# DECENTRALIZED NONLINEAR CONTROLLER DESIGN FOR MULTIMACHINE POWER SYSTEMS VIA BACKSTEPPING

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# Abstract

In this paper, a backstepping-based decentralized control scheme is proposed for transient stability enhancement of multi-machine power systems. The design is based on two stages: First, an equivalent single-machine infinite-bus model (SMIB) is developed with respect to each machine. Then, apart from each SMIB model, a decentralized nonlinear control scheme based on a backstepping technique is designed which guarantees asymptotic stability of the overall interconnected power system. Some simulation results demonstrate the effectiveness of this approach.

#### **1** Introduction

Power systems are increasingly brought to operate at high power transmission levels for some economical or environmental reasons, such as deregulation of the energy market. This requires the control system to have the ability to compensate potential instabilities and poorly damped power angle oscillations, as networks load is expected to increase in the future. In lot of cases, transient stability limits are more constraining than the steady-state limits. This requires the control system to have the ability to regulate the system under diverse operating conditions. Unfortunately, power systems are some very nonlinear systems; the behavior of conventional linear controllers, such as power system stabilizers, that are designed on the basis of some linearized power system models ("small signal models") is significantly affected by changes in operating conditions. In this paper, we will focus our attention on the transient stability enhancement of multimachine power system by means of a backstepping control design. This paper presents an extension to the multi-machine case of the backstepping controller designed for SMIB systems proposed in [2].

A great deal of attention has been paid to the application of linear control theory to power systems [11], [6], [1]. However, power system stabilizer design based on some linearized models is not adequate in presence of large disturbances: when a

large fault occurs, a linear controller may not preserve stability. Recently, to overcome this problem, several authors (see for example, [7], [8], [4]) have applied nonlinear control theory. Most of these nonlinear controller designs for power systems are based on differential geometry approach. The so-called Direct Feedback Linearization (DFL) approach was applied to design a voltage regulator [9], [10].

The paper is devoted to the design of a new nonlinear controller for a multi-machine power system, by using a backstepping approach. Our goal is to improve transient stability of the overall power system under the effect of a symmetrical three phase short circuit fault. With the backstepping methodology, the design of both feedback control laws and associated Lyapunov functions is systematic. Strong properties of global or regional stability are built into the nonlinear system in a fixed number of steps, which is never higher than the system order. While feedback linearization methods require precise models and often cancel some useful nonlinearities, backstepping designs offer a choice of design tools for accommodation of uncertain nonlinearities [3].

This paper is organized as follows: Section 2 describes the nonlinear dynamics of a multi-machine power systems. In section 3, we consider the design of a decentralized nonlinear control scheme for a multi-machine power system based on both equivalent circuit (SMIB model) design and the use of backstepping. Section 4 presents some simulation results and comparisons. Finally section 5 sums up some conclusions.

# 2 Nonlinear dynamics of multi-machine power systems

The classical model of a synchronous machine may be used to study the dynamics of power systems when the system dynamics largely dependent on the stored kinetic energy of the rotating masses.

Mechanical equations of each generator *i* 

$$\dot{\delta}_i(t) = \omega_i(t) \tag{1}$$

$$\dot{\omega_i}(t) = -\frac{D_i}{2H_i}\omega_i(t) + \frac{\omega_0}{2H_i}(P_{mi}(t) - P_{ei}(t)) \quad (2)$$

with

- $\delta_i$  power angle of machine *i*;
- $\omega_i$  relative speed of machine *i*;
- $P_{mi}$  mechanical input power of machine i;
- $P_{ei}$  electrical output power of machine *i*;
- $\omega_0$  synchronous machine speed;
- $D_i$  per-unit damping contant of machine *i*;
- $H_i$  inertia constant(in sec) of machine *i*;

#### Electrical equations of each generator *i*

$$\dot{E'_{qi}}(t) = \frac{-1}{T'_{doi}} \{ E'_{qi} + (x_{di} - x'_{di}) I_{di} - k_{ci} U_{fi} \}$$
(3)

$$\dot{E'_{di}}(t) = \frac{-1}{T'_{qoi}} \{ E'_{di} - (x_{qi} - x'_{qi})I_{qi} \}$$
(4)

with

- $E'_{qi}$  transient EMF in quadrature axis of machine *i*;
- $E_{di}^{i}$  transient EMF in direct quadrature axis of machine *i*;
- $I_{qi}^{i}$  quadrature component armature current of machine *i*;
- $I_{qi}$  direct component armature current of machine *i*;
- $T'_{d0i}$  direct axis transient short-circuit time constant of machine *i*:
- $T'_{qoi}$  quadrature axis transient short-circuit time constant of machine *i*;
- $u_{fi}$  input of the SCR amplifier of the generator of machine *i*;
- $k_{ci}$  gain of excitation amplifier of machine *i*;

#### Additional electrical equations

$$P_{ei} = E'_{di}I_{di} + E'_{ai}I_{qi} \tag{5}$$

$$V_{di} = E'_{di} + x'_{qi}I_{qi} - r_{ai}I_{di}$$
(6)

$$V_{qi} = E'_{qi} - x'_{di}I_{di} - r_{ai}I_{qi}$$
(7)

$$V_{ti} = \sqrt{V_{di}^2 + V_{qi}^2}$$
 (8)

with

- $V_{ti}$  generator terminal voltage of machine *i*;
- $V_{qi}$  quadrature component of  $v_t$  of machine i;
- $V_{di}$  direct component of  $v_t$  of machine *i*;

In order to study the transient stability of a multi-machine power system according to the Park model, it is necessary to derive the d-q components of each generator expressed in the coordinate frame d-q. Therefore, for each direct and quadrature component of line *i* current, we get:

$$I_{di} = G_{ii}E'_{di} + \sum_{i \neq k, k=1}^{n} F_{G+B}(\delta_{ik})E'_{dk} - B_{ii}E'_{qi} - \sum_{i \neq k, k=1}^{n} F_{B-G}(\delta_{ik})E'_{qk}$$
$$I_{qi} = B_{ii}E'_{di} + \sum_{i \neq k, k=1}^{n} F_{B-G}(\delta_{ik})E'_{dk} + G_{ii}E'_{qi} + \sum_{i \neq k, k=1}^{n} F_{G+B}(\delta_{ik})E'_{qk}$$

where

$$F_{G+B}(\delta_{ik}) = G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}$$

$$F_{B-G}(\delta_{ik}) = B_{ik} \cos \delta_{ik} - G_{ik} \sin \delta_{ik}$$
  
with  
$$G_{ik} : \text{ real part of } \overline{Y}$$
  
$$B_{ik} : \text{ imaginary part of } \overline{Y}$$
  
$$\delta_{ik} = \delta_i - \delta_k$$

By substituting the above equations into equations (3), (4) and (11), we obtain the state equations of a multimachine system with respect to the following state variables:  $\delta$ ,  $\omega$ ,  $E'_d$  and  $E'_q$ .

#### Dynamics of a multimachine power system

For each generator i of the network the state equations are given by:

$$\begin{split} \delta_{i}(t) &= \omega_{i}(t) \\ \dot{\omega}_{i}(t) &= -\frac{K_{Di}}{2H_{i}}\omega_{i}(t) + \frac{\omega_{0}}{2H_{i}}\left\{P_{moi} - G_{ii}\left(E_{di}^{'2} + E_{qi}^{'2}\right) \\ -E_{di}^{'}\left(\sum_{i \neq k, k=1}^{n} F_{G+B}(\delta_{ik})E_{dk}^{'} - \sum_{i \neq k, k=1}^{n} F_{B-G}(\delta_{ik})E_{qk}^{'}\right) \\ -E_{qi}^{'}\left(\sum_{i \neq k, k=1}^{n} F_{B-G}(\delta_{ik})E_{dk}^{'} + \sum_{i \neq k, k=1}^{n} F_{G+B}(\delta_{ik})E_{qk}^{'}\right)\right\} \\ \dot{E}_{di}(t) &= -\frac{1}{T_{q0i}^{'}}E_{di}^{'}(t) + \frac{x_{qi} - x_{qi}^{'}}{T_{q0i}^{'}}\left(B_{ii}E_{di}^{'} + G_{ii}E_{qi}^{'}\right) \\ + \frac{x_{qi} - x_{qi}^{'}}{T_{q0i}^{'}}\left(\sum_{i \neq k, k=1}^{n} F_{B-G}(\delta_{ik})E_{dk}^{'} + \sum_{i \neq k, k=1}^{n} F_{G+B}(\delta_{ik})E_{qk}^{'}\right) \\ \dot{E}_{qi}(t) &= -\frac{1}{T_{d0i}^{'}}E_{qi}^{'}(t) + \frac{x_{di} - x_{di}^{'}}{T_{d0i}^{'}}\left(G_{ii}E_{di}^{'} - B_{ii}E_{qi}^{'}\right) \\ + \frac{x_{di} - x_{di}^{'}}{T_{d0i}^{'}}\left(\sum_{i \neq k, k=1}^{n} F_{G+B}(\delta_{ik}E_{dk}^{'}\right) \end{split}$$

$$-\sum_{i\neq k,k=1}^{n} F_{B-G}(\delta_{ik}) E'_{qk} + \frac{1}{T'_{d0i}} K_{ci} U_{fi}$$
(9)

#### **3** Decentralized nonlinear controller design

A classical study of transient stability will be presented here on a nine-bus power system composed of three generators and three loads. This approach can be obviously extended to a nmachine power system. A one-line impedance diagram for the system is given in fig. 1. In order to apply the backstepping method [2, 5], we compute:

- The equivalent circuit from point of view of bus N°1 for generator # 1
- The equivalent circuit from point of view of bus N°2 for generator # 2
- The equivalent circuit from point of view of bus N°3 for generator # 3



Figure 1: Nine-bus system impedance diagram; all impedances are in p.u. on a 100-MVA base

in three different situations corresponding to pre-fault, duration fault and post-fault, as defined by fig. 2.



Figure 2: Equivalent circuit (SMIB model) with respect to each generator

For this equivalent circuit, we have used three different impedances  $R_{th} + jX_{th}$  with respect to pre-fault, duration fault and post-fault situations. However for  $V_{th}$  we have only considered the pre-fault value.

In order to apply the backstepping method, we have changed the state variables ( $\delta$ ,  $\omega$  and  $E_q$ ) of [2] into new variables:  $\delta$ ,  $\omega$  and  $P_e$ . With regard to these new variables, the set of state equations for each equivalent circuit is given by:

$$\delta(t) = \omega(t)$$

$$\dot{\omega}(t) = -\frac{D}{2H}\omega(t) + \frac{\omega_0}{2H} \left(P_{m0} - P_e(t)\right)$$

$$\dot{P}_e(t) = -\left(\frac{C_\lambda}{T'_{do}}M_1 + M_2\right)E_q - A_\lambda M_1 V_s \omega \sin \delta$$

$$+ B_\lambda M_1 V_s \omega \cos \delta + M_3 + M_1 K_c \frac{C_\lambda}{T'_{do}} u_f$$
(10)

The definition of the parameters are:

$$E_q = A_\lambda V_s \cos \delta + B_\lambda V_s \sin \delta + C_\lambda E'_q$$

$$A_\lambda = \frac{x_{qs}(x_d - x'_d)}{\lambda_x} \quad B_\lambda = \frac{R_L(x_d - x'_d)}{\lambda_x} \quad C_\lambda = \frac{-(R_L^2 + x_{qs} x_{ds})}{\lambda_x}$$

$$\lambda_x = x_{qs}(x_d - x'_d) - x_{qs} x_{ds} - R_L^2$$

$$x_{qs} = X_{TH} + x_q + x_t \quad x_{ds} = X_{TH} + x_d + x_t \quad R_L = R_{TH}$$

 $x_t$ : transformer reactance

$$\begin{split} M_1 &= K_4 V_s \cos \delta + K_3 V_s \sin \delta + 2K_6 E_q \\ M_2 &= (K_3 \omega V_s \cos \delta - K_4 \omega V_s \sin \delta) E_q \\ M_3 &= (K_1 - K_2) \omega V_s^2 \sin 2\delta + K_5 \omega V_s^2 \cos 2\delta \\ K_1 &= \frac{R_L}{x_{qs}^2} (1 - 2R_L B_s + R_L^2 B_s^2) \qquad K_2 = R_L^2 A_s^2 \end{split}$$

$$K_{3} = \frac{2R_{L}}{x_{qs}} (B_{s} - R_{L}(A_{s}^{2} + B_{s}^{2}) + \frac{x_{qs}}{R_{x}})$$

$$K_{4} = -2R_{L}(A_{s}^{2} + B_{s}^{2}) + \frac{R_{L}}{R_{x}} \quad R_{x} = x_{qs}x_{ds} + R_{L}^{2}$$

$$K_{5} = \frac{2R_{L}}{x_{qs}} (R_{L}(A_{s}^{2} + B_{s}^{2}) - B_{s}) + \frac{x_{d} - x_{q}}{R_{x}}$$

$$K_{6} = R_{L}(A_{s}^{2} + B_{s}^{2}) \quad K_{7} = R_{L}B_{s}^{2} - \frac{R_{L}}{R_{x}}$$

$$A_{s} = x_{qs}/R_{x} \quad B_{s} = R_{L}/R_{x}$$

We can expresse  $P_e$  as follows:

$$P_{e} = K_{1}V_{s}^{2}\sin^{2}\delta + K_{2}V_{s}^{2}\cos^{2}\delta + K_{3}E_{q}V_{s}\sin\delta + K_{4}E_{q}V_{s}\cos\delta + K_{5}V_{s}^{2}\cos\delta\sin\delta + k_{6}E_{q}^{2} + K_{7}V_{s}^{2}$$
(11)

and the terminal voltage:

$$V_t = \left\{ \left[ \frac{x_q x_{qs}}{R_x} V_s \sin \delta + \frac{R_L x_q}{R_x} (E_{q1} - V_s \cos \delta) \right]^2 + \left[ \left( \frac{R_x - x_q x_{qs}}{R_x} \right) E_{q1} + \frac{x_q x_{qs}}{R_x} V_s \cos \delta + \frac{R_L x_q}{R_x} V_s \sin \delta \right]^2 \right\}^{.5}$$
(12)

where

$$\begin{split} E_{q1} &= \left( -B_{\gamma} + \sqrt{B_{\gamma}^2 + 4\frac{R_L}{R_x}P_e} \right) / \left( \frac{2R_L}{R_x} \right) \\ B_{\gamma} &= \frac{x_{qs}}{R_x} V_s \sin \delta - \frac{R_L}{R_x} V_s \cos \delta \end{split}$$

and  $V_s = V_{th}$  is the Thevenin voltage of each equivalent circuit in the pre-fault situation. We also supposed for simplicity that the variation of input mechanical power is zero  $(P_m(t) = P_{m0})$ .

First, we put the state equations in a strict-feedback form:

$$\dot{x_1} = x_2$$
 (13)  
 $\dot{x_2} = -Ex_2 - Fx_3$  (14)

$$\dot{x_{3}} = -\left(\frac{C_{\lambda}}{T_{do}'}M_{1} + M_{2}\right)E_{q} - A_{\lambda}M_{1}V_{s}x_{2}\sin(x_{1} + \delta_{0}) + B_{\lambda}M_{1}V_{s}x_{2}\cos(x_{1} + \delta_{0}) + M_{3} + M_{1}K_{c}\frac{C_{\lambda}}{T_{do}'}u_{f}$$
(15)

$$x_1 = \Delta \delta(t), \ x_2 = \omega(t), \ x_3 = \Delta P_e(t), \ E = \frac{D}{2H}, \ F = \frac{\omega_0}{2H}.$$

With regard to (11) it must be noted that  $E_q$  is a fonction of  $P_e$ and  $\delta$ . If we define the functions  $f, f_1, f_2, g, g_1, g_2$  as follows:

we get the following strict feedback form:

$$\begin{aligned}
\dot{x_1} &= f(x_1) + g(x_1)x_2 \\
\dot{x_2} &= f_1(x_1, x_2) + g_1(x_1, x_2)x_3 \\
\dot{x_3} &= f_2(x_1, x_2, x_3) + g_2(x_1, x_2, x_3)u_f(t)
\end{aligned}$$
(16)

The main advantage of the backstepping method is the simultaneous derivation of both the Lyapunov function and the control law [5]. According to [2] and the configuration of state equations (13), (14) and (15) we can use the strict-feedback form of backstepping method as following. If we choose the Lyapunov function candidate  $v = \frac{1}{2}x_1^2$ , we can state the virtual control law for (13):  $\alpha = -x_1$ . Then we seek for a control law  $x_2 = \alpha_1$  which stabilizes the first two state equations, where  $x_2$  is viewed as a control input and where the related Lyapunov function is  $v_1 = v + \frac{1}{2}(x_2 - \alpha)^2$ .  $\alpha_1$  is such that  $\dot{v}_1 = -x_1^2 - c_1(x_2 - \alpha)^2$ . Finally, we obtain the control law for each generator as follows:

$$u_f = \frac{1}{g_2} \left\{ -c_2(x_3 - \alpha_1) - \frac{\partial v_1}{\partial x_2} g_1 - f_2 + \left( \frac{\partial \alpha_1}{\partial x_1} \quad \frac{\partial \alpha_1}{\partial x_2} \right) \left( \begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \end{array} \right) \right\}$$

With the Lyapunov function  $v_2 = v_1 + \frac{1}{2}(x_3 - \alpha_1)^2$  such that  $\dot{v}_2 = -x_1^2 - c_1(x_2 - \alpha)^2 - c_2(x_3 - \alpha_1)^2$ , and where  $c_1, c_2 > 0$ ,

$$\alpha_1 = \frac{1}{g_1} \left\{ -(c_1+1)(x_1+x_2) - f_1 \right\}$$

Under the realistic assumption that, we can determine which of the parallel transmission lines is disconnected (by use of some sensor pre-assigned on each of the lines), we can compute the power angle, after disconnecting the faulted line. When we fix the terminal voltage to 1 *p.u.*, we can solve nonlinear equation (12) for each generator in order to compute a new  $\delta$  ( $P_e$  remains unchanged). We denote the post-fault power angle as  $\delta_{pos}$  and we define  $\tilde{\delta} = \delta(t) - \delta_{pos}$ .

 $\delta_{pos}$  is now introduced in the regulation error dynamics and the control law instead of  $\delta$ :

$$\begin{aligned} \dot{\tilde{\delta}} &= \tilde{\omega} \\ \dot{\tilde{\omega}} &= -\frac{K_D}{2H} \tilde{\omega} - \frac{\omega_0}{2H} \tilde{P}_e \\ \dot{\tilde{P}}_e &= -\left(\frac{C_\lambda}{T_{do}'} M_{1pos} + M_{2pos}\right) E_{qpos} \\ -A_\lambda M_{1pos} V_s \omega \sin(\delta_{pos} + \tilde{\delta}) + B_\lambda M_{1pos} V_s \omega \cos(\delta_{pos} + \tilde{\delta}) \\ +M_{3pos} + M_{1pos} K_c \frac{C_\lambda}{T_{do}'} (\tilde{u}_f + u_{f0-pos}) \end{aligned}$$
(17)

Where

$$\widetilde{P}_e = \Delta p_e \qquad \widetilde{\omega} = \omega$$

Index *pos* means that the related relations are expressed in terms of  $\delta_{pos}$  instead of  $\delta$ . We also compute  $U_{f0-pos}$  corresponding to the new equilibrium state induced by  $\delta_{pos}$ . Simulation results given hereafter, show clearly the improvement brought by this methodology.

# 4 A three-machine example

In this section, the transient and steady state responses obtained with the backstepping method are simulated and compared with the responses obtained with PSS controllers. We have simulated the closed-loop behavior of the system with two different control schemes:

- without auxiliary controllers (only with conventional PSS and AVR),

- with the here-proposed controller with  $c_1 = 1, c_2 = 80$ . The kinds of faults that we consider in this paper are the symmetrical three-phase short circuit faults occurring on any line. The fault is situated at the generator bus.

#### The case study

The three-machine system described by Fig.1 is chosen to demonstrate the effectiveness of the proposed backstepping controller. The system parameters used in the simulation are the following:

The physical limit of the exciter is:  $max|k_cu_f(t)| = 7pu$  for each generator.

#### Fault sequence (permanent fault)

- stage 1 : The system is in the pre-fault steady state.
- stage 2 : A fault occurs at t = 0.5sec on line 5-7 near the bus # 7.
- stage 3 : The fault is removed by opening the breakers of
- the faulted line at t = 0.57 sec
- stage 4 : The system is in a post-fault state.

We have supposed that generator 1 is the reference generator. The transient response obtained with the backstepping method is improved compared to the PSS controller response (see fig. 6,7, 8, 9). The variations of electrical power, terminal voltage, and power angle for each machine are given by fig. (3), (4) and (5) at the occurence of the short circuit when the backstepping-based controllers are used.



Figure 3: *Electrical power output variation - short circuit between line 5-7* 

If we change the position of fault on the other lines, for example line 4-6 (near bus # 4), we can see that there is always asymptotic stability for the variables of machines. (see fig. 10 and 11).



Figure 4: Terminal voltage variations of each machine - short circuit between line 5-7



Figure 5: Power angle variations of each machine - short circuit between line 5-7

# 5 Conclusions

In this paper, the idea of transient stability enhancement via backstepping nonlinear control of single-machine infinite-bus power system proposed in [2] has been extended to the multi-machine case. The backstepping technique has been extended to the multi-machine power system via the design of an equivalent circuit with respect to each generator. A new power system controller has been proposed in this paper to achieve both transient stability enhancement and good post-fault performance of the generator terminal voltage  $V_t(t)$ . It represents a realistic alternative to the usual AVR/PSS scheme. A simple design procedure has been proposed. The performance of this controller has been tested through different simulation scenarios and in comparison with three existing control schemes. The simulation results show that:

• Both transient enhancement and good post-fault perfor-



Figure 6: Power angle variations of generator # 2 - short circuit between line 5-7



Figure 7: Power angle variations of generator # 3 - short circuit between line 5-7

mance of both the generator terminal voltage power angle and electrical power can be achieved;

- The performance of this controller is independent of the operating point;
- Good transient enhancement is obtained, if the faulted line changes.

Further researches will be devoted to the stability analysis of this decentralized control scheme.



Figure 8: Terminal voltage variations of generator # 2 - short circuit between line 5-7



Figure 9: Terminal voltage variations of generator # 3 - short circuit between line 5-7

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Figure 10: Terminal voltage variations of each machine - short circuit between line 4-6



Figure 11: Power angle variations of each machine - short circuit between line 4-6

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