EQUIVALENT CONDITIONS FOR EXPONENTIAL STABILITY FOR A SPECIAL CLASS OF CONSERVATIVE LINEAR SYSTEMS

G. WEISS*, **M. Tucsnak**†

* Dept. of Electrical and Electronic Engineering
Imperial College London
Exhibition Road, London SW7 2BT, UK
e-mail: G.Weiss@ic.ac.uk
† Department of Mathematics
University of Nancy-I, POB 239
Vandoeuvre les Nancy 54506, France
e-mail: Marius.Tucsnak@iecn.u-nancy.fr

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Abstract

Let A_0 be a possibly unbounded positive operator on the Hilbert space H, which is boundedly invertible. Let C_0 be a bounded operator from $\mathcal{D}(A_0^{\frac{1}{2}})$ (with the norm $\|z\|_{\frac{1}{2}}^2 = \langle A_0 z, z\rangle$) to another Hilbert space U. It is known that the system of equations

$$\ddot{z}(t) + A_0 z(t) + \frac{1}{2} C_0^* C_0 \dot{z}(t) \,=\, C_0^* u(t)\,,$$
$$y(t) \,=\, -C_0 \dot{z}(t) + u(t)\,,$$

determines a well-posed linear system Σ with input u and output y, input and output space U and state space $X = \mathcal{D}(A_0^{\frac{1}{2}}) \times H$. Moreover, Σ is conservative, which means that a certain energy balance equation is satisfied both by the trajectories of Σ and by those of its dual system. In this paper we present various conditions which are equivalent to the exponential stability of such a systems. Among the equivalent conditions are exact controllability and exact observability. Denoting

$$V(s) \,=\, \left(s^2 I + \frac{s}{2} C_0^* C_0 + A_0\right)^{-1}\,,$$

we also obtain that the system is exponentially stable if and only if $s \to A_0^{\frac{1}{2}} V(s)$ is a bounded $\mathcal{L}(H)$-valued function on the imaginary axis. This is also equivalent to the condition that $s \to sV(s)$ is a bounded $\mathcal{L}(H)$-valued function on the imaginary axis (or equivalently, on the open right half-plane).

1 Introduction and main results

In our paper [15] we have investigated a class of conservative linear systems with a special structure, described by a second order differential equation (in a Hilbert space) and an output equation, see (1.1) and (1.3) below. Such systems occur frequently in applications, such as wave equations and beam equations, see for example [1], [4], [14], [15] and the references therein. For this reason, it is useful to have criteria for their exponential stability.

We recall the construction from the paper [15], in order to be able to state the new results. Let H be a Hilbert space, and let $A_0 : \mathcal{D}(A_0) \to H$ be a self-adjoint, positive and boundedly invertible operator. We introduce the scale of Hilbert spaces H_α, $\alpha \in \mathbb{R}$, as follows: for every $\alpha \geq 0$, $H_\alpha = \mathcal{D}(A_0^\alpha)$, with the norm $\|z\|_\alpha = \|A_0^\alpha z\|_H$. The space $H_{-\alpha}$ is defined by duality with respect to the pivot space H as follows: $H_{-\alpha} = H_\alpha^*$ for $\alpha > 0$. Equivalently, $H_{-\alpha}$ is the completion of H with respect to the norm $\|z\|_{-\alpha} = \left\|A_0^{-\alpha} z\right\|_H$. The operator A_0 can be extended (or restricted) to each H_α, such that it becomes a bounded operator

$$A_0 : H_\alpha \to H_{\alpha-1} \qquad \forall\, \alpha \in \mathbb{R}\,.$$

Let C_0 be a bounded linear operator from $H_{\frac{1}{2}}$ to U, where U is another Hilbert space. We identify U with its dual, so that $U = U^*$. We denote $B_0 = C_0^*$, so that $B_0 \in \mathcal{L}(U, H_{-\frac{1}{2}})$. The class of systems studied in [15] and also here is described by

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} z(t) + A_0 z(t) + \frac{1}{2} B_0 \frac{\mathrm{d}}{\mathrm{d}t} C_0 z(t) \,=\, B_0 u(t)\,, \tag{1.1}$$

$$z(0) \,=\, z_0\,, \qquad \dot{z}(0) \,=\, w_0\,, \tag{1.2}$$

$$y(t) \,=\, -\frac{\mathrm{d}}{\mathrm{d}t} C_0 z(t) + u(t)\,, \tag{1.3}$$

where $t \in [0, \infty)$ is the time. The equation (1.1) is understood as an equation in $H_{-\frac{1}{2}}$, i.e., all the terms are in $H_{-\frac{1}{2}}$. Most of the linear equations modelling the damped vibrations of elastic structures can be written in the form (1.1), where z stands for the displacement field and the term $B_0 \frac{\mathrm{d}}{\mathrm{d}t} C_0 z(t)$, informally written as $B_0 C_0 \dot{z}(t)$, represents a viscous feedback damping. The signal $u(t)$ is an external input with values in U (often a displacement, a force or a moment acting on the boundary) and the signal $y(t)$ is the output (measurement) with values in U as well. The state $x(t)$ of this system and its state space X are defined by

$$x(t) = \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}, \qquad X = H_{\frac{1}{2}} \times H .$$

We use the standard notation for certain function spaces, such as $\mathcal{H}^n(0,\infty;W)$, $\mathcal{H}^n_{loc}(0,\infty;W)$, $C^n(0,\infty;W)$ and $BC^n(0,\infty;W)$ (with $n \in \{0,1,2,\ldots\}$). BC stands for "bounded and continuous". We write C instead of C^0.

We assume that the reader understands the concepts of a well-posed linear system and of a conservative linear system, see for example [12], [14], [15, Sections 1,3,4]. The first main result of [15] has been the following:

Theorem 1.1 *With the above assumptions, the equations* (1.1)–(1.3) *determine a conservative linear system* Σ*, in the following sense:*

There exists a conservative linear system Σ *whose input and output spaces are both* U *and whose state space is* X*. If* $u \in L^2([0,\infty),U)$ *is the input function,* $x_0 = \begin{bmatrix} z_0 \\ w_0 \end{bmatrix} \in X$ *is the initial state,* $x = \begin{bmatrix} z \\ w \end{bmatrix}$ *is the corresponding state trajectory and* y *is the corresponding output function, then*

(1)
$$z \in BC(0,\infty;H_{\frac{1}{2}}) \cap BC^1(0,\infty;H) \cap \mathcal{H}^2_{loc}(0,\infty;H_{-\frac{1}{2}}).$$

(2) *The two components of* x *are related by* $w = \dot{z}$.

(3) $C_0 z \in \mathcal{H}^1(0,\infty;U)$ *and the equations* (1.1) *(in* $H_{-\frac{1}{2}}$*) and* (1.3) *(in* U*) hold for*
almost every $t \geq 0$ *(hence,* $y \in L^2([0,\infty),U)$*).*

If $\dot{z}$ is a continuous function of t, with values in $H_{\frac{1}{2}}$ (see Theorems 1.2 and 1.4 in [15] for sufficient conditions for this), then (1.1) and (1.3) can be rewritten as

$$\ddot{z}(t) + A_0 z(t) + \frac{1}{2} B_0 C_0 \dot{z}(t) = B_0 u(t), \tag{1.4}$$

$$y(t) = -C_0 \dot{z}(t) + u(t). \tag{1.5}$$

We introduce the space $Z_0 = H_1 + A_0^{-1} B_0 U$, which is a Hilbert space if we define on it a suitable norm, see [15, Theorem 1.2]. We can rewrite the equations (1.4), (1.5) as a first order system as follows:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= \overline{C}x(t) + u(t), \end{cases} \tag{1.6}$$

where

$$A = \begin{bmatrix} 0 & I \\ -A_0 & -\frac{1}{2}B_0 C_0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ B_0 \end{bmatrix}, \tag{1.7}$$

$$\mathcal{D}(A) = \left\{ \begin{bmatrix} z \\ w \end{bmatrix} \in H_{\frac{1}{2}} \times H_{\frac{1}{2}} \;\middle|\; A_0 z + \frac{1}{2} B_0 C_0 w \in H \right\}, \tag{1.8}$$

$$\overline{C} : Z_0 \times H_{\frac{1}{2}} \to U, \qquad \overline{C} = [0 \ \ -C_0]. \tag{1.9}$$

We denote by C the restriction of $\overline{C}$ to $\mathcal{D}(A)$. A is the generator of a strongly continuous semigroup of contractions on X, denoted $\mathbb{T} = (\mathbb{T}_t)_{t\geq 0}$. For the concepts of semigroup generator, control operator, observation operator and transfer function of a well-posed linear system, we refer to [11], [12].

We denote by $\mathbb{C}_\omega$ the open right half-plane in $\mathbb{C}$ where Re $s > \omega$. We know from [15, Proposition 5.3] that for any $s \in \rho(A)$ (in particular, for any $s \in \mathbb{C}_0$), the operator $s^2 I + A_0 + \frac{s}{2} B_0 C_0 \in \mathcal{L}(H_{\frac{1}{2}}, H_{-\frac{1}{2}})$ has a bounded inverse denoted $V(s)$:

$$V(s) = \left(s^2 I + A_0 + \frac{s}{2} B_0 C_0 \right)^{-1} \in \mathcal{L}(H_{-\frac{1}{2}}, H_{\frac{1}{2}}). \tag{1.10}$$

The following proposition is a restatement of a part of Theorem 1.3 in [15].

Proposition 1.2 *With the notation of Theorem* 1.1 *and* (1.7)–(1.10)*, the semigroup generator of* Σ *is* A*, its control operator is* B *and its observation operator is* C*. The transfer function of* Σ *is given for all* $s \in \mathbb{C}_0$ *by*

$$\mathbf{G}(s) = \overline{C}(sI - A)^{-1} B + I = I - C_0 s V(s) B_0$$

and we have $\|\mathbf{G}(s)\| \leq 1$ *for all* $s \in \mathbb{C}_0$.

Now we have all the necessary ingredients to state the new results of this paper. The following theorems use various controllability, observability and stability concepts. The precise definition of these concepts is given in Section 2.

Theorem 1.3 *With the above notation, the following assertions are equivalent:*

(1) *The pair* (A,B) *is exactly controllable (in some finite time).*

(2) *The pair* (A,C) *is exactly observable (in some finite time).*

(3) *The semigroup* $\mathbb{T}$ *is exponentially stable.*

(4) *The pair* (A,B) *is optimizable.*

(5) *The pair* (A,C) *is estimatable.*

(6) *We have* $\sup_{s\in\mathbb{C}_0} \|A_0^{\frac{1}{2}} V(s)\|_{\mathcal{L}(H)} < \infty$.

(7) *We have* $\sup_{s\in\mathbb{C}_0} \|sV(s)\|_{\mathcal{L}(H)} < \infty$.

(8) *For a dense subset of* $\mathbb{R}$*, denoted* E*, we have* $iE \subset \rho(A)$ *and*

$$\sup_{\omega\in E} \|A_0^{\frac{1}{2}} V(i\omega)\|_{\mathcal{L}(H)} < \infty .$$

(9) *For a dense subset of* $\mathbb{R}$*, denoted* E*, we have* $iE \subset \rho(A)$ *and*

$$\sup_{\omega\in E} \|\omega V(i\omega)\|_{\mathcal{L}(H)} < \infty .$$

The equivalence of (1)–(5) remains valid for *every* conservative system. This fact, the theorem above, the other new theorems stated in this paper and various other results will be proved in the journal version of this paper, see [9].

By a well-known theorem of Jan Prüss and Huang Falun, an operator semigroup $\mathbb{T}$ with generator A is exponentially stable if and only if $(sI-A)^{-1}$ is uniformly bounded on $\mathbb{C}_0$. In the specific case of the semigroup generated by A from (1.7)-(1.8), the resolvent $(sI-A)^{-1}$ can be written as a 2×2 matrix of operators:

$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s}[I-V(s)A_0] & V(s) \\ -V(s)A_0 & sV(s) \end{bmatrix}.$$

Thus, to verify that the stability condition (3) in Theorem 1.3 holds, we would have to verify that the four entries of this 2×2 matrix are all uniformly bounded on $\mathbb{C}_0$. However, conditions (6) and (7) in Theorem 1.3 tell us that, in fact, we only have to verify one entry: one of those in the second column in the matrix. Conditions (8) and (9) tell us that, in fact, the boundedness of one of these two entries of $(sI-A)^{-1}$ has to be verified only on a dense part of the imaginary axis, and we can still conclude exponential stability.

The version of this theorem corresponding to bounded B and C, i.e., with $C_0 \in \mathcal{L}(H,U)$, is in Liu [5, Sections 2-3] (without conditions (4)–(7) and (9)). Using the boundedness of C_0 (and hence also of B_0), Liu was able to give in [5, Theorem 3.4] also other, Hautus-type conditions which are equivalent to the exponential stability of $\mathbb{T}$. For unbounded C_0, we were only able to obtain a Hautus-type estimate as a necessary condition for exponential stability, see Proposition 2.7.

We mention that semigroups of the type discussed in this paper do not necessarily satisfy the spectrum determined growth condition. For a counterexample (a damped wave equation on a compact manifold) see Lebeau [4].

Recall that $\|z\|_0$ denotes the norm of z in H. In the proof of Theorem 1.3 (more precisely, to show that (6)$\Longrightarrow$(3)) we use the following proposition, which is of independent interest. For bounded C_0, this proposition follows easily from [5, Theorem 3.4], but for unbounded C_0 the proof is more delicate (it will be given in the journal version of this paper). Related results for a bounded (but not necessarily positive) operator in place of $C_0^*C_0$ were given in Liu, Liu and Rao [6].

Proposition 1.4 *With the above notation, if C_0 is bounded from below, in the sense that there exists a $c>0$ such that*

$$\|C_0 z\|_U \geq c\|z\|_0 \qquad \forall\, z \in H_{\frac{1}{2}},$$

then $\mathbb{T}$ is exponentially stable.

2 Some background concerning the concepts used in the first section

In this section we recall some controllability, observability and stability concepts, quoting the relevant literature. We also list the more interesting new results which are needed as intermediate steps in the proofs of our main results.

Throughout this section, U, X and Y are Hilbert spaces and $A : \mathcal{D}(A) \to X$ is the generator of a strongly continuous semigroup $\mathbb{T} = (\mathbb{T}_t)_{t\geq 0}$ on X. The space X_1 is $\mathcal{D}(A)$ with the norm $\|z\|_1 = \|(\beta I - A)z\|$, where $\beta \in \rho(A)$ is fixed, while X_{-1} is the completion of X with respect to the norm $\|z\|_{-1} = \|(\beta I - A)^{-1}z\|$. We assume that the reader understands the concept of an admissible (in particular, infinite-time admissible) control operator for $\mathbb{T}$. If $B \in \mathcal{L}(U, X_{-1})$ is admissible, then for every $\tau \geq 0$ we denote by Φ_τ the operator

$$\Phi_\tau u = \int_0^\tau \mathbb{T}_{t-\sigma} B u(\sigma)\,\mathrm{d}\sigma. \tag{2.1}$$

We have $\Phi_\tau \in \mathcal{L}(L^2([0,\infty),U),X)$. If B is admissible, then for every $x_0 \in X$ and every $u \in L^2([0,\infty),U)$, the function $x(t) = \mathbb{T}_t x_0 + \Phi_t u$ is called the *state trajectory* corresponding to the initial state x_0 and the input function u. We have $x \in \mathcal{H}^1_{loc}(0,\infty;X)$ and $\dot{x}(t) = Ax(t) + Bu(t)$ (equality in X_{-1}) for almost every $t \geq 0$. If, moreover, B is infinite-time admissible, then we denote

$$\tilde{\Phi} u = \lim_{\tau\to\infty} \int_0^\tau \mathbb{T}_t B u(t)\,\mathrm{d}t, \tag{2.2}$$

and we have $\tilde{\Phi} \in \mathcal{L}(L^2([0,\infty),U),X)$.

Similarly, we assume that the reader understands the concepts of an admissible (in particular, infinite-time admissible) observation operator for $\mathbb{T}$, also presented in Section I.2. If $C \in \mathcal{L}(X_1, Y)$ is admissible, then we denote by Ψ the unique continuous operator from X to $L^2_{loc}([0,\infty),Y)$ such that

$$(\Psi x_0)(t) = C\mathbb{T}_t x_0 \qquad \forall\, x_0 \in \mathcal{D}(A). \tag{2.3}$$

In particular, if C is infinite-time admissible, then $\Psi \in \mathcal{L}(X, L^2([0,\infty),Y))$. Recall that B is an (infinite-time) admissible control operator for $\mathbb{T}$ if and only if B^* is an (infinite-time) admissible observation operator for $\mathbb{T}^*$.

Definition 2.1 Let A be the generator of a strongly continuous semigroup $\mathbb{T}$ on X and let $B \in \mathcal{L}(U, X_{-1})$ be an admissible control operator for $\mathbb{T}$.

The pair (A,B) is *exactly controllable* in time $T>0$, if for every $x_0 \in X$ there exists a $u \in L^2([0,T],U)$ such that $\Phi_T u = x_0$.

(A,B) is *exactly controllable* if the above property holds for some $T>0$.

(A,B) is *optimizable* if for any $x_0 \in X$, there exists $u \in L^2([0,\infty),U)$ such that the state trajectory corresponding to x_0 and u is in $L^2([0,\infty),X)$.

Clearly, exact controllability implies optimizability. Optimizability is one possible generalization of the concept of stabilizability, as known from finite-dimensional control theory. We refer to [13] for details on optimizability and estimatability.

Now we introduce the corresponding observability concepts via duality.

Definition 2.2 Suppose that $C \in \mathcal{L}(X_1, Y)$ is an admissible observation operator for $\mathbb{T}$ (equivalently, C^* is an admissible control operator for the adjoint semigroup $\mathbb{T}^*$). We say that (A, C) is *exactly observable* (in time T) if (A^*, C^*) is exactly controllable (in time T). (A, C) is called *estimatable* if (A^*, C^*) is optimizable.

Let Ψ be the operator defined in (2.3) and for every $\tau \geq 0$ put $\Psi_\tau = \mathbf{P}_\tau \Psi$. Then (A, C) is exactly observable in time $T > 0$ if and only if Ψ_T is bounded from below.

Recall that the *growth bound* of a strongly continuous semigroup $\mathbb{T}$ is $\omega_0(\mathbb{T}) = \lim_{t\to\infty} \frac{1}{t} \log \|\mathbb{T}_t\| = \inf_{t>0} \frac{1}{t} \log \|\mathbb{T}_t\|$, see for example Pazy [7]. The semigroup $\mathbb{T}$ is *exponentially stable* if its growth bound is negative: $\omega_0(\mathbb{T}) < 0$.

Let $\mathbb{T}$ be a strongly continuous semigroup on X, with generator A. A well-known spectral mapping result of Prüss [8, p. 852] implies that if the function $\|(sI - A)^{-1}\|$ is bounded on $\mathbb{C}_0$, then $\mathbb{T}$ is exponentially stable. A little later and independently, this result was explicitly stated and proved by Huang Falun [2]. A short proof was given in Weiss [10, Section 4]. Here we need a result which is closely related to the one just mentioned, without being an obvious consequence of it. The result is very slightly more general than another result of Huang Falun, see [2, Theorem 3]. Moreover, the proposition below gives an estimate for the growth bound $\omega_0(\mathbb{T})$.

Proposition 2.3 *Let $\mathbb{T}$ be a strongly continuous semigroup on X with generator A. Assume that $\omega_0(\mathbb{T}) \leq 0$ and E is a dense subset of $\mathbb{R}$ such that $iE \subset \rho(A)$ and*

$$\|(i\omega I - A)^{-1}\| \leq M \qquad \forall\, \omega \in E,$$

for some $M > 0$. Then $\mathbb{T}$ is exponentially stable, more precisely, $\omega_0(\mathbb{T}) \leq -\frac{1}{M}$.

Proposition 2.4 *Suppose that $X, \mathbb{T}, A, U$ and B are as in Definition 2.1. Then the following three statements are equivalent:*

(1) $\mathbb{T}$ *is exponentially stable.*

(2) (A, B) *is optimizable, $\mathbb{C}_0 \subset \rho(A)$ and, for some $M > 0$,*

$$\left\|(sI - A)^{-1}B\right\|_{\mathcal{L}(U,X)} \leq M \qquad \forall\, s \in \mathbb{C}_0 .$$

(3) (A, B) *is optimizable, $\omega_0(\mathbb{T}) \leq 0$, there exists a dense subset of $\mathbb{R}$, denoted E,*
such that $iE \subset \rho(A)$ and, for some $M > 0$,

$$\left\|(i\omega I - A)^{-1}B\right\|_{\mathcal{L}(U,X)} \leq M \qquad \forall\, \omega \in E.$$

Recall that for any well-posed linear system Σ with input function u, state trajectory x and output function y,

$$\begin{bmatrix} x(\tau) \\ \mathbf{P}_\tau y \end{bmatrix} = \Sigma_\tau \begin{bmatrix} x(0) \\ \mathbf{P}_\tau u \end{bmatrix}, \tag{2.4}$$

where $\mathbf{P}_\tau$ denotes the truncation of a function to $[0, \tau]$ and

$$\Sigma_\tau = \begin{bmatrix} \mathbb{T}_\tau & \Phi_\tau \\ \Psi_\tau & \mathbb{F}_\tau \end{bmatrix}. \tag{2.5}$$

We denote the input, state and output spaces of Σ by U, X and Y, respectively. Then the operators Σ_τ appearing above are bounded from $X \times L^2([0, \tau], U)$ to $X \times L^2([0, \tau], Y)$, which means that for some $c_\tau \geq 0$

$$\|x(\tau)\|^2 + \int_0^\tau \|y(t)\|^2\, \mathrm{d}t \leq c_\tau^2 \left(\|x(0)\|^2 + \int_0^\tau \|u(t)\|^2\, \mathrm{d}t \right).$$

As explained in Section I.1, the system Σ is *conservative* if the operators Σ_τ are unitary, from $X \times L^2([0, \tau], U)$ to $X \times L^2([0, \tau], Y)$. This implies that for any input function $u \in \mathcal{H}^1(0, \infty; U)$ and any initial state $x(0) = x_0 \in X$ with $Ax_0 + Bu(0) \in X$, the function $\|x(t)\|^2$ is in $C^1[0, \infty)$ and

$$\frac{\mathrm{d}}{\mathrm{d}t}\|x(t)\|^2 = \|u(t)\|^2 - \|y(t)\|^2 \qquad \forall\, t \geq 0, \tag{2.6}$$

see [15, Proposition 4.3]. Conversely, if (2.6) holds for both the system Σ and for its dual system Σ^d, then Σ is conservative, see [15, Corollary 4.4].

Proposition 2.5 *Let Σ be a conservative linear system with input space U, state space X, output space Y, semigroup $\mathbb{T}$, control operator B, observation operator C and transfer function $\mathbf{G}$. Then the following statements are true:*

(1) $\mathbb{T}$ *is a semigroup of contractions.*

(2) B *is infinite-time admissible.*

(3) C *is infinite-time admissible.*

(4) $\|\mathbf{G}(s)\| \leq 1$ *for all $s \in \mathbb{C}_0$.*

Proposition 2.6 *With the notation of Proposition 2.5 and denoting the generator of $\mathbb{T}$ by A, for each $\tau > 0$, the following statements are equivalent:*

(1) *The pair (A, B) is exactly controllable in time τ.*

(2) *The pair (A, C) is exactly observable in time τ.*

(3) $\|\mathbb{T}_\tau\| < 1$ *(in particular, $\mathbb{T}$ is exponentially stable).*

Proposition 2.7 *With the above notation, if $\mathbb{T}$ is exponentially stable, then denoting $M = \sup_{\omega\in\mathbb{R}} \|(i\omega I - A)^{-1}\|_{\mathcal{L}(X)}$ we have, for every $z \in H_{\frac{1}{2}}$ and every $\omega \in [0, \infty)$,*

$$\|(\omega^2 I - A_0)z\|_{-\frac{1}{2}} + \frac{\omega}{2}\|B_0 C_0 z\|_{-\frac{1}{2}} \geq \frac{1}{M}\|z\|_0 .$$

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References

[1] K. Ammari, M. Tucsnak and Z. Liu, Decay rates for a beam with pointwise force and moment feeedback, *Mathematics of Control, Signals and Systems* **15** (2002), pp. 229–255.

[2] Huang Falun, Characteristic conditions for exponential stability of linear dynamical systems in Hilbert spaces, *Annals of Diff. Equ.* **1** (1985), pp. 43–56.

[3] B. Jacob and H. Zwart, Exact observability of diagonal systems with a finite dimensional output operator, *Syst. & Control Letters* **43** (2001), pp. 101–109.

[4] G. Lebeau, Equation des ondes amorties, in *Algebraic and Geometric Methods in Mathematical Physics*, editors: A. Boutet de Monvel and V. Marchenko, Kluwer Academic Publishers, 1996, pp. 73–109.

[5] K. Liu, Locally distributed control and damping for the conservative systems, *SIAM J. Control and Optim.* **35** (1997), pp. 1574–1590.

[6] K. Liu, Z. Liu and B. Rao, Exponential stability of an abstract nondissipative linear system, *SIAM J. Control and Optim.* **40** (2001), pp. 149–165.

[7] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer-Verlag, New York, 1983.

[8] J. Prüss, On the spectrum of C_0-semigroups, *Trans. American Math. Society* **284** (1984), pp. 847–857.

[9] M. Tucsnak and G. Weiss, How to get a conservative well-posed linear system out of thin air, Part II: Controllability and stability, *SIAM J. Control and Optimization*, to appear in 2003.

[10] G. Weiss, Weak L^p-stability of a linear semigroup on a Hilbert space implies exponential stability, *Journal of Diff. Equations* **76** (1988), pp. 269–285.

[11] G. Weiss, Transfer functions of regular linear systems. Part I: Characterizations of regularity, *Trans. American Math. Society* **342** (1994), pp. 827–854.

[12] G. Weiss, Regular linear systems with feedback, *Mathematics of Control, Signals and Systems* **7** (1994), pp. 23–57.

[13] G. Weiss and R. Rebarber, Optimizability and estimatability for infinite-dimensional linear systems, *SIAM J. Control and Optimization* **39** (2001), pp. 1204–1232.

[14] G. Weiss, O.J. Staffans and M. Tucsnak, Well-posed linear systems -a survey with emphasis on conservative systems, *Applied Mathematics and Computer Science* **11** (2001), pp. 101–127.

[15] G. Weiss and M. Tucsnak, How to get a conservative well-posed linear system out of thin air, Part I: Well-posedness and energy balance, *ESAIM COCV* **9** (2003), pp. 247–274.