of the indirect algorithm we note a bias of the parameters.

We note a faster convergence of the direct algorithm compared to the indirect one. From the point of view of performances, the direct algorithm is better than the indirect one. The direct algorithm has, moreover,a simpler structure than the indirect one.

Let consider now that the frequency of the sinusoidal disturbance varies continuously and let use a chirp disturbance signal (linear swept-frequency signal) between 25 and $47Hz$. We present the results obtained in real-time on the active suspension system using the direct adaptive control method, the tests being done as follows: Start-up in closed-loop. After 5 seconds

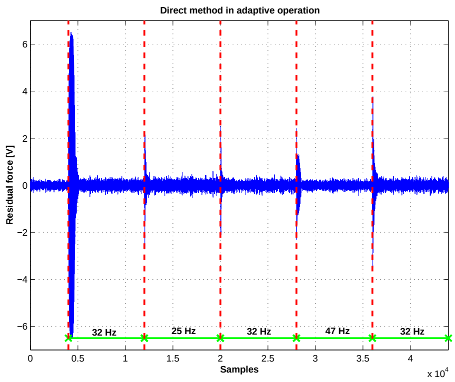

Figure 7: Time domain real time results with the direct adaptation method

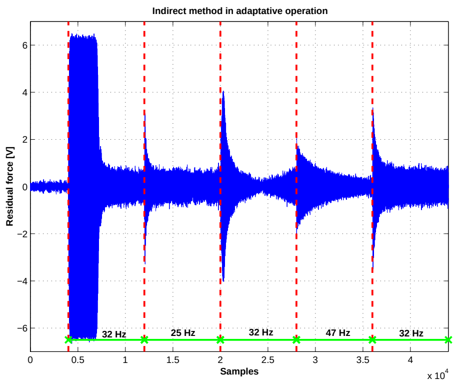

Figure 8: Time domain real time results with the indirect adaptation method

(4000 samples) a sinusoidal disturbance of $25Hz$ (constant frequency) is applied on the shaker. Once the controller is applied, the adaptation algorithm works permanently and the controller is updated (direct approach) at each sampling instant. From 10 to 15 seconds we apply a chirp between 25 and $47Hz$ (linear-swept frequency). After 15 seconds we apply a $47Hz$ (constant frequency) sinusoidal disturbance and we stop the tests after 20 seconds (16000 samples). The time-domain results obtained in open and in closed-loop are presented in figure 9, where we represented the residual force as a function of time. We can remark that the performances obtained are very good.

5 Conclusions

It has been shown in this paper that indeed direct adaptive control has a very promising feature for adaptive rejection of unknown disturbances. However a number of theoretical issues remain to be studied. In particular, despite good simulation and real time results, the robustness of the performance with respect to variations of the plant model (which are not taken into account by the adaptation loop) should be studied in detail.

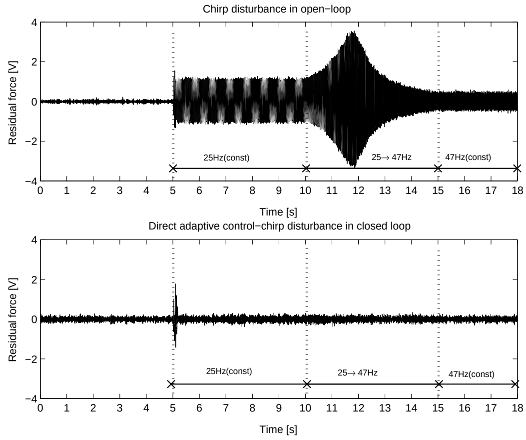

Figure 9: Real-time results obtained with the direct adaptive method and a chirp disturbance: (a) Open-loop; (b) Closed-loop

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