# DISCONTINUOUS REGULATOR FOR A CLASS OF LINEAR TIME DELAYED SYSTEMS

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**Abstract**

This paper applies the decomposition block control technique to design a discontinuous regulator that guarantees asymptotic reference tracking for a class of linear delayed systems with disturbances. This class of systems is those presented in so-called Block Controllable Form with Delay. The block control technique is used to derive a sliding manifold on which the motion of the closed-loop system is stable, and the tracking error is zeroed. Example of the application of the proposed control strategy design is illustrated.

## 1 Introduction

Stabilization of time-delay system (TDS) via continuous feedback control [8,9,23] has been extensively studied using various techniques such as $H_\infty$ control, Riccati equation and Linear Matrix Inequality approaches (see [2,5,11,12,13,18]). In order to introduce the robustness property in the closed-loop system, sliding mode discontinuous controllers [21] for TDS have been designed in [6,7,10,14,19,20].

Another problem of major importance in control theory is that of synthesizing feedback controllers to achieve asymptotic tracking of prescribed reference output while rejecting disturbance [3]. The problem in question is to find, for every reference output function $y_{ref}(\cdot)$ in a prescribed family of functions, a control law, such that the corresponding response $y(\cdot)$ of the plant satisfies

$$\lim_{t\to\infty}(y(t)-y_{ref}(t))=0\,.$$

The solution of this problem includes two subproblems: first, the stabilization of the system in the absence of perturbation, and second, eliminating the effect of a perturbation. In [3] a complete solution for multivariable, linear, time-invariant, systems without delays and with continuous control was presented based on the existence of a solution of a set of linear matrix equations. On the other hand, in [20] the sliding mode regulator problem was introduced and existence conditions for its solution were derived for undelayed systems. The underlying idea is to design a sliding surface on which the dynamics of the system is constrained to evolve by means of a discontinuous control law, instead of designing a continuous stabilizing feedback, as in the case of the classical regulator problem. The sliding surface contains the steady-state surface, and the dynamics of the systems tend asymptotically, along the sliding surface, to the steady-state behavior.

In this paper, we consider a linear system with delay in state and control with both matched and unmatched perturbations. First, a sliding mode regulator problem for TDS is formulated. In order to solve this problem the block control principle [1], is applied. To achieve this, a special state representation, referred as the Block Controllable Form with Delay (or BCD-form), consisting of a set of blocks, will be used [15]. Using the block control technique, a sliding manifold is designed such that the sliding mode dynamics are invariant with respect to delay and perturbations. Note that designing a sliding controller without taking in the account the delay may cause chattering or even instability of the closed-loop system dynamics [4].

## 2 State Feedback Sliding Mode Regulation Problem

Let us consider a multivariable, linear, time-invariant TDS subject to an external disturbance, governed by

$$\dot{x}(t)=Ax(t)+Cx(t-\tau)+Bu(t)+Du(t-\tau)+Pw(t) \quad (1)$$

$$y(t)=Mx(t) \quad (2)$$

with the reference signal to be tracked

$$y_{ref}(t)=Qw(t) \quad (3)$$

and an exosystem

$$\dot{w}(t)=Sw(t) \quad (4)$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, $y \in R^p$ is the output, and $w \in R^q$ is the exogenous signal representing the reference and/or disturbance signals.

For the delay system (1)-(4), the **Sliding Mode Regulation Problem with Delays (SMRPD)** can be stated as the problem of finding, if possible, a sliding manifold

$$s(x(t), x(t-\tau), \ldots, x(t-q_0\tau),\ w(t),\ w(t-\tau), \ldots, x(t-q_1\tau) = 0 \tag{5}$$

and discontinuous feedback

$$u(t) = \sum_{i=1}^{p_1} K_{1,i} u(t-i\tau) + \sum_{j=0}^{p_2} K_{2,j} x(t-j\tau) + + \sum_{k=0}^{p_3} K_{3,k} w(t-k\tau) + K_4 sign(s) \tag{6}$$

such that:

**SD)** The closed loop system (1) and (6) in the absence of perturbations is asymptotically stable, i.e., for any initial condition $\varphi(t) \in [t_0 - \tau, t_0] \to R^n$ the corresponding solution $x(t)$ of (1) and (6) satisfies

$$x(t) \to 0 \text{ as } t \to \infty .$$

**RD)** The output tracking error goes asymptotically to zero, i.e.

$$\lim_{t\to\infty} (y(t) - y_{ref}(t)) = 0 . \tag{7}$$

Trying to apply the classical regulation theory to solve the SMRPD would imply to go through solving a set of matrix equations that depend on the delay terms, which could be a not simple task. In this paper, we propose an alternative way of finding a solution by transforming the delay system into a special form using the block control technique and then using a discontinuous control law.

### 3 Block Representation

The key idea of the block control technique is the use of a change of coordinates to transform the system (1) to the so-called Block Controllable Form with Disturbances (or BCD-form), consisting of *r* blocks:

$$\begin{aligned} \dot{x}_1(t) &= A_{11}x_1(t) + C_{11}x_1(t-\tau) + B_1 v_1(t) + P_1 w(t), \\ v_1(t) &= x_2(t) + \Pi_1 x_2(t-\tau) \end{aligned} \tag{8a}$$

$$\begin{aligned} \dot{x}_i(t) &= \sum_{j=1}^{i} [A_{i,j} x_j(t) + C_{i,j} x_j(t-\tau)] + B_i v_i(t) + P_i w(t), \\ v_i(t) &= x_{i+1}(t) + \Pi_i x_{i+1}(t-\tau), \ i = 2,\ldots,r-1 \end{aligned} \tag{8b}$$

$$\begin{aligned} \dot{x}_r(t) &= \sum_{j=1}^{r} [A_{r,j} x_j(t) + C_{r,j} x_j(t-\tau)] + B_r v_r(t) + P_r w(t), \\ v_r(t) &= u(t) + \Pi_r u(t-\tau) \end{aligned} \tag{8c}$$

with the output

$$y(t) = \sum_{i=1}^{r} M_i x_i(t) \tag{9}$$

where $\bar{x} = (x_1, \ldots, x_r)^T$, $x_i \in R^{n_i}$, $M = [M_1 \ \cdots \ M_r]$, $rankB_i = n_i$, $i = 1,\ldots,r$, and $\sum_{i=1}^{r} n_i = n$.

The integers $n_1, n_2, \cdots, n_r$ set the structure of the system, and we assume that they satisfy the condition

$$n_1 = n_2 = \cdots = n_r = m .$$

In this paper, we will assume that the initial system is transformable to the form (8a)-(8c). The transformation and the conditions, under which the system (1) can be reduced to the BCD-form (8a)-(8c) in the absence of disturbances, are derived in [15]. The modified transformation for the perturbed TDS (1) is presented in [16]. Note that the case when the output vector coincides with the state vector of the first block (8a), that is, $y(t) = x_1(t)$ was considered in [17].

### 4 Delay and Disturbances Block Cancellation

As in the classical setup for state feedback regulator problem we assume first that the state *x*(*t*) and disturbance *w*(*t*) are measurable. The design procedure for obtaining a discontinuous control law, which ensures asymptotic regulation of the output tracking error, will be divided in two steps. First, exploiting the block control technique [9], the system (8) will be transformed into a desired form, and a sliding surface will be constructed. Then, a discontinuous control law will be designed to make attractive this surface. For, we need to assume the steady state existence. The following result incorporates this condition.

**<u>THEOREM</u>**: *Assume that*

*(i)* *The system (1) is transformable to the BCD-form (8a)-(8c) with the output (9);*

*(ii)* *All eigenvalues of matrices* $\Pi_i$, $i = 1,\ldots,r$ *are located inside the open unit circle;*

*(iii)* *There exist matrices* $\Gamma_i$, $i = 1,\ldots,r$, *that solve the following matrix equations:*

$$A_{11}\Gamma_1 + C_{11}\Gamma_1 e^{-\tau S} + B_1[\Gamma_2 + \Pi_1\Gamma_2 e^{-\tau S}] = \Gamma_1 S - P_1 \tag{10a}$$

$$\sum_{j=1}^{i} \left[A_{i,j}\Gamma_j(t) + C_{i,j}\Gamma_j e^{-\tau S}\right] + B_i\left[\Gamma_{i+1} + \Pi_i\Gamma_{i+1} e^{-\tau S}\right] = \Gamma_i S - P_i \quad i = 2,\ldots,r-1 \tag{10b}$$

$$\sum_{i=1}^{r} M_i\Gamma_i - Q = 0 . \tag{11}$$

*Then the SMRP is solvable.*

The procedure for proving this theorem is constructive and is therefore included in the main text.

**Proof.** If the conditions (10a)-(10b) and (11) hold, then defining the new state as

$$\varepsilon_i = x_i(t) - \Gamma_i w(t), \quad i = 1,...,r \tag{12}$$

the system (8a)-(8c) can be represented as

$$\dot{\varepsilon}_1(t) = A_{11}\varepsilon_1(t) + C_{11}\varepsilon_1(t-\tau) + B_1\bar{v}_1(t),$$
$$\bar{v}_1(t) = \varepsilon_2(t) + \Pi_1\varepsilon_2(t-\tau) \tag{13a}$$

$$\dot{\varepsilon}_i(t) = \sum_{j=1}^{i}[A_{i,j}\varepsilon_j(t) + C_{i,j}\varepsilon_j(t-\tau)] + B_i\bar{v}_i(t),$$
$$\bar{v}_i(t) = \varepsilon_{i+1}(t) + \Pi_i\varepsilon_{i+1}(t-\tau), \; i = 2,...,r-1 \tag{13b}$$

$$\dot{\varepsilon}_r(t) = \sum_{j=1}^{r}[A_{r,j}\varepsilon_j(t) + C_{r,j}\varepsilon_j(t-\tau)] + B_r v_r(t) + \overline{P}_r w,$$
$$v_r(t) = u(t) + \Pi_r u(t-\tau) \tag{13c}$$

with the tracking error

$$e(t) = \sum_{i=1}^{r} M_i\varepsilon_i(t) + \left[Q - \sum_{i=1}^{r} M_i\Gamma_i\right] w \tag{14}$$

where $\overline{P}_r = A_{r,r}\Gamma_r + C_{r,r}\Gamma_r e^{-\tau S} + P_r - \Gamma_r S$. Here, we used $w(t-\tau) = e^{-\tau S}w(t)$.

The desired form and sliding manifold can be obtained in the following iterative procedure that consists in $r$ steps.

Step 1. We put $z_1(t) = \varepsilon_1(t)$, and then the first block (13a) can be presented as

$$\dot{z}_1(t) = \sum_{j=0}^{1} D^1_{1,j}\varepsilon_1(t-j\tau) + B_1 v_1(t) \tag{15}$$

where $D^1_{10} = A_{11}$, $D^1_{11} = C_{11}$. Now, we define the following desired dynamics for the first transformed block as

$$\dot{z}_1(t) = \Lambda_1 z_1(t) + z_2(t) \tag{16}$$

where $\Lambda_1$ is a matrix with desired eigenvalues, and $z_2 \in R^{n_2}$ is a vector of new variables. From equation (15) and desired dynamics (16), the following transformation between $z_2(t)$ and $v_1(t)$ or $\varepsilon_2(t)$, is derived:

$$z_2(t) = \sum_{j=0}^{1} D^1_{1,j}\varepsilon_1(t-j\tau) + B_1 v_1(t) - \Lambda_1 z_1 \tag{17a}$$

$$v_1(t) = \varepsilon_2(t) + \Pi_1\varepsilon_2(t-\tau). \tag{17b}$$

Step 2. Taking the derivative of (17a) along of the trajectories of (8a)-(8b), the second block can be represented as

$$\dot{z}_2(t) = \sum_{j=0}^{2}\left[D^1_{2,j}\varepsilon_1(t-j\tau) + D^2_{2,j}\varepsilon_2(t-j\tau)\right] + \overline{B}_2 v^1_2(t) \tag{18a}$$

where the entries of matrices, $D^i_{2,j}$, $i = 1,2$, $j = 0,1,2$, depend on the parameters of the systems (8a)-(8c) and (4), $\overline{B}_2 = B_1 B_2$, and

$$v^1_2(t) = [x_3(t) + \Pi_2 x_3(t-\tau)] + B_2^{-1}\Pi_1 B_2[x_3(t-\tau) + \Pi_2 x_3(t-2\tau)]$$

or

$$v^1_2(t) = v_2(t) + B_2^{-1}\Pi_1 B_2 v_2(t-\tau),$$
$$v_2(t) = x_3(t) + \Pi_2 x_3(t-\tau). \tag{18b}$$

Now we define the desired dynamics for $z_2(t)$ similar to (16), that is

$$\dot{z}_2(t) = \Lambda_2 z_2(t) + z_3(t)$$

where $z_3 \in R^{n_3}$, and $\Lambda_2$ is a matrix with desired eigenvalues. Using this equation and (18a)-(18b), the transformation between $z_3(t)$ and $v^1_2(t)$ or $\varepsilon_3(t)$ can be obtained of the form

$$z_3(t) = \sum_{j=0}^{2}\left[D^1_{2,j}\varepsilon_1(t-j\tau) + D^2_{2,j}\varepsilon_2(t-j\tau)\right] + \overline{B}_2 v^1_2(t) - \Lambda_2 z_2(t) \tag{19}$$

and so on. This procedure can be performed iteratively obtaining on the $p^{th}$ step, $p = 3,...,r-1$, the following recursive transformation:

$$z_{p+1}(t) = \sum_{j=0}^{p}\sum_{i=1}^{p} D^i_{p,j}\varepsilon_i(t-j\tau) + \overline{B}_p v^1_p(t) - \Lambda_p z_p(t) \tag{20a}$$

where $\overline{B}_k = B_1 \cdots B_k$, $\Lambda_k \subset R^{n_k \times n_k}$ is a desired matrix, and

$$\begin{aligned}
v^1_p(t) &= v^2_p(t) + B_p^{-1}\cdots B_2^{-1}\Pi_1 B_2 \cdots B_k v^2_p(t-\tau)\\
v^2_p(t) &= v^3_p(t) + B_{p-1}^{-1}\cdots B_3^{-1}\Pi_2 B_3 \cdots B_{p-1} v^3_p(t-\tau)\\
&\vdots\\
v^{k-1}_p(t) &= v_p(t) + B_p^{-1}\Pi_{p-1}B_p v_p(t-\tau)\\
v_p(t) &= x_{p+1}(t) + \Pi_p x_{p+1}(t-\tau).
\end{aligned} \tag{20b}$$

On the last step the system (8a)-(8c) is described in the new variables in the desired form

$$\dot{z}_i(t) = \Lambda_i z_i(t) + z_{i+1}(t), \qquad i = 1,...,r-1$$
$$\dot{z}_r(t) = \sum_{j=0}^{r}\sum_{i=1}^{r} D^i_{r,j}\varepsilon_i(t-j\tau) + \sum_{i=0}^{r-1} F_{r,i}w(t-i\tau) + \overline{B}_r v^1_r(t) \tag{21a}$$

where

$z = (z_1, \cdots, z_r)^T$, $z_i \in R^{n_i}, i = 1,\cdots r$, $\overline{B}_r = B_1 \cdots B_r$ and

$$\begin{aligned}
v^1_r(t) &= v^2_r(t) + B_r^{-1}\cdots B_2^{-1}\Pi_1 B_2 \cdots B_r v^2_r(t-\tau)\\
v^2_r(t) &= v^3_r(t) + B_{r-1}^{-1}\cdots B_3^{-1}\Pi_2 B_3 \cdots B_{r-1} v^3_r(t-\tau)\\
&\vdots\\
v^{r-1}_r(t) &= v_r(t) + B_r^{-1}\Pi_{r-1}B_r v_r(t-\tau)\\
v_r(t) &= u(t) + \Pi_r u(t-\tau).
\end{aligned} \tag{21b}$$

A natural choice of the sliding manifold (5), is

$$s = z_r = 0 \tag{22}$$

$$z_r(t) = \sum_{j=0}^{r-1}\sum_{i=1}^{r-1} D_{r-1,j}^{i}\varepsilon_i(t-j\tau) + \overline{B}_{r-1}v_{r-1}^{1}(t) - \Lambda_{r-1}z_{r-1}(t)$$

that is derived from (20a). The desired discontinuous dynamics are then defined as

$$\dot{s}(t) = k_r sign(s) \tag{23}$$

with $k_r < 0$. The discontinuous control law calculated from (21a) and (23) is

$$v_r^1(t) = -\overline{B}_r^{-1}\left[\sum_{j=0}^{r}\sum_{i=1}^{r} D_{k,j}^{i}x_i(t-j\tau) + \sum_{i=0}^{r-1}F_{k,i}w(t-i\tau)\right] + k_r\overline{B}_r^{-1}sign[s(t)] \tag{24}$$

and it provides sliding mode on (22) in a finite time. This motion is described in the new variables $z_1(t),\cdots,z_{r-1}(t)$ by the following $(n-n_r)^{th}$ order system:

$$\begin{aligned} \dot{z}_i(t) &= \Lambda_i z_i(t) + z_{i+1}(t), \quad i = 1,\cdots,r-2 \\ \dot{z}_{r-1}(t) &= \Lambda_{r-1}z_{r-1}(t) \end{aligned} \tag{25}$$

with the desired dynamics.

The stability of the closed loop system (8a)-(8c) and (24) is defined first by eigenvalues of the system (25) that can be chosen arbitrarily, and second, by the property of the state and control internal dynamics, presented by (17b), (18b), (20b) and (21b), respectively. It follows that the state and control internal dynamics are asymptotically stable if the condition (*ii*) of the Theorem holds. To show this, the control internal dynamics (21b) can be represented in discrete time as

$$\begin{aligned} u_k &= -\Pi_r u_{k-1} + v_{r,k} \\ v_{r,k} &= -B_r^{-1}\Pi_{r-1}B_r v_{r,k-1} + v_{r,k}^{r-1} \\ &\vdots \\ v_{r,k}^3 &= -B_{r-1}^{-1}\cdots B_3^{-1}\Pi_2 B_3\cdots B_{r-1}v_{r,k-1}^3 + v_{r,k}^2 \\ v_{r,k}^2 &= -B_r^{-1}\cdots B_2^{-1}\Pi_1 B_2\cdots B_r v_{r,k-1}^2 + v_{r,k}^1 \end{aligned} \tag{26}$$

where

$u_k = u(kT)$, $u_{k-1} = u((k-1)T)$, $v_{r,k}^i = v_{r,k}^i(kT)$,
$v_{r,k-1}^i = v_{r,k-1}^i((k-1)T)$, $i = 1,...r-1$, and $v_{1,k} = v_1(kt)$,
$T = \tau$, $t = kT$, $k = 0,1,...$.

We may observe that the system (26) is stable if the condition (*ii*) holds. In this case, the output $u_k$ of system (26), is bounded for any bounded input $v_{r,k}^1$. In the same manner it is possible to show stability for the state internal dynamics (17b), (18b) and (20b). If the matrices $\Lambda_i$, $i = 1,\cdots,r-1$ in the sliding mode dynamics (25) are Hurwitz, then

$$\lim_{t\to\infty} z_i(t) = 0,\ i = 1,...,r-1$$

and by condition (*ii*) of the Theorem, we have

$$\lim_{t\to\infty} \varepsilon_i(t) = 0,\ i = 1,...,r-1$$

on the manifold $\varepsilon_r(t) = 0$. Hence, the requirement SD) is fulfilled. Finally, if the condition (11) holds the tracking error (14) converges asymptotically to zero, and therefore the requirement RD) is also fulfilled.

## 5 Example

Consider the following second order system with delay in control and state that is in the BCD form:

$$\begin{aligned} &\dot{x}_1 = x_1(t) + x_1(t-\tau) + P_1w(t) + v_1(t), \\ &v_1(t) = x_2(t) + \pi_1x_2(t-\tau) \end{aligned} \tag{27a}$$

$$\begin{aligned} &\dot{x}_2 = x_1(t) + x_1(t-\tau) + x_2(t) + x_2(t-\tau) + P_2w(t) + v_2(t), \\ &v_2(t) = u(t) + \pi_2u(t-\tau) \end{aligned} \tag{27b}$$

with the output

$$y(t) = Mx(t) \tag{28}$$

where $x = [x_1, x_2]^T$, $M = [1\ \ 1]$, $P_1 = P_2 = [1\ \ 0]$, and parameters $\pi_1$ and $\pi_2$ which satisfy the condition (*ii*) of the Theorem, that is, $|\pi_1| < 1$ and $|\pi_2| < 1$. The reference signal $y_{ref} = Qw(t)$, $Q = [1\ 0]$ is generated from the exosystem

$$\dot{w}(t) = Sw(t), \quad S = \begin{bmatrix} 0 & \alpha \\ -\alpha & 0 \end{bmatrix}, \ \alpha > 0 \tag{29}$$

where $w = (w_1, w_2)^T$. Now we apply the block control technique described in the Section 4. Defining the steady state for $x_1(t)$ and $x_2(t)$ as $\Gamma_1w(t)$ and $\Gamma_2w(t)$, respectively, we introduce the following tracking errors:

$$\varepsilon_1(t) = x_1(t) - \Gamma_1w(t) \ \text{ and } \ \varepsilon_2(t) = x_2(t) - \Gamma_2w(t).$$

The parameters of the matrices $\Gamma_1 = [\gamma_{11}\ \ \gamma_{12}]$ and $\Gamma_2 = [\gamma_{21}\ \ \gamma_{22}]$ are calculated as a solution of the following equations:

$$\Gamma_1 + \Gamma_1e^{-\tau S} + \Gamma_2 + \pi_1\Gamma_2 = \Gamma_1S - P_1 \ \text{ and } \ \Gamma_1 + \Gamma_2 = Q$$

of the form

$\gamma_{11} = \pi_1[(\pi_1 - 1)(e_{11}e_{22} - e_{12}e_{21}) + \alpha e_{12} + 2e_{22}] - 2e_{22}$,
$\gamma_{12} = -\gamma_{22} = -\pi_1(\alpha e_{11} + 2e_{12}) - 2(\alpha - e_{12})$,
$\gamma_{21} = (\pi_1 - 1)(e_{11}e_{22} - e_{12}e_{21} - \alpha e_{21} - 2e_{22}) + \alpha(\alpha - e_{12})$,

$$e^{-\tau S} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} = \begin{bmatrix} \cos(\alpha\tau) & -\sin(\alpha\tau) \\ \sin(\alpha\tau) & \cos(\alpha\tau) \end{bmatrix}.$$

Then the system (27a)-(27b) is represented in $\varepsilon_1(t)$ and $\varepsilon_2(t)$ as

$$\dot{\varepsilon}_1(t) = \varepsilon_1(t) + \varepsilon_1(t-\tau) + v_1(t) \tag{30a}$$

$$\dot{\varepsilon}_2(t) = \varepsilon_1(t) + \varepsilon_1(t-\tau) + \varepsilon_2(t) + \varepsilon_2(t-\tau) + \overline{P}_2 w(t) + v_2(t) \tag{30b}$$

where $\overline{P}_2 = \Gamma_1 + \Gamma_2 + P_2 - \Gamma_2 S + (\Gamma_1 + \Gamma_2)e^{-\tau S}$.

The change of variables similar to (17a)

$z_1(t) = \varepsilon_1(t)$,

$z_2(t) = \varepsilon_1(t) + \varepsilon_1(t-\tau) + \varepsilon_2(t) + \pi_2 \varepsilon_2(t-\tau) - k_1 z_1(t)$, $k_1 < 0$

reduces the system (30a)-(30b) to

$$\dot{z}_1(t) = k_1 z_1(t) + z_2(t)$$

$$\dot{z}_2(t) = \sum_{J=0}^{2} d_{2,j}^{1} \varepsilon_1(t - j\tau) + \sum_{J=0}^{2} d_{2,j}^{2} \varepsilon_2(t - j\tau) + \overline{P}_2 w_2(t) + \pi_1 \overline{P}_2 w_2(t-\tau) + v_2^1(t)$$

where

$d_{20}^1 = 2 - k_1$, $d_{11}^1 = 2 - k_1 + \pi_1$, $d_{12}^1 = 1 + \pi_1$, $d_{20}^2 = 2 - k_1$, $d_{21}^2 = 2\pi_1 - \pi_1 k_1 + 2$ and $d_{22}^2 = 2\pi_1$.

Then the control

$$v_2^1(t) = -\sum_{j=0}^{2} d_{2,j}^{1} \varepsilon_1(t - j\tau) - \sum_{j=0}^{2} d_{2,j}^{2} \varepsilon_2(t - j\tau) - \overline{P}_2 w_2(t) - \overline{P}_2 w_2(t-\tau) + k_2 sign[\overline{z}_2(t)],$$

with $k_2 < 0$ guarantees the sliding mode motion on $z_2 = 0$ described by $\dot{z}_1(t) = k_1 z_1(t)$.

**Simulation results.**

For this example we selected the following parameters: $\alpha = 1$, $\pi_1 = 0.4$, $\pi_2 = 0.5$, $\tau = 0.5$, $k_1 = -5$, $k_2 = -5$. Figures 1 and 2 show responses for the output *y*, reference $w_1$ and tracking error $e = y - y_{ref}$, respectively. Responses for the sliding variable $z_2$ and control input $u$ are depicted in Figures 3 and 4, respectively.

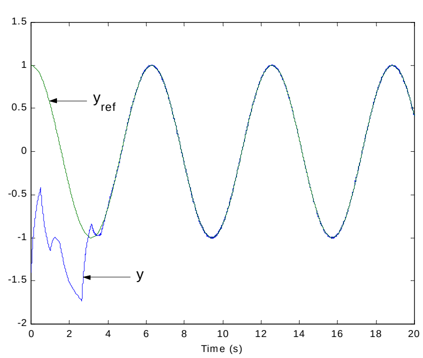


Fig. 1. The plant output $y = x_1 + x_2$ and reference $y_{ref} = w_1$.

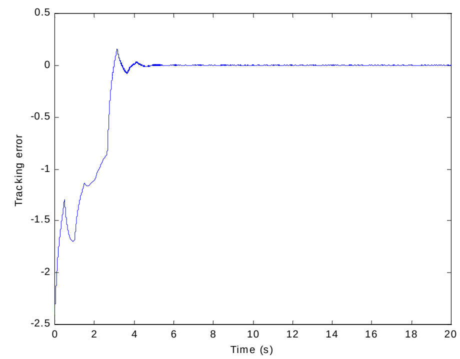


Fig. 2. The tracking error $e = y - y_{ref}$.

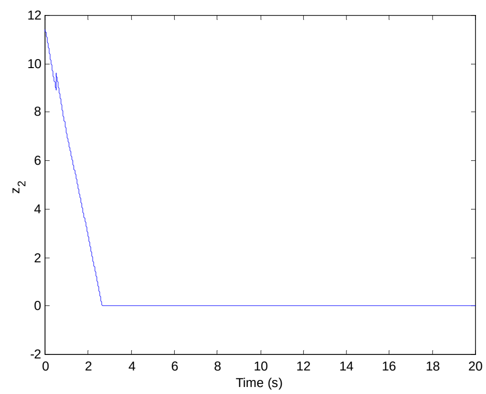


Fig. 3. The switching function $z_2$.

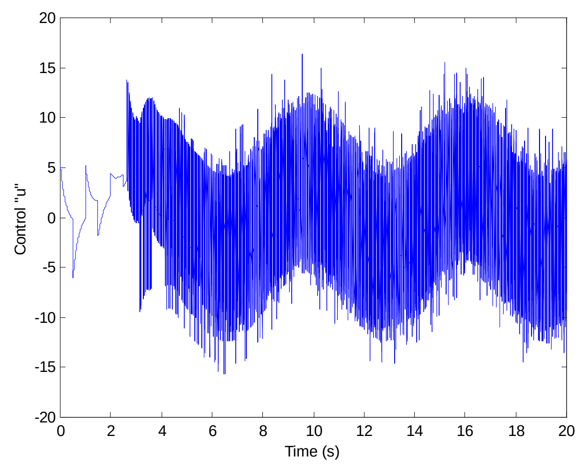


Fig. 4. The control $u$

## 6. Conclusions

In this work the sliding mode regulator problem with delay is introduced, and conditions for the existence of a solution for a class of multivariable linear TDS presented in BCD form and which satisfies a controllability condition on the non delay part of the TDS, are derived. Based on this form, and using the block control technique, a discontinuous feedback which ensures trajectory tracking, is designed. The simulation results confirm the effectiveness of the proposed method. Possible extension to the class of TDS for which the controllability condition may include the delay part as well, is object of further research.

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