ON HAZARDS OF USING FUNDAMENTAL ANTI-WINDUP TECHNIQUE FOR H_2 STATE-SPACE CONTROLLERS WITH AN EXPLICIT OBSERVER

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Abstract

This paper considers H_2 controllers implemented using an explicit observer for systems subjected to input magnitude saturation. It is stressed that the approach of only focusing on controller windup, by using fundamental observer anti-windup technique where the observer is fed by the saturated control signal, could imply severely deteriorated closed-loop characteristics and even sustained oscillations.

1 Introduction

All control system applications are somehow subjected to physical limitations imposing constraints. In particular, control object input limitations are of special interest and common occurrences. Among others, limitations could be due to actuator saturation which is referred to as *input saturation*.

Design methods and their corresponding control laws that takes non-linearities, such as input saturation, into account are quite complicated. Therefore, presence of non-linearities are usually neglected during control synthesis and consequently, the control object input is, in general, different from the controller output. This mismatch cause *controller windup* and could lead to performance degradation and even instability.

Windup problems were originally encountered when using PI/PID controllers (called *integral windup*) to deal with linear systems subjected to input saturation. Later on, it was recognized by Doyle *et al.* ([2]) that integral windup is only a special case of the more general mismatching problem mentioned above.

Numerous methods to deal with this inconsistency between the control object input and the state of the controller when, for example, the actuators saturates, exist. In the case of state-space formulation of a feedback controller where it is viewed as a combination of an observer and a state feedback, many authors ([6, 4, 11, 12]) recommend feeding the observer with the saturated control signal instead of the control signal computed by the state feedback. This technique is here referred to as fundamental anti-windup for state-space feedback controllers with an explicit observer.

Furthermore, it has been stressed by Hippe *et al.*, Öhr *et al.*, Kothare *et al.*, and Rönnbäck ([6, 5, 10, 7, 9]) that it is not always enough to focus on controller windup. These authors use an extended interpretation of *windup* which has been adopted in this paper and could be summarised as follows: Windup is the degradation of the closed-loop performance and stability margins due to e.g. input saturation.

Even though the inadequacy of only focusing on controller windup has been pointed out, it has never before been mentioned that the commonly recommended fundamental anti-windup observer approach mentioned above could actually lead to severely deteriorated closed-loop characteristics and even limit cycles. This was discovered while working with the design of an active engine vibration isolation system subjected to actuator constraints ([8]) and is further explored in this paper.

The aim of this paper is to stress the potentially hazardous consequences of using the fundamental and widely used anti-windup technique related to the case of feedback controllers utilising state feedback and an explicit observer.

The organisation of this paper is as follows: Section 2 describes the fundamental anti-windup technique for controllers based on observer and state feedback while Section 3 presents the *describing function theory* used to predict the presence of limit cycles in feedback systems containing static non-linearities. Section 4 introduces the specific SISO control object and requirements specifications used for studying the effects of input saturation. An H_2 controller design giving rise to the input saturation characteristics is presented in Section 5, whereas Section 6 shows some simulations demonstrating the effect of input saturation with and without anti-windup compensation. Finally, the paper is concluded in Section 7.

2 Fundamental Anti-windup for State-Space Controllers with an Explicit Observer

A H_2 controller can be implemented with an internal structure identical to the structure of an LQG controller, i.e. with a state observer and state feedback according to (1).

$$\begin{aligned} \dot{\hat{x}} &= \overline{A}\hat{x} + \overline{B}\bar{u} + L(y - \overline{D}\bar{u} - \overline{C}\hat{x}) \\ \bar{u} &= -K\hat{x} \end{aligned} \tag{1}$$

Here, the controller states represents the estimated control object internal states appended with some extra states due to implementation of the weighting functions used in the design of an H_2 controller. In the case of possibly saturated actuators the control signal applied to the control object input, will generally differ from the control signal $\bar{u}$ computed in (1).

Let the non-linear function $f(\bar{u})$ represent saturation defined as

$$f(\bar{u}) = \begin{cases} u_{low} & \bar{u} \leq u_{low} \\ \bar{u} & u_{low} < \bar{u} < u_{high} \\ u_{high} & \bar{u} \geq u_{high} \end{cases} \tag{2}$$

Focusing on controller windup with windup interpreted as consequences due to the controller being unaware of actuator saturation, the fundamental *anti-windup* technique is to feed the observer with the measured, or estimated, applied control object input as described by (3).

$$\begin{aligned} \dot{\hat{x}} &= \overline{A}\hat{x} + \overline{B}\bar{u} + L(y - \overline{D}\bar{u} - \overline{C}\hat{x}) \\ \bar{u} &= f(-K\hat{x}) \end{aligned} \tag{3}$$

Figures 1 and 2 describe schematically those principally different ways of implementing the controller, i.e. without and with anti-windup compensation respectively.

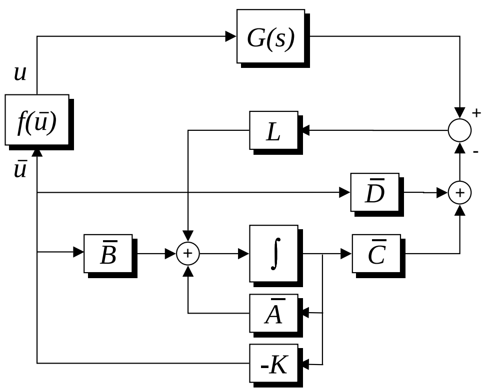

Figure 1: Implementation of a H_2 controller using the computed control force for state estimation

3 Describing Function Analysis

Describing function analysis [3, 1] is used to predict the presence of limit cycles in a feedback system containing a static non-linearity. It is an approximate method and cannot be used to prove the presence of a limit cycle but is normally used to give an indication of the closed-loop system behaviour. The set-up for describing function analysis is shown in Figure 3.

The describing function theory could be summarised as

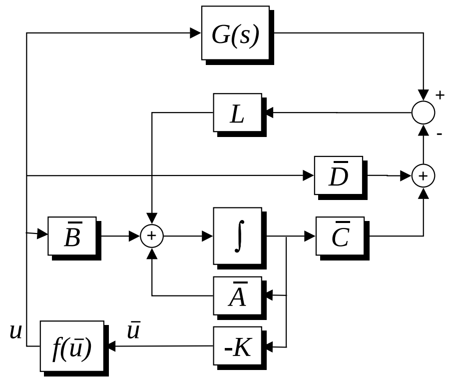

Figure 2: Implementation of a H_2 controller by fundamental anti-windup technique when the feedback controller is formulated using an explicit observer

follows [3]: If there is a linear system ($\widetilde{G}(s)$) and a static non-linearity in a closed-loop negative feedback system, the condition for a limit cycle is given by (4),

$$Y_f(C)\widetilde{G}(i\omega) = -1 \tag{4}$$

where $Y_f(C)$ is the describing function for the non-linearity $f(\bar{u})$ in Figure 3, C and ω are the amplitude and frequency respectively of the oscillating signal $\bar{u}$. The amplitude and frequency of an oscillation could also be identified graphically since (4) correspond to the intersection between $\widetilde{G}(i\omega)$ and $-1/Y_f(C)$.

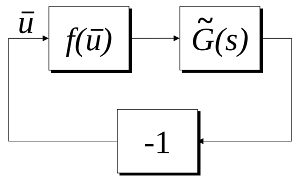

Figure 3: The set-up for describing function analysis. The function $f(\bar{u})$ representing the non-linear saturation characteristics is described by (2)

Assuming that the actuators saturate at $\pm$S N, the expression for the describing function corresponding to (2) becomes [3]

$$Y_f(C) = \begin{cases} \frac{2}{\pi}(\arcsin\frac{S}{C} + \frac{S}{C}\sqrt{1-(\frac{C}{S})^{-2}} & C > S \\ 1 & C \leq S \end{cases} \tag{5}$$

To investigate the closed-loop characteristics of the two different controller implementations illustrated in Figures 1 and 2 using graphical describing function analysis, the Nyquist diagram of $\widetilde{G}(s)$ is required. However, the linear system $\widetilde{G}(s)$ will be different for the two implementations. $\widetilde{G}$ has been derived for the two cases and for state estimation using the computed control signal according to Figure 1, $\widetilde{G}$ is

$$\widetilde{G}(s) = K(sI - \overline{A} + \overline{B}K - L\overline{D}K + L\overline{C})^{-1}LG(s) \qquad (6)$$

where $G(s)$ is the transfer function of a SISO control object. For an implementation according to Figure 2 (i.e. with anti-windup compensation), $\widetilde{G}$ is

$$\widetilde{G}(s) = G_1(s) + G_2(s)G(s) \qquad (7)$$

where

$$\begin{aligned} G_1(s) &= K(sI - \overline{A} + L\overline{C})^{-1}(\overline{B} - L\overline{D}) \\ G_2(s) &= K(sI - \overline{A} + L\overline{C})^{-1}L \end{aligned} \qquad (8)$$

4 A SISO Vibration Isolation Example

Consider the vibration isolation system presented in Figure 4. The one translational DOF mass is suspended using a spring and a damper. It is excited primarily (disturbed) by a force F_p in the direction of the spring and by a secondary force F_s, i.e. the controller output, applied between the mass and the receiver according to the figure. The control object output is produced by a sensor measuring the total force applied to the receiver in the direction of the control force, F_s.

Linearising the control object around static equilibrium, i.e. assuming a constant angle α, the transfer function presented in (9) is obtained.

$$G(s) = \frac{-(s^2 + 1.23s + 920.5)}{(s^2 + 2.004s + 1500)} \qquad (9)$$

The objective of a controller for the described system is to isolate the vibrations of the mass from the receiver by minimising the forces transmitted to the receiver in the direction of the control signal. Moreover, the mass suspension illustrated in Figure 4 should be able to carry high static loads.

The system described above is an invented one, only used here to illuminate the potential risk of using the fundamental anti-windup technique. However, similar conditions could be found in reality, e.g. when dealing with active engine vibration isolation [8].

5 H_2 Design and Input Saturation

According to Section 4 the closed-loop requirements for the SISO vibration isolation system considered correspond to a

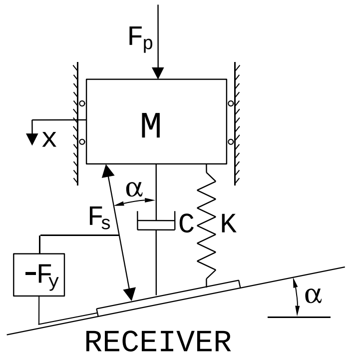

Figure 4: One kinematical DOF system

low sensitivity in a frequency range above DC. Such requirements specifications could be met by using H_2 synthesis with a weighting function W_S for the sensitivity and another one W_U for the transfer function from an output disturbance to the control object input, presented in (10) and (11), respectively. The resulting H_2 controller is of 6th order and the corresponding sensitivity S and complementary sensitivity T are presented in Figure 5.

$$W_S = \frac{(1 + s/(2 \cdot \pi \cdot 3))}{(1 + s/(2 \cdot \pi \cdot 20))(1 + s/(2 \cdot \pi \cdot 100))^2} \qquad (10)$$

$$W_U = \frac{50(1 + s/(2 \cdot \pi \cdot 5))}{(1 + s/(2 \cdot \pi \cdot 0.01))} \qquad (11)$$

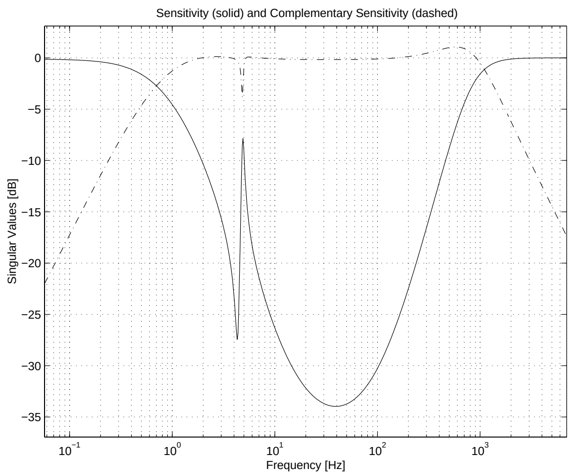

Figure 5: S (solid) and T (dashed) corresponding to (10) and (11)

To investigate the closed-loop characteristics in the presence of input saturation, the graphical describing function analysis described in Section 3, has been applied. Figure 6 shows the

Nyquist diagram of $\widetilde{G}$ when the controller is implemented according to Figure 1 and hence, there is no indication of instability. On the other hand, Figure 7 displays the intersection (corresponding to approximately 5.75 Hz) with the negative real axis to the left of -1 indicating a limit cycle, when the controller is implemented according to Figure 2, i.e. when using the fundamental anti-windup technique.

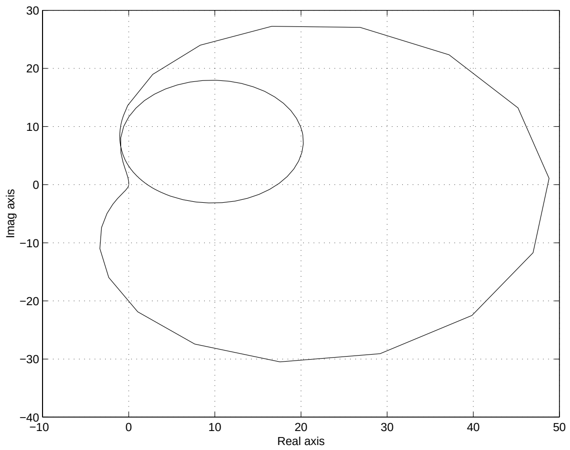

Figure 6: Nyquist diagram of $\widetilde{G}$ given by (6) corresponding to (10) and (11), without reflection with respect to the real axis

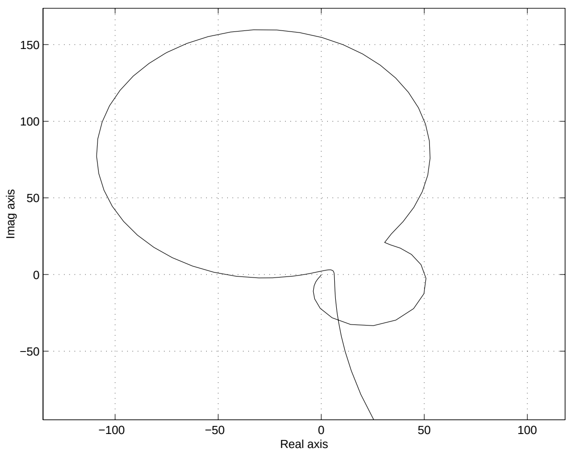

Figure 7: Nyquist diagram of $\widetilde{G}$ given by (7) corresponding to (10) and (11), without reflection with respect to the real axis

6 Simulations

To verify the results obtained using describing function analysis, simulations have been carried out. The figures below present the results from two different closed-loop simulations corresponding to the two different controller implementations. The control object is exited by a 10 Hz sinusoidal signal with an amplitude of 10 N and the actuator is assumed to be saturating at ± 5 N. The simulation outcomes correspond to the analysis results presented in Section 5.

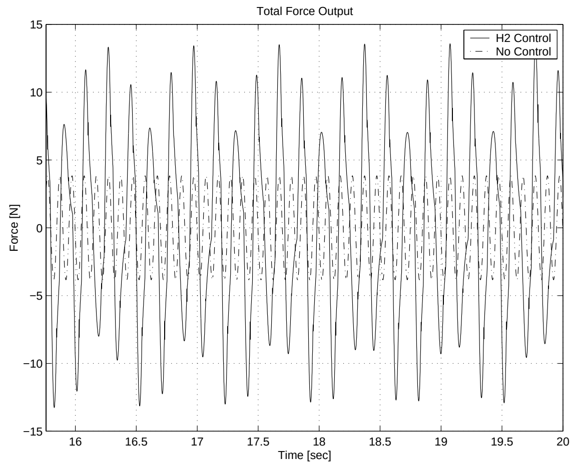

Figure 8: Control object output, with and without control. The controller is implemented according to Figure 2

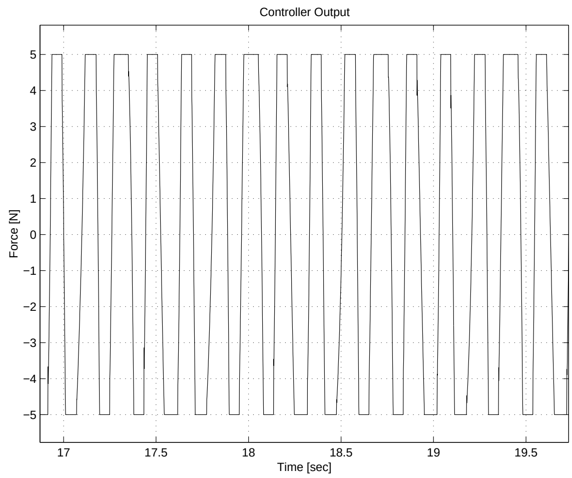

Figure 9: Applied control force with controller implemented according to Figure 2

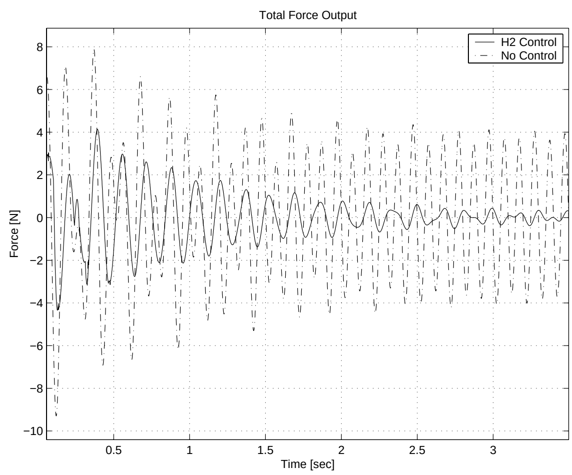

Figure 10: Control object output, with and without control. The controller is implemented according to Figure 1

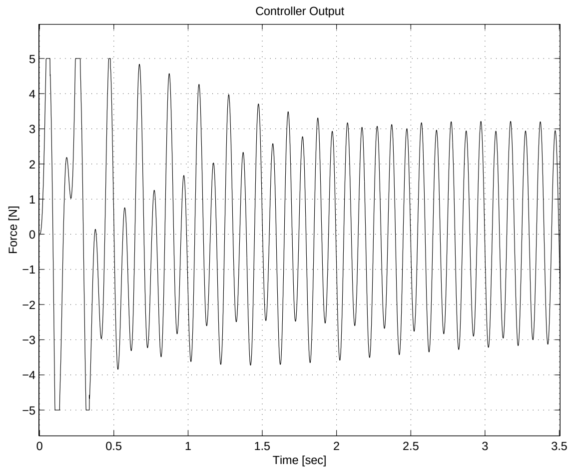

Figure 11: Applied control force with controller implemented according to Figure 1

7 Conclusions

A vibration isolation example has been considered to highlight the possibly hazardous effects of using fundamental anti-windup technique for a H_2 controller implemented using an explicit observer. It has been shown that closed-loop performance and stability characteristics could be substantially deteriorated. At the same time as controller windup is eliminated by feeding the observer with the applied control signal, sustained oscillations, (i.e. limit cycles), could be obtained.

For stable systems H_2 design methodology generates controllers for which the corresponding loop gains are guaranteed not to encircle -1 in a Nyquist diagram, i.e. to be stable or at least on the margin of stability. When a H_2 controller that causes saturated actuators is implemented using fundamental anti-windup technique, the effective loop transfer function ($\widetilde{G}$ in Figure 3) enclosing the non-linear function representing the saturation might cross the real axis to the left of -1. For stable control objects this usually implies closed-loop instability according to Nyquist stability criteria, and in the case of input saturation it indicates the presence of a limit cycle.

It is found to be easy to design a closed-loop system with the unusual saturation effects for a control object with one pole/zero pair where the zero corresponds to a lower frequency. Considering the transfer function of such control object, this characteristic implies a phase shift near plus 180 degrees followed by a negative one of near 180 degrees. This phase characteristics are easily given to $\widetilde{G}$ in Figure 3, creating conditions for intersection with the real axis to the left of -1 indicating the presence of a limit cycle.

Finally, when using the fundamental anti-windup technique to deal with H_2 controllers and input saturation, limit cycles appear only for certain specific combinations of control object characteristics and choices of H_2 weighting functions. It remains to explain exactly when and why these sustained oscillations occur.

Acknowledgements

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