# MODELLING AND OBSERVER-BASED FAULT DETECTION FOR AN AUTOMOTIVE DRIVE-TRAIN

J.A.F. Vinsonneau\*, D.N. Shields\*, P.J. King<sup>†</sup>, K.J. Burnham\*

\* School of Mathematical and Information Science, Coventry University, Coventry CV1 5FB. U.K. Tel: +44 (0)2476 888972, fax: +44 (0)2476 888052, Email: d.n.shields@coventry.ac.uk

<sup>†</sup> Jaguar Cars Ltd, Coventry, U.K.

**Keywords:** Drive-train modelling, automotive, nonlinear system, fault detection, observers

### Abstract

Observer-based fault detection and modelling are considered for the drive-train of a Jaguar car with an automatic transmission. Test data is used for the modelling and use is made of nonlinear polynomials, where various structures are compared and assessed. A robust fault detection observer (RFDO), consisting of an observer which generates a residual, for the class of nonlinear models emerging from the modelling, is proposed consisting of polynomial nonlinearities up to degree three and a cubic term with respect to the state and input. The observer and the residual are decoupled from unknown inputs. Three fault scenarios are considered, each with a different set of measurements, to test the effectiveness of the RFDO design.

# 1 Introduction

Automotive emission regulations and the requirement for improved fuel economy have driven innovation in powertrain design and control for more than three decades. Low Emission Vehicle (LEV) II regulations, running from 2004 until 2010, require on-board diagnostics to be more sophisticated, detecting emission problems relating to any sensor or component of the engine [2]. A means of detecting problems early is to make use of a fault detection method based on a system model ([7, 9]). This forms part of the theory of FDI recently introduced in most modern cars to control and diagnose engine malfunctions ([8, 4]). Of the great variety of methods used in the literature where system knowledge is available, the most common is the model-based method for generating residuals and forming decision logic ([3]).

This paper considers improvements in the form of better modelling and a new FDI method for a complete car drive-train. Section 2 considers the modelling of the drive-train of the car. Section 3 develops a RFDO strategy for fault detection. Section 4 is an application of sections 2 and 3. Variables used in this paper are defined in Table 1.

## 2 Drive-train model development

The modelling of the drive-train of the car is considered. The dynamic equations describing the manifold pressure, the en-

	Description	units	scaling units
$u_1$	Throttle angle, $\theta$	[deg]	$u_{1N}=u_1/50$
$u_2$	Braking force $F_{braking}$	[kN]	$u_{2N} = u_2/2.5$
$u_3$	Mass of car $m_{veh}$	[tons]	$u_{3N} = u_3/2.5$
$u_4$	Road gradient $r_g$	[deg]	$u_{4N}=u_4/30$
$u_5$	Spark Advance $\sigma$	[deg]	$u_{5N}=u_5/50$
$u_6$	Air/Fuel ratio $A/F$	[-]	$u_{6N} = u_6/26$
$u_7$	Gear ratio $R_{tr}$	[-]	$u_{7N} = u_7/4$
$u_{8N}$	$u_{1N}u_{7N}$		
$x_1$	Manifold pressure $P_m$	[bar]	$x_{1N} = x_1/1$
$x_2$	Engine speed $w_e$	[rad/s]	$x_{2N} = x_2/500$
$x_3$	Wheel speed $w_w$	[rad/s]	$x_{3N} = x_3/150$
$y_1$	Man. press. meas.	[bar]	$y_{1N}=y_1/1$
$y_2$	Engine speed meas.	[rad/s]	$y_{2N} = y_2/500$
$y_3$	Wheel speed meas.	[rad/s]	$y_{3N} = y_3/150$
$R_d$	Ratio of differential	[-]	
$R_r$	Rolling radius	[m]	
$R_{tr}$	Torque ratio of TC	[-]	
$\eta_{df}$	Efficiency of differential	[95%]	
$J_e$	Inertia of engine	$[kg.m^2]$	
$J_v$	Inertia of vehicle	$[kg.m^2]$	
$T_e$	Torque of engine	[N.m]	
$T_i$	Load torque	[N.m]	

Table 1: Nomenclature.

gine speed and the wheel speed are expressed using nonlinear polynomials. Figure (1) shows a realistic Simulink model of the drive-train, which is derived by breaking each of the many dynamic and passive engine components into blocks. Those corresponding to the torque converter, gear box and differential do not have equivalent explicit differential equations for their description. For example, the gear ratio  $R_{tr}$  is linked to  $R_d$  and  $W_w$  through two tables, where one of which is logical in form, and the differential relationship is not that of a simple explicit differential equation. The problem is partly avoided by assuming that  $R_{tr}$  is a measured input (since its information is known). Inputs  $u_5$  and  $u_7$  are similarly taken as measured values. In Fig. (1), they are derived from classified control algorithms and are not available in explicit form. Thus, the actual inputs that can be manipulated in Fig. (1) are  $u_1$  and  $u_4$ . The car can be driven automatically (simulated) by varying  $u_1$ and  $u_2$ . The model in Fig. (1) has been verified to be accurate compared with data collected from road tests. An objective for Jaguar Cars is, assuming Fig. (1) represents now a real car, to determine if a RFDO is effective. Since the Simulink model



Figure 1: Drive-train diagram of an automatic transmission.

	$sx_{1-1}$	$sx_{1-2}$	$\mathbf{s}\mathbf{x}_{1-3}$	$sx_{1-4}$
$R_T^2$	0.9977	0.9970	0.9995	0.8538
IAE	0.0060	0.0065	0.0024	0.0495
YIC	-8.2833	-9.4690	-13.1617	-1.8389
	$x_{1N}$	$x_{1N}$	$x_{1N}$	$x_{1N}$
	$u_{1N}$	$u_{1N}$	$x_{2N}$	$x_{2N}$
	$u_{1N}^2$	$u_{1N}^2$	$u_{1N}$	$u_{1N}$
$\chi_1^N$	$u_{1N}^3$	$u_{1N}^3$	$u_{1N}^2$	$u_{1N}^2$
	$x_{2N}x_{1N}$	$u_{1N} x_{1N}$	$u_{1N}^3$	$u_{1N}^3$
	$x_{2N}x_{1N}^2$	$u_{1N} x_{1N}^2$	$u_{1N} x_{1N}$	$u_{1N}x_{1N}$
	$x_{1N} x_{2N}^2$	$x_{2N}x_{1N}$	$u_{1N} x_{1N}^2$	
		$x_{2N} x_{1N}^2$	$x_{2N}x_{1N}$	
			$x_{1N}^2 x_{2N}$	
			$x_{1N}x_{2N}^2$	

Table 2: Comparison of polynomial structures for (1):  $x_{1,N+1} = \chi_1^N \theta_1.$ 

is not appropriate for observer design, alternative polynomial modelling is considered.

Considered first is the state equation for the manifold pressure, obtained by applying conservation of mass inside the volume of the manifold. The pressure is driven by  $u_1$  and  $x_2$ , as shown in [5] and [10] as,

$$\dot{x}_1 = \frac{RT}{V_m} \bigg[ f(u_1, x_1) - h(x_2, x_1) \bigg], \tag{1}$$

where f(.) and g(.) are some nonlinear functions. Equation (1) is discretized. A restricted comparison of modelling structure is shown in Table 2. Parameter vector  $\theta_1$  was estimated using a Least-Square (LS) algorithm applied on half of the data set and tested on the full set, where the fitness was assessed using three criteria indices, where  $\chi_1^N$  is defined in Table 2. Three comparison indices are used to assess the model: the integral-absolute-error (IAE) divided by the number of samples (the smaller the better), the multiple correlation coefficient  $(R_T^2)$  (the closer to 1 the better) and the Young Criteria (YIC) (the smaller the better). The structure  $sx_{1-3}$  gives better results,

	$sx_{2-1}$	$sx_{2-2}$	$sx_{2-3}$	$sx_{2-4}$
$R_T^2$	0.8599	0.8825	0.9019	0.9632
IAE	0.0244	0.0249	0.0204	0.0118
YIC	-6.5001	-5.3882	-6.9622	-7.7050
	$x_{2N}$	$x_{2N}$	$x_{2N}$	$x_{2N}$
	$u_{5N}$	$u_{5N}$	$x_{1N}$	$x_{1N}$
$\chi_2^N$	$u_{6N}$	$u_{6N}$	$u_{7N} x_{3N}$	$x_{1N}^{2}$
	$x_{1N}$	$x_{1N}$		$x_{1N}x_{2N}$
	$u_{7N}x_{3N}$	$u_{7N}x_{3N}$		$u_{7N}x_{3N}$
		$u_{7N} x_{3N}^2$		$u_{5N}x_{2N}$
				$u_{5N} x_{2N}^2$

Table 3: Comparison of polynomial structures for (2),  $x_{2,N+1} = \chi_2^N \theta_2.$ 

according to the indices, for which  $\theta_1 = [1.014, 0.093, -0.034, 2.163, -1.701, 1.134, -1.451, -0.641, 0.188, -0.289].$ 

The output torque of the engine is characterized by the driving torque  $T_e$  resulting from the combustion, and the external load from the torque converter,  $T_i$  [5]:

$$\dot{x}_2 = rac{1}{J_e}igg[T_e(u_5,u_6,x_1,x_2) - T_i(u_2,u_7,x_2,x_3)igg]$$
 (2)

Equation (2) is discretized and fitted with polynomials using a LS algorithm. A restricted set of structures is shown in Table 3. Polynomials structure  $sx_{2-4}$  shows the best fit, as indicated by all criteria indices, for which  $\theta_2$ =[0.968, 0.017, 0.036, -0.118, 0.118, -0.065, 0.243];

The transmission of the torque and revolution speed through the torque converter is expressed by a complex nonlinear relation, due to the fluid coupling [6], and represented in Fig. (1).

$$\dot{\omega}_w = \frac{1}{J_v} \bigg[ R_d T_{out} \eta_{df} - T_{vl} \bigg], \tag{3}$$

with  $T_{out}$  the output torque of the gear box. Equation (3) in state form is written

$$\dot{x}_3 = rac{1}{J_v(u_3)}igg[r(u_7,x_2,x_3) - T_{vl}(u_2,u_3,u_4,x_3)igg] ~,~(4)$$

	$sx_{3-1}$	$sx_{3-2}$	$sx_{3-3}$	$sx_{3-4}$
$R_T^2$	0.8484	0.9879	0.9915	0.9977
IAE	0.0447	0.0132	0.0105	0.0052
YIC	-5.1696	-9.0063	-5.5145	-6.8980
	$x_{3N}$	$x_{3N}$	$x_{3N}$	$x_{3N}$
	$x_{2N}$	$x_{2N}$	$x_{2N}$	$x_{2N}$
	$u_{2N}$	$u_{2N}$	$u_{2N}$	$u_{2N}$
	$u_{2N}x_{2N}$	$u_{2N}x_{2N}$	$u_{2N}x_{2N}$	$u_{2N}x_{2N}$
$\chi_3^N$		$u_{2N}x_{3N}$	$u_{2N}x_{3N}$	$u_{2N}x_{3N}$
		$u_{7N}x_{3N}$	$u_{7N}x_{3N}$	$u_{7N}x_{3N}$
			$u_{8N}x_{3N}$	$u_{8N} x_{3N}$
			$u_{8N}x_{2N}x_{3N}$	$u_{8N}x_{2N}x_{3N}$
				$u_{8N}^2 x_{3N}$
				$u_{8N}^2 x_{2N} x_{3N}$

Table 4: Restricted comparison of polynomial structures, for system (4).

where  $J_v(.)$ , r(.) and  $T_{vl}(.)$  are nonlinear functions. Equation (4) is discretized and fitted using polynomials. A comparison of stuctures is shown in Table 4. Attention is drawn to structure  $sx_{3-2}$ , giving the best polynomial fit with respect to YIC, and also structure  $sx_{3-4}$  actually the best fit with respect to both  $R_T^2$  and IAE. Structure  $sx_{3-4}$  was therefore selected for which  $\theta_3$ =[1.000, 0.006, -0.002, 0.004, -0.002, -0.021, -0.137, 0.245, 1.912, -3.662].

Combining models in (1), (2) and (4) together, a compact polynomial model for the SI engine with automatic transmission in derived.  $= [u_{12}, u_{12}^2, u_{13}^3, u_{13}, u_{13},$ 

$$\underline{x}_{k} = [u_{1N}, u_{1N}, u_{1N}, u_{2N}, u_{5N}, u_{7N}, u_{8N}, (u_{8N})] ,$$

$$\underline{x}_{k+1} = A\underline{x}_{k} + E_{a}d_{k} + K_{a}f_{k} + B\underline{u}_{k} + u_{k}(1)A_{ux}^{1}\underline{x}_{k} + u_{k}(4)A_{ux}^{4}\underline{x}_{k} + u_{k}(5)A_{ux}^{5}\underline{x}_{k} + u_{k}(6)A_{ux}^{6}\underline{x}_{k} + u_{k}(7)A_{ux}^{7}\underline{x}_{k} + u_{k}(8)A_{ux}^{8}\underline{x}_{k} + x_{k}(1)A_{1xx}\underline{x}_{k} + u_{k}(5)x_{k}(2)A_{uxx}^{5}\underline{x}_{k} + u_{k}(7)x_{k}(2)A_{uxx}^{7}\underline{x}_{k} + u_{k}(8)x_{k}(2)A_{uxx}^{8}\underline{x}_{k} + x_{k}(1)x_{k}(1)A_{xxx}^{1}\underline{x}_{k} + u_{k}(8)x_{k}(2)A_{uxx}^{8}\underline{x}_{k} + x_{k}(1)x_{k}(1)A_{xxx}^{1}\underline{x}_{k} + x_{k}(2)x_{k}(2)A_{xxx}^{2}\underline{x}_{k},$$

$$(5)$$

$$\underline{y}_{k} = C\underline{x}_{k} + K_{s}f_{k}.$$

#### **3** Nonlinear robust fault detection observer

Here, the polynomial model developed in section 2 is used to design a nonlinear full order observer. The work is a development of that in [9] and [1], where different nonlinear models were used. Consider the discrete-time nonlinear model

$$\underline{x}_{k+1} = A\underline{x}_{k} + E_{a}d_{k} + K_{a}f_{a_{k}} + B\underline{u}_{k} + \sum_{i_{0}=1}^{m} u_{k}^{i_{0}}A_{ux}^{i_{0}}\underline{x}_{k} + \sum_{i_{1}=1}^{n} x_{k}^{i_{1}}A_{xx}^{i_{1}}\underline{x}_{k} + \sum_{i_{0}=1}^{m} \sum_{i_{1}=1}^{n} u_{k}^{i_{0}}x_{k}^{i_{1}}A_{uxx}^{i_{0}i_{1}}\underline{x}_{k} + \sum_{i_{1}=1}^{n} \sum_{i_{2}=1}^{n} x_{k}^{i_{1}}x_{k}^{i_{2}}A_{xxx}^{i_{1}i_{2}}\underline{x}_{k},$$
(7)
$$\underline{y}_{k} = C\underline{x}_{k} + K_{s}f_{sk},$$
(8)

with state  $\underline{x}_k \in \mathbb{R}^n$ , input  $\underline{u}_k \in \mathbb{R}^m$ , output  $\underline{y}_k \in \mathbb{R}^p$  and disturbance  $d_k \in \mathbb{R}^q$ . Here,  $A, B, C, K_a, K_s, E_a, A_{ux}^{i_0} A_{xx}^{i_1}$ ,  $A_{uxx}^{i_0i_1}$  and  $A_{xxx}^{i_1i_2}$  are constant matrices of appropriate dimensions. A nonlinear state observer is designed, of the form

$$z_{k+1} = F z_k + J u_k + H y_k + \sum_{i_0=1}^m u_k^{i_0} H_{ux}^{i_0} \underline{y}_k$$
  
+ 
$$\sum_{i_1=1}^p y_k^{i_1} H_{xx}^{i_1} \underline{y}_k + \sum_{i_0=1}^p \sum_{i_1=1}^p u_k^{i_0} y_k^{i_1} H_{uxx}^{i_0i_1} \underline{y}_k$$
  
+ 
$$\sum_{i_1=1}^p \sum_{i_2=1}^p y_k^{i_1} y_k^{i_2} H_{xxx}^{i_1i_2} \underline{y}_k, \qquad (9)$$

where  $z_k \in \mathbb{R}^d$  is a linear estimate of  $Tx_k$ . A fault detection signal, also called residual, linear in both  $z_k$  and  $y_k$ , is defined as  $\epsilon_k = L_1 z_k + L_2 y_k$ , where  $\epsilon_k \in \mathbb{R}^{d_0}(\{d_0, d\} \ge 1\}, L_1 \in \mathbb{R}^{(d_0 \times d)}$  and  $L_2 \in \mathbb{R}^{(d_0 \times d)}$ . The observer error is given by  $e_k = z_k - Tx_k$ . Without loss of generality, it is assumed  $C = [I_p \quad 0_{n-p}]$ .

Result: Let (10-21) hold true:

TTI In C

$$TA - FT = HC, (10)$$

$$J = TB, \tag{11}$$

$$TE_a = 0, (12)$$

$$L_1 T + L_2 C = 0, (13)$$

$$H_{ux}^{i_0}C - TA_{ux}^{i_0} = 0; i_0 = 1, ..., m,$$
(14)

$$H_{xx}^{i_1}C - TA_{xx}^{i_1} = 0; i_1 = 1, \dots, p,$$
(15)

$$I A_{xx} = 0; i_1 = p + 1, \dots, n, \tag{10}$$

$$H_{uxx}^{i_0i_1}C - TA_{uxx}^{i_0i_1} = 0; i_0 = 1, ..., m, i_1 = 1, ..., p,$$
(17)

 $TA_{uxx}^{i_{0}i_{1}} = 0; i_{0} = 1, ..., m, i_{1} = p + 1, ..., n, (18)$ 

$$IA_{xxx}^{+} = 0; \quad \{i_1, i_2\} = p+1, \dots, n,$$
 (20)

$$|\lambda_i(F)| < 1. \tag{21}$$

Then  $e_k$  and  $\epsilon_k$  are decoupled from  $d_k$  and satisfy the form

$$e_{k+1} = Fe_k + TK_a f_k + W_k(u_k, y_k, f_{a_k})$$
(22)

$$\varepsilon_k = L_1 e_k + L_2 K_s f_{s_k}, \tag{23}$$

where  $W_k(u_k, y_k, 0) = 0$  ( $W_k$  not detailed here).

The following numerical algorithm is given for solving Equations (10)-(21). For convenience, without loss of generality, the constant matrices in (7-8) are partitioned as

$$A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}, \tag{24}$$

$$A_{ux}^{i_0} = \begin{bmatrix} A_{1ux}^{i_0} & A_{2ux}^{i_0} \end{bmatrix},$$
(25)

$$A_{1x}^{i_1} = \begin{bmatrix} A_{1xx}^{i_1} & A_{2xx}^{i_1} \end{bmatrix}, \tag{26}$$

$$A_{uxx}^{i_0i_1} = [A_{1uxx}^{i_0i_1} \quad A_{2uxx}^{i_0i_1}], \tag{27}$$

$$A_{xxx}^{i_1i_2} = \begin{bmatrix} A_{1xxx}^{i_1i_2} & A_{2xxx}^{i_1i_2} \end{bmatrix},$$
(28)

where  $\{A_1, A_{1ux}^{i_0}, A_{1xx}^{i_1}, A_{1uxx}^{i_0i_1}, A_{1xxx}^{i_1i_2}\} \in \mathbb{R}^{n \times p}$  and  $\{A_2, (8) \quad A_{2ux}^{i_0}, A_{2xx}^{i_1}, A_{2uxx}^{i_0i_1}, A_{2xxx}^{i_1i_2}\} \in \mathbb{R}^{n \times (n-p)}$ .

Matrix T is also partitioned into the form

$$T = \begin{bmatrix} T_1 & T_2 \end{bmatrix} \tag{29}$$

where  $T_1 \in \mathbb{R}^{d \times p}$  and  $T_2 \in \mathbb{R}^{d \times (n-p)}$ . The equations (10), (13), in addition to (14), (15), (17) and (19) are correspondingly partitioned into the form

$$TA_1 - FT_1 = H \tag{30}$$

$$TA_2 - FT_2 = 0 (31)$$

$$L_1 T_1 - L_2 = 0 \tag{32}$$

$$L_1 T_2 = 0 \tag{33}$$

$$TA_{1ux}^{i_0} = H_{ux}^{i_0} \tag{34}$$

$$TA_{2ux}^{i_{2ux}} = 0; i_0 = 1, ..., m$$
(35)

$$TA_{1xx}^{i_1} = H_{xx}^{i_1}$$
(36)  
$$TA_{1x}^{i_1} = 0; i_1 = 1, ..., n$$
(37)

$$IA_{2xx} = 0; i_1 = 1, ..., p$$
 (37)

$$TA_{1uxx}^{i_0i_1} = H_{uxx}^{i_0i_1} \tag{38}$$

$$TA_{2uxx}^{i_0i_1} = 0; i_0 = 1, ..., m, i_1 = 1, ..., p$$
 (39)

$$TA_{1xxx}^{i_1i_2} = H_{xxx}^{i_1i_2} \tag{40}$$

$$TA_{2xxx}^{i_1i_2} = 0; \{i_1, i_2 = 1\}, ..., p$$
(41)

Equations (31), (35), (37), (39) and (41) can be merged into one equation,

$$T[A_{2ux}^{i_0}, A_{2xx}^{i_1}, A_{2uxx}^{i_0i_1}, A_{2xxx}^{i_1i_2}, E_a] = TZ = 0$$
(42)

Matrix T is solved in (42) as  $\begin{bmatrix} T_1 & T_2 \end{bmatrix} = MU_{z2}^T$ , where  $U_{z2}$  is obtained from the SVD of Z,

$$Z = \begin{bmatrix} U_{z1} & U_{z2} \end{bmatrix} \begin{bmatrix} \Sigma_{z1} \\ 0 \end{bmatrix} \begin{bmatrix} V_{z1} & V_{z2} \end{bmatrix}^T, \quad (43)$$

and M is an arbitrary  $d \times r$  matrix with r, the order of the left null space for Z, being given as

$$r = n - rank\{Z\},\tag{44}$$

where  $r \ge 1$  is needed. The observer order, d, is chosen as d = r.  $U_{z2}^T$  is partitioned into two matrices

$$U_{z2}^T = \begin{bmatrix} N_1 & N_2 \end{bmatrix},\tag{45}$$

where  $N_1 \in \Re^{r \times p}$  and  $N_2 \in \Re^{r \times (n-p)}$ . Then equation (29) is equivalent to the following two equations  $T_1 = MN_1$  and  $T_2 = MN_2$ . If the following conditions are satisfied,

$$TA_2(I_n - T_2^+ T_2) = 0, (46)$$

$$MU_{z2}^{*}A_{2}(I_{n} - (MN_{2})^{+}(MN_{2})) = 0, \qquad (47)$$

the observer matrix F has the following general form such that (31) is satisfied,

$$F = TA_2(T_2)^+ (48)$$

$$= MU_{z2}^{T}A_{2}(MN_{2})^{+} + W(I_{d} - MN_{2}(MN_{2})^{+})$$
(49)

$$=A^* + WC^*, \tag{50}$$

where W is a  $d \times d$  arbitrary matrix. The eigenvalues of W are designed such that R is stable. (.)<sup>+</sup> represents the pseudo

inverse of (.).

The matrix H is calculated from (30) and J from (11). Equations (32) and (33) have solutions,

$$L_1 = W_1 U_{n2}^T, \ L_2 = -L_1 T_1, \tag{51}$$

where  $U_{n2}$  is from the SVD of  $T_2$ 

$$T_2 = \begin{bmatrix} U_{n1} & U_{n2} \end{bmatrix} \begin{bmatrix} \Sigma_n \\ 0 \end{bmatrix} \begin{bmatrix} V_{n1} & V_{n2} \end{bmatrix}^T, \quad (52)$$

where  $W_1$  is  $a\Phi \times r_n$ , chosen arbitrary, with

$$r_n = d - rank\{T_2\} \ge d - (n - p).$$
 (53)

The dimension of the residual vector is chosen  $\Phi = r_n$ . Given d,  $\Phi$ , T, F,  $L_1$  and  $L_2$ , the matrices J, H,  $H_{ux}^{i_0}$ ,  $H_{xx}^{i_1}$ ,  $H_{uxx}^{i_0i_1}$  and  $H_{xxx}^{i_1i_2}$  are obtained from (30), (32), (34), (36) (38) and (40).

Detectability: In order that  $\epsilon_k$  is affected by faults, the following sufficient conditions are deriveable,

$$L_1 F^i T K_a \neq 0, \quad \forall i, i \leq d, (f_{ak} \neq 0, f_{sk} = 0)$$
  
$$L_1 F^i W_k \neq 0, \quad \forall i, i \leq d, \forall k (f_{ak} = 0, f_{sk} \neq 0)$$

<u>Remark:</u> The size of Z increases with respect to the complexity of the system. The freedom for obtaining robustness of the RFDO decreases with the increase of the number of the nonlinear terms.

## 4 Applications

For models (1) and (2), data was taken from an Jaguar car model "XJ8 Saloon", normally aspirated 4 litre, V8 engine, automatic gearbox, for a general driving cycle and with normal atmospheric conditions, and temperature. The sampling rate was 50ms. Data for (4) was partially simulated in Simulink as some real data was lacking (gear selection, braking force).

#### 4.1 Computational algorithm

The flow-chart in Fig. 2 represents the computational algorithm described in Section 3. Three classes of output, with p < n, are considered: for each, n = 3, p = 2 and  $E_a = 0$ . The form of C affects the partition of matrices A, B,  $A_{ux}$ ,  $A_{xx}$ ,  $A_{uxx}$  and  $A_{xxx}$ , and thus the constructed matrix Z.

**Class 1: only**  $x_1$  and  $x_2$  measured.  $y = [x_1 \ x_2 \ 0]'$ . For this case, in (42),  $rank\{Z\} = 2$  and the observer and residual can be designed.

**Class 2: only**  $x_2$  and  $x_3$  measured.  $y = \begin{bmatrix} x_2 & x_3 & 0 \end{bmatrix}'$ . For this case, in (42),  $rank\{Z\} = 2$  and the observer and residual can be designed.

**Class 3: only**  $x_1$  and  $x_3$  measured.  $y = \begin{bmatrix} x_3 & x_1 & 0 \end{bmatrix}'$ . Here, (42) and (44) imply an observer cannot be designed. A novel recovery solution is proposed for this (see Fig. 2). It consists of reducing the rank of Z by resetting (judiciously) certain coefficients in the matrices in (42). This hopefully ensures r > 0 and a residual can be designed. The downside is that the designed



Figure 2: flowchart for RFDO.

residual is affected by an increase in modelling error. For the application here it is seen that

In equation (54),  $Z_{(1,2)}$  is the only element in row 1, and is identified as  $\theta_3(4)$ . The latter is reset and  $\theta_3$  refitted as in Table 4 as  $\theta'_3$ =[1.0004, 0.0046, -0.0007, **0**, -0.0023, -0.0180, -0.1043, 0.1825, 2.1824, -4.207]. Then  $rank\{Z\} = 2$  and from (44), r = 2.

For all classes the approximation of the model by a polynomial leads to some modelling error and offset. The modelling error here is considered as noise and is filtered with a low-pass fil-

Outputs	$\epsilon$	mean value of residuals (offset)
$y_1,y_2$	$\overline{\epsilon}_{12}$	-2.4991e-5
$y_2, y_1$	$\overline{\epsilon}_{21}$	+2.4991e-5
$y_1, y_3$	$\overline{\epsilon}_{13}$	-0.0135
$y_3, y_1$	$\overline{\epsilon}_{31}$	+0.0135
$y_2, y_3$	$\overline{\epsilon}_{23}$	-5.9247e-4
$y_3, y_2$	$\overline{\epsilon}_{32}$	-5.9247e-4

Table 5: Offset for residuals for different classes of y.

ter. An offset is inevitable, however, and will not vanish. Its amplitude can be estimated from the mean value of non-faulty residuals, as shown in Table 5. An offset is here subtracted from the corresponding residual.

#### 4.2 Results

For each class described in section 4.1, the construction of Z leads to a first order observer. Condition (13) is satisfied, and (53) holds for each case.

Three multiplicative sensors faults were created on measurement, with an amplitude of 30%.  $f_1^s$  was added to  $y_{1N}$  for  $t \in [50:150], f_2^s$  to  $y_{2N}$  for  $t \in [200:300]$ , and  $f_3^s$  to  $y_{3N}$ for  $t \in [400 : 500]$  as shown in Fig. 3. A leak, a component fault  $f_1^a$ , corresponding to a hole (Ø3mm) in the manifold and an actuator fault  $f_2^a$ , corresponding to a loss of 50Nm in the torque converter were also simulated in a Simulink environment, where the Simulink model was taken as the real car. Modified data was saved to a file.  $f_1^a$  and  $f_2^a$  were applied for  $t \ge 500$ , as represented in Fig. 3. The mean value was taken off,  $\epsilon = |\epsilon - \epsilon|$ , where  $\epsilon = mean \ value$  (off-set). Then  $\epsilon_1$  was filtered using a low-pass filter with a cut-off frequency at 1Hz, to attenuate high frequency effects due to bad modelling. The level of thresholds were fixed at 5% of each sensor fault. Figure 4 shows the non-faulty measured scaled value for the manifold pressure, the engine speed and the wheel speed.

Scenario 1: class 1 and residual  $\epsilon_{12}$ . Two sensor faults  $f_1^s$ ,  $f_2^s$  and an actuator fault,  $f_1^a$  are applied. Fig. 5 (top) shows the residual after computation, which responds to both sensor faults and the leak. However, the leak is not always detected, especially for low engine speeds and high manifold pressures, where modelling noise masks the fault.

Scenario 2: class 3 and residual  $\epsilon_{13}$ . Two sensor faults  $f_1^s$ ,  $f_3^s$  and an actuator fault,  $f_1^a$  are applied. In Fig. 5 (middle), the observer aims to estimate the engine speed using  $y_3$ . Modelling errors are too big and the leak cannot be detected. The residual responds to  $f_3^s$ , but not to  $f_1^s$ . A spike appears at t = 200s. This is not caused by a fault but is a wrong-flagging (caused by modelling error).

Scenario 3: class 2 and residual  $\epsilon_{23}$ . Here,  $f_2^s$ ,  $f_3^s$  and  $f_2^a$  are applied. Fig. 5 (bottom) shows the response of the residual. A spike appears due to modelling error at time t = 200s (not caused by a fault). Evidence shows that  $f_3^s$  ( $400 \le t \le 500$ ) and  $f_2^a$  (t > 500s) are detected for high wheel and engine



Figure 4: Outputs measured.

speeds. The rersidual is oscillatory in nature for t > 500.

Comments on Fig. 5. The horizontal lines indicated are not threshold lines but are placed for readability. For senario 1 the effect of modelling error is smaller and all faults are detecable. For senarios 2 and 3 the effect of modelling error is much larger, deemed mainly due to unmodelled nonlinearities within the torque converter, thus contributing to spikes in the residuals (t=200s). The residual performances shown are the same for other data sets.



Figure 5: Filtered  $\check{\epsilon}_{12}$ ,  $\check{\epsilon}_{13}$  and  $\check{\epsilon}_{23}$ , respectively  $\epsilon_{12f}$ ,  $\epsilon_{13f}$  and  $\epsilon_{23f}$ . ( $\check{\epsilon}_{12} = | \epsilon_{12} - \bar{\epsilon}_{12} |$ ,  $\check{\epsilon}_{13} = | \epsilon_{13} - \bar{\epsilon}_{13} |$  and  $\check{\epsilon}_{23} = | \epsilon_{23} - \bar{\epsilon}_{23} |$ ).

## 5 Conclusions

Effective modelling is shown by using real data, using polynomial nonlinearities and using a judicious relabelling of certain quantities as known measured inputs. Theory for residual design is given for a discretized drive-train model, including a novel design modification. The application of the design theory to a Jaguar car shows the importance of accurate modelling. Three fault senarios, each with a different output class (p < n), are considered and residuals are generated for five faults (not simultaneous). Originality of the work consists in improved modelling and a new FDI design for systems of the from (5). The RFDO here can be used to isolate faults, as in [9], using a bank of observers.

### References

- Sharon Ann Ashton. Methods for Fault Detection and Isolation With Application to Hydraulic Pipelines. PhD thesis, Coventry University, Dec 1999.
- [2] Jeffrey A. Cook, Jing Sun, and J. W. Grizzle. Opportunities in automotive powertrain control applications. *Proceedings of the 2002 IEEE International Conference on Control Applications*, 2002.
- [3] P. M. Frank, S. X. Ding, and B. Köppen-Seliger. Current developments in the theory of FDI. In *IFAC Fault Detection, Supervision and Safety for Technical Processes*, pages 16–27, Budapest, Hungary, 2000.
- [4] R. Isermann. Diagnosis methods for electronic controlled vehicles. *Vehicle Systen Dynamics*, 36(2-3):77– 117, 2001.
- [5] U. Kienche and L. Nielsen. Automotive Control System
   For Engine, Driveline and Vehicle. Springer-Verlag, Berlin, Germany, first edition, 2000.
- [6] G. Lechner and H. Naunheimer. Automotive Transmissions - Fundamentals, Selection, Design and Application. Springer-Verlag, Berlin, Germany, first edition, 1999.
- [7] R. J. Patton and J. Chen. Observer based fault detection and isolation: Robustness and applications. *Control En*gineering Practice, 5(5):671–682, 1997.
- [8] G. Rizzoni, Y. Kim, and A. Soliman. Estimation problems in engine control and diagnosis. In *SAFEPROCESS'00*, pages 124–130, 14-16 June, Budapest, Hungary, 2000. IFAC.
- [9] D. N. Shields, S. A. Ashton, and S. Daley. Design of nonlinear observers for detecting fault in hydraulic subsea pipelines. *Control Engineering Practice*, 9:pp. 297– 311, 2001.
- [10] J. A. F. Vinsonneau, D. N. Shields, P. J. King, and K. J. Burnham. Improved SI engine modelling techniques with application to fault detection. In *Proc. of the IEEE Int. Conference on Control Applications*, Sept. 18-20, Glasgow, Scotland, UK., 2002.