A GENERAL FRAMEWORK FOR ROBUST ANTI-WINDUP SCHEMES

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Abstract

A general framework for designing antiwindup control systems is developed. More traditional anti-windup schemes generally suffer from several limitations, including the lack of (a priori) robustness respect to plant uncertainty, and also have a considerable heuristic component. The design method proposed in this paper provides an accurate graphic design procedure that guarantees closed loop global stability, in the presence of saturation-type non-linearities and plant uncertainty. The case of SISO LTI plant is considered.

1. Introduction

Actuator saturation is a common and significant non-linearity in practical control systems. If input constraints are not taken into account, harmful effects on system performance and stability may appear. If, in addition, the controller is unstable or contains integrators the known phenomenon referred to as integral windup may occur. When the control signal saturates, the feedback is broken and then the controller continues integrating the tracking error, providing larger control signals resulting in undesirable large overshoots or even driving the system to unstability. Control systems design with hard constraints has become a very active research area, see for example [3]. Another important aspect in control practice is plant uncertainty. Most of the techniques included in the anti-windup field does not explicitly consider plant uncertainty in the (LTI) plant, this being an important practical drawback. It seems to be fundamental to obtain robust control systems with acceptable performance under uncertainty and design constraints. In this direction, Quantitative Feedback Theory (QFT) ([6]) provides a natural frequency domain framework for designing robust controllers. And for the linear mode, linear QFT can be directly used. In this paper, the previous work in [2] is extended to cope with rate saturation, and a general framework is proposed, to take the uncertainty into account in the design procedure and to assure the global stability of the system. This framework can be applied to some common anti-windup schemes, whose parameters are usually heuristically fixed in practice. The problem considered is based on a three degrees of freedom (3DoF) control scheme (Figure 1), and consists in designing H to account for the saturation element in a control loop. The compensators F and G are considered to be previously designed for the linear mode. As shown in [5] , an actuator model is assumed, so that this model is inserted after the controller and before the actuator. The paper is organized as follows. In section 2 the method proposed in [2] is reviewed for amplitude saturation case and is extended to deal with the rate saturation problem. In section 3 a general framework to cope with the saturation problem in systems with uncertainty, from a frequency domain viewpoint is proposed. Finally some conclusions are outlined in section 4.

2. Main result

In this section, we summarize two design methods that take plant uncertainty into account when saturation elements exist in a control system. In section 2.2 the most important results in [2] are reviewed without proof, related to amplitude saturation case. Analogously, in section 2.3, the rate saturation case is analyzed. In both cases, emphasis is made on closed loop stability of the nonlinear operation mode. Stability based on multipliers ([4]) is used as a basic result.

2.1. Preliminaries

The following notation is introduced: RH_∞ is the set of proper (bounded at infinity) complex variable rational functions with real coefficients without poles in the closed right half plane (right half plane including the origin), and R_0H_∞ is the same set but including functions with poles in the origin. A memoryless non-linearity $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ belongs to $sector[\epsilon, k]$ if $\varphi(0) = 0$ and $\epsilon \leq \varphi(x)x \leq kx^2 \; \forall x \in \mathbb{R}$ with $\epsilon > 0$. A memoryless non-linearity $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ belongs to $slope[\epsilon, k]$ if $\varphi(0) = 0$ and $\epsilon \leq (\varphi(y_1) - \varphi(y_2))/(y_1 - y_2) \leq k \; \forall y_1, y_2 \in \mathbb{R}$, with $y_1 \neq y_2$ and $\epsilon > 0$.

Consider the feedback system of Figure 1without the nonlinear block, that is considering $N = 1$. Here P is a SISO LTI plant belonging to some set $\wp$, G is a feedback compensator and F is a precompensator. This simplified feedback system is the typical two degrees of freedom structure, commonly used in QFT. The following assumptions are used in the rest of this work regarding this simplified feedback system: i) the closed loop system is stable for all P in $\wp$, ii) the open loop transfer function $L(s) = P(s)G(s)$ must belong to R_0H_∞ for all $P \in \wp$, iii) $G(s)$ and $H(s)$ must belong to R_0H_∞, iv)the number of integrators in H must be greater or equal than the number of integrators in G, and less than the number of integrators in G plus one, and less or equal than the number of integrators in G plus the number of integrators in P and v)$(1 + H)^{-1}$ has the same number of poles and zeros,its poles must belong to the open LHP (not including the origin), and its zeros must belong to the closed LHP. Note that typical Type 0, Type I or Type II plants are allowed, and that integrators in the compensator G, designed in a first step, are allowed too.

2.2. Systems with amplitude saturation

Figure 1 shows the basic control setup, where the block P represents a SISO LTI plant with transfer function $P(s)$ in a set $\wp \subset R_0H_\infty$. The compensator consists of three blocks: $F(s)$, the precompensator, $G(s)$, the feedback compensator, and $H(s)$, the anti-windup compensator. An ideal saturation is assumed. In order to analyze system stability, is more convenient to transform the system to the equivalent system (from the stability point of view) of Figure 2, consisting of a feedback interconnection of a linear block K, including all linear dynamics present in the loop, and the non-linear block N. K and R are systems with transfer functions given by

$$R(s) = F(s)G(s)/(1 + H(s)) \quad (1)$$
$$K(s) = (P(s)G(s) - H(s))/(1 + H(s)) \quad (2)$$

For design purposes, X is defined as

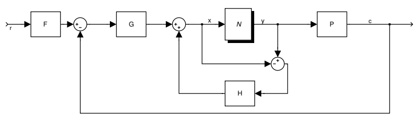


Figura 1: A 3DoF control scheme with actuator amplitude saturation.

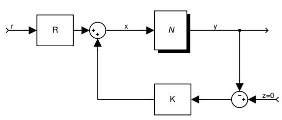
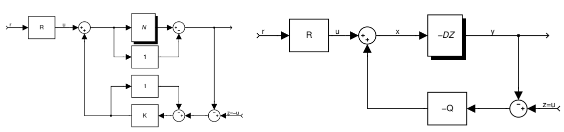


Figura 2: System with a feedback interconnection between a linear block K and a non-linear block N.



(a) (b)

Figura 3: A single transformation for the system in Figure 2.

$$X(s) \triangleq 1 + K(s) = (1 + P(s)G(s))D(s) \quad (3)$$

where $D(s) = (1 + H(s))^{-1}$. When D is designed, the computation of the anti-windup compensator H is straightforward. One basic goal is to define regions in NP such that if X^0 lies within them, then closed loop stability may be guarantied. Although it will be not detailed here, an algorithm to design H has been proposed in [2] using QFT ideas. It is supposed that the pair of compensators $\{F, G\}$ was designed in a previous step using (for example) QFT. The algorithm is strongly based on the following result:

Lemma 2.1 *Assume that*

$$X(s) \in RH_\infty \; \forall P \in \wp \quad (4)$$

$$N \textit{ is static, odd, and belonging to slope}[0, 1] \quad (5)$$

and, there exists a LTI multiplier (possibly non causal) $Z(s)$, with impulse response $z(t)$ satisfying $\|z(t)\|_1 < 1$, and

$$X(j\omega) \neq ja/(1 - Z(j\omega)) \; \forall \omega > 0 \; \forall a \in \mathbb{R} \; \forall P \in \wp \quad (6)$$

then the feedback system is globally stable system for all plants P in $\wp$. If N is even, then additionally $z(t) > 0$ for all $t \in \mathbb{R}$.

If there exists integrators in P, the above result can not be directly used. But it is possible to circumvent this problem by means of blocks transformations. See Figure 3(a). This system is equivalent to the system of Figure 3(b), that consists of a linear block R and a feedback interconnection between a linear block Q and a non-linear block DZ with

$$DZ(x) = \begin{cases} 0 & \text{if } x(t) \in [y_{min}, y_{max}] \\ -x + y_{min} & \text{if } x < y_{min} \\ -x + y_{max} & \text{if } x > y_{max} \end{cases} \quad (7)$$

and

$$Q(s) = \frac{K(s)}{1 + K(s)} \quad (8)$$

Lemma 2.2 *Assume that*

$$X(s) \in R_0H_\infty \; \forall P \in \wp \quad (9)$$

$$N \textit{ is static, odd, and belonging to slope}[0, 1] \quad (10)$$

and, there exists a LTI multiplier (possibly non causal) $Z(s)$, with impulse response $z(t)$ satisfying $\|z(t)\|_1 < 1$, and

$$1/X(j\omega) \neq ja/(1 - Z(j\omega)) \; \forall \omega > 0 \; \forall a \in \mathbb{R} \; \forall P \in \wp \quad (11)$$

then the feedback system is globally stable system for all plants P in $\wp$. If N is even, then additionally $z(t) > 0$ for all $t \in \mathbb{R}$.

2.3. Systems with rate saturation

In this section the robust stability problem of QFT designs for linear systems with actuator rate saturation is analyzed, using a similar reasoning as in previous section. Using a 3DoF scheme, the design method previously proposed may be used in order to stabilize systems with actuator rate saturation and uncertain plants with one or zero integrators. In this case K and R blocks in Figure 2 are transfer functions given by

$$\begin{aligned} R(s) &= F(s)G(s)s/(1+sH(s)) && (12)\\ K(s) &= (P(s)G(s)-sH(s))/(1+sH(s)) && (13) \end{aligned}$$

If simple variable changes of variable are performed, $H_v(s) = sH(s), G_v(s) = sG(s)$ and $P_v(s) = s^{-1}P(s)$, the following transfer functions K_v and R_v result

$$\begin{aligned} R_v(s) &= F(s)G_v(s)/(1+H_v(s)) && (14)\\ K_v(s) &= (P_v(s)G_v(s)-H_v(s))/(1+H_v(s)) && (15) \end{aligned}$$

As it may be observed, equations (14) and (15) are equivalent to (1) and (2) respectively, so that all the development in section 2.2 is also applicable to the rate saturation problem, taking K_v and R_v in equations (15) and (14) respectively, $G_v(s) = sG(s), P_v(s) = s^{-1}P(s)$, and when the function $D = (1+H_v)^{-1}$ has been shaped, computing $H(s)$ as $H_v(s)/s$. However, in the amplitude saturation problem the maximum number of integrators in plant was two, and in the rate case, as $P_v = s^{-1}P$, the number of integrators in the plant must be restricted to one.

2.4. QFT solution to the stability problem

Here we briefly discuss how QFT can be used to efficiently solve the global stability problem according to the conditions given in the above Lemmas. The basic idea is the shaping of $D(s)$ in such a way that the condition given by (6) or (11), depending on the case, is satisfied. QFT can give an efficient and simple graphical solution to that equations. Once $D(s)$ is obtained, the anti-windup compensator H can be directly obtained. First, we define a nl-template $\Im_{nl}(\omega)$ (template for non-linear mode) as a set of complex values, given by $\Im_{nl}(\omega) \triangleq \{1 + P(j\omega)G(j\omega) : P \in \wp\}$. This set will allow the designer to deal with the uncertainty in an explicit form in the process design for the shaping of anti-windup compensator H in the second step of classical anti-windup paradigm, where the non-linear operation mode is taken into account. For a nominal $P^0 \in \wp$, and taking $D = 1$ in (3), a nominal point is obtained. In particular, $X^0(j\omega) = 1 + P^0(j\omega)G(j\omega)$ represents the nominal point in the nl-template for a frequency ω. Note that shifting of nl-template for frequency ω in NP is equivalent to add $|D(j\omega)|$ decibels and $Angle(D(j\omega))$ degrees to each point in $\Im_{nl}(\omega)$, or equivalently to the nominal $X^0(j\omega)$, assuming this as representative of the rest of points. So the introduction of poles and zeros in D in (3) is equivalent to shift the nl-templates in NP. Based on these ideas, regions on NP can be easily computed that allow a efficient shaping (or design) of $D(s)$, an thus of H. A more detailed algorithm is out of the scope of this work and will be given elsewhere.

3. A general framework

3.1. Conventional Anti-Windup technique (CAW)

The control scheme used in this technique is shown in Figure 4, where H is, in general, a transfer function, and the dynamic

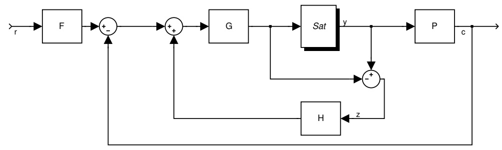


Figura 4: CAW Scheme.

of the element Sat is given by an ideal saturation. Two cases are considered: (Case 1) H is a single gain, and (Case 2) H has more complex dynamics. Transforming the system in Figure 4 into an equivalent one from a stability point of view, the system in Figure 2 is obtained, where R and K blocks are given by

$$\begin{aligned} R &= FG/(1+GH) && (16)\\ K &= G(H-P)/(1+GH) && (17) \end{aligned}$$

Plant without integrators

Supposing that P has no poles at the origin, it is clear that although G contains integrators, K in equation (17) will be a type 0 transfer function. So that, the result in Lemma 2.1 may be used, without including any additional system transformation. Another condition that must be guaranteed to apply the result in Lemma 2.1 is that K in equation (17) is stable[1]. For this purpose necessarily P, H and $G/(1+GH)$ must be stable (the stability of $G/(1+GH)$ is a necessary condition for the closed loop global stability of system). The function that will be used to carry out the shaping, in NP, taking the uncertainty of the plant into account is:

$$X(s) \triangleq 1 - K(s) = \frac{G(s)}{1+G(s)H(s)}P(s) \qquad (18)$$

So that, the nl-template in this case is the same as the corresponding *l-template*, and the $X^0(j\omega)$ nominal function shaping (from equation (18) taking P^0 in place of P) would be carried out shifting the nl-template using the function $D = G(1+GH)^{-1}$. When this function has been shaped, then the computation of compensator H is straightforward

$$H(s) = D^{-1}(s) - G^{-1}(s) \qquad (19)$$

If H is a single gain (case 1), it is clear that the number of poles and zeros of D is conditioned by the linear controller G, designed in the design stage for the linear operation mode. Variations in H will imply only a variation in the location of poles of D, but not a variation in its number (supposing that there are not cancellations). So, the root locus seems to be an adequate tool for this case. The designer would may start with left half plane of the root locus of $1 + GH$, and choose poles that shift the $\Im_{nl}(\omega)$ to an allowed zone for each frequency ω. The value for H is chosen directly from the root locus. For this case the shaping of function D is not very flexible. The system output $\hat{c}$ in Figure 4 is given by the expression

$$\hat{c} = \hat{r}\frac{FPG}{1+PG} + \hat{z}\frac{P(1+GH)}{1+PG} \qquad (20)$$

[1]A rational function of complex variable is stable if all of its poles are located in opened left half plane

where it may be observed that $\hat{c}$ is influenced by two terms, one due to the linear operation mode, and other due to the non-linear operation mode ($\hat{c}$ denotes the Laplace Transform of the signal c). If H has a more complex dynamic (case 2), as it may be deduced from equation (20), necessarily H must be stable. If a block H with an integrator is designed, the system may be closed loop stable, but an offset will appear in the output. In order to guarantee the stability of H, from equation (19) all zeros in G and D must be minimum phase zeros (in order to achieve null offset, zeros in the origin are not valid either). With respect to realizability of H, from equation (19), if functions G and D have a proper inverse then H will be proper, in other case the excess m of poles over zeros of D and G must be the same, and in addition the $dG \cdot nD$ product must have its m first terms the same as the m first terms of the $nG \cdot dD$ product, where nG, dG and nD, dD are the numerator and denominator of G and of D respectively. So, it may be deduced that the D shaping is more complex (with respect to realizability of the resulting compensator H) when G has not proper inverse. Another possibility, in case 2, consists in the imposition of the next structure for the dynamic in H:

$$H(s) = \frac{H_1(s)}{G(s)} \tag{21}$$

so, from equations (18) and (21) $X = (1 + H_1)^{-1}PG$ is obtained. So that the nl-template will be given by the set $\{PG : P \in \wp\}$, and the shaping will be carried out with $D_1 = (1 + H_1)^{-1}$. Obviously the excess of poles over zeros in D_1 must be zero, which guarantees a proper function H_1. It is clear that in order to have H proper, a necessary condition is that the excess of poles over zeros in G must be less or equal than the excess of poles over zeros in H_1.

Plant with one integrator

If the plant P contains exactly one integrator, the transformation in Figures 3(a) and 3(b) will have to be carried out, applying after that the result in Lemma 2.2. In this case H will be given by the expression

$$H(s) = D(s) - G^{-1}(s) \tag{22}$$

When H is a single gain (case 1), for the plant without integrators, it is clear that the number of poles and zeros in D is conditioned by the linear controller G designed in the design stage for the linear operation mode. Variations in H will imply only a variation in the location of zeros of D, but the number of zeros is not affected (supposing that there not exist cancellations). So, as previously the designer could start from the left half plane of the root locus of $1 + GH$, in order to assure the stability of R in (16), and choose zeros that achieve displacements of $\Im_{nl}(\omega)$ to an allowed zone for each frequency ω. The value for H is chosen directly from the root locus. If the dynamics of H is more complex (case 2), as previously it was indicated, H must be stable. If H is designed with an integrator, the system may be closed loop stable, but an offset will appear in the output. In order to guarantee the stability of H, from (22) all of zeros in G must be of minimum phase (in order to obtain a null offset zeros in origin are not valid either), and poles of D must belong to the opened left half plane. With respect to the realizability of H, from equation (22), only if inverse functions of G y D are proper then H will be proper. Another possibility, in case 2, consists in fixing the structure for H as in equation (21), so that from equations (18) and (21) $X = (1 + H_1)^{-1}PG$ is obtained. So, the nl-template will be given by the set $\{(PG)^{-1} : P \in \wp\}$, and the shaping will be carried out using $D_1 = 1 + H_1$. Obviously the excess of poles over zeros in D_1 must be zero, which guarantees the properness of H_1. It is clear that in order to achieve a proper H a necessary condition is that the excess of poles over zeros in G is less or equal than the excess of poles over zeros in H_1. If the plant is unstable a result based on Circle Criterion Case #1 in [2] may be used to probe the local stability of the closed loop system. The function D is strongly influenced by G in the proposed two step anti-windup design, so that in some situations it may be adequate to redesign the controller G for the linear operation mode.

3.2. Hanus conditioning technique

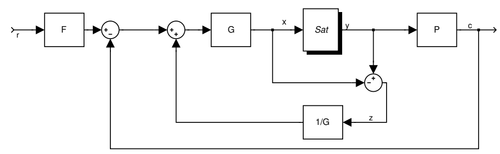


Figura 5: Scheme for the Hanus Conditioning.

In this case the number of degrees of freedom for the anti-windup design is zero. Transforming the system in Figure 5 into an equivalent one from a stability point of view, a scheme as that shown in Figure 2 is obtained, where R and K blocks are given by

$$R = 0.5FG \tag{23}$$
$$K = 0.5(1 - PG) \tag{24}$$

The stability of R is a necessary condition for the closed loop global stability, so that all of poles in G must belong to the open LHP. In order to apply the result in Lemma 2.1 without additional system transformations, K must be stable, so that P must be stable too (unless the Circle Criterion Case #1 is used, where P is allowed to have poles in closed RHP). If P contains an integrator, it is necessary to apply the transformation in Figures 3(a) and 3(b) to the transformed Hanus scheme in Figure 2, applying after that, the result in Lemma 2.2. The function X in this case will be given by the expression

$$X(s) = \frac{1}{2} - K(s) = \frac{P(s)G(s)}{2} \tag{25}$$

Obviously in this case it is not adequate to work with a two step anti-windup design because there are not enough degrees of freedom. Here, in the design stage for the controller G for the linear operation mode, the non-linear operation mode must be taken into account too. For example, applying the result in Lemma 2.1 for multiplier $W(s) = 0$ (equivalent to Circle

Criterion Case #2) to the scheme in Figure 5, the condition $Real(P(j\omega)G(j\omega)) < 3\,\forall\omega > 0\,\forall P \in \wp$ is enough to assure the global stability of the system. This condition must be transformed into a restriction over L^0 so that for all plants in $\wp$ the condition is satisfied. This new restriction must be added to the set of restrictions in NP over function L^0 and then the controller G must be obtained (nominal loop shaping stage in QFT). In [1], where the Circle Criterion is used to probe the stability for the system in Figure 1 with $H = 0$, the condition $Real(P(j\omega)G(j\omega)) > -1\,\forall\omega > 0\,\forall P \in \wp$ is derived. Depending on the particular problem the scheme in Figure 1 with $H = 0$ and the restriction $Real(P(j\omega)G(j\omega)) > -1\,\forall\omega > 0\,\forall P \in \wp$, or the scheme in Figure 5 with the restriction $Real(P(j\omega)G(j\omega)) < 3\,\forall\omega > 0\,\forall P \in \wp$ will be used (supposing that in both cases the Circle Criterion is used to assure the global stability of system). Notice that the scheme handled in this section is a particular case of Case 2 in the proposed scheme in the previous section, taking $H = 1/G$.

3.3. Anti-reset Windup technique

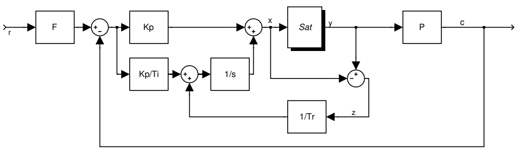


Figura 6: *Anti-Reset Windup* scheme.

Due to the importance of PID controllers in practice applications, the scheme handled in this section is one of the most extended in the industry, to deal with the saturation problem. In general, the design of an anti-windup mechanism for PID controllers has a strong heuristic component. First of all a PID controller is designed and after that, a mechanism is included to reduce the effect of the controller integrator when saturation occurs. In this section some guides to design this type of anti-windup mechanisms are introduced, so that the system stability in presence of saturation and uncertainty in the plant is assured, following the same techniques that in previous sections. Transforming the scheme in Figure 6 into an equivalent scheme, from a stability point of view again, a system as in Figure 2 is obtained, where R and K blocks are given by

$$R = \frac{FK_pT_r(T_is+1)}{T_i(T_rs+1)} \tag{26}$$

$$K = \frac{T_i - K_pT_r(T_is+1)P}{T_i(T_rs+1)} \tag{27}$$

Clearly the stability of R is assured if $T_r > 0$ (necessary condition), and the stability of K is conditioned to the stability of P. In this case the X function is defined by

$$X(s) = 1 - K(s) = \frac{T_r}{T_rs+1}\left[s + K_p\left(\frac{1+T_is}{T_i}\right)P(s)\right] \tag{28}$$

So, the new nl-template that will be handled in the shaping is

$$\Im_{nl}(\omega) = \left\{ j\omega + K_p\left(\frac{1+T_ij\omega}{T_i}\right)P(j\omega) :\ P \in \wp \right\} \tag{29}$$

and the function that will be used to shift $\Im_{nl}(\omega)$, so that the nominal $X^0(j\omega)$ (with X given by equation (28)) lies within allowed zones in NP, will be

$$D(j\omega) = \frac{T_r}{T_rj\omega+1} \tag{30}$$

where the function D in equation (30) depends only on adjustable parameter T_r in the scheme in Figure 6. With respect to the stability of the plant, in order to apply results in previous Lemmas on global stability, the number of integrators in P must be less or equal than one, and the rest of poles in P must belong to open LHP. If the plant has poles in RHP, the result based on Circle Criterion Case #1 in [2] may be used to assure the system local stability.

3.4. Horowitz Scheme

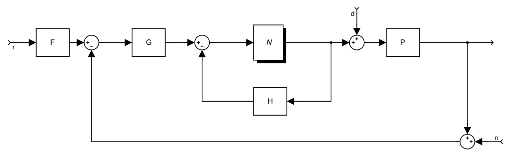


Figura 7: The Horowitz 3DoF control scheme.

In what follows, the application of the results in previous Lemmas to assure global stability, to the scheme proposed by Horowitz in [5] is analyzed. The anti-windup Horowitz scheme is shown in Figure 7. Using a simple transformation it is possible to obtain the scheme in Figure 2, where R and K blocks in this case are given by

$$R = FG \tag{31}$$

$$K = PG + H \tag{32}$$

A necessary condition for the closed loop global stability of the system is that the number of integrators in G must be zero, due to the structure of R in (31). It is supposed that the number of integrators in P is one, as in [5]. As was indicated previously, when the number of integrators in P is greater than zero, a problem appears because it is not possible to apply directly the stability criterion based on multipliers, due to the fact that the function K in the equation (32) does not satisfy the condition in (4). As was done previously, the problem is solved applying the transformation in Figure 3(a), resulting in the equivalent system in Figure 3(b), where the linear block is a dead zone satisfying condition (5) if $|y_{min}| = |y_{max}|$ or DZ even if $|y_{min}| \neq |y_{max}|$. Function X is defined as

$$X(s) \triangleq (1 + K(s)) = (1 + P(s)G(s) + H(s)) \tag{33}$$

with $P \in \wp$. With the aim of $X(s) \in RH_\infty\,\forall P \in \wp$, all of zeros in $X(s)$ must belong to LHP, which is directly satisfied from the stability requirement of function $(1 + L_n)^{-1}$ driving the commutation from non linear to linear mode operation (see [5] for more details). As a consequence, using the Zames-Falb result, if condition (6) (with $-Q$ in place of K) is satisfied for some LTI $Z(s)$ multiplier with impulse response $z(t)$ with $\|z(t)\|_1 < 1$, then the global stability of the system can be concluded if $|y_{min}| = |y_{max}|$. If $|y_{min}| \neq |y_{max}|$ then in addition

Method	$X_1(H)$	$X_2(P,G)$
Section 2	$\frac{1}{1+H(s)}$	$1+P(s)G(s)$
CAW (Case 3)	$\frac{1}{1+H_1(s)}$	$P(s)G(s)$
Hanus Condit.	1	$\frac{P(s)G(s)}{2}$
Antir. Windup	$\frac{T_r}{T_r s+1}$	$s+K_p\left(\frac{1+T_i s}{T_i}\right)P(s)$
Horowitz	$\frac{1}{1+H(s)}$	$1+P(s)G_l(s)$

Tabla 1: Values of functions X_1 and X_2 for some traditional schemes.

Method	$X_1(H,G)$	$X_2(P,G)$
CAW (Case 1)	$\frac{G(s)}{1+G(s)H}$	$P(s)$
CAW (Case 2)	$\frac{G(s)}{1+G(s)H(s)}$	$P(s)$

Tabla 2: Values of functions X_1 and X_2 for cases 1 and 2 in the CAW scheme.

$z(t)$ must be greater than zero for all t in $\mathbb{R}$. In this case, H must be designed so that previous condition is satisfied. From (33) and (8) the previous condition is transformed in the restriction on X shown in Lemma 2.2. Using the result in Lemma 2.2 with function X given by (33), it is easy to probe that the solution proposed by Horowitz in [5], where plant uncertainty is not taken into account, results in a globally stable system.

Note 3.1 Obviously, the existence of a stabilizer compensator $\{F, G, H\}$ in all of previous schema is conditioned to the uncertainty present in plant, and to the particular dynamic fixed for D in each case.

3.5. A general method

The common procedure in all of previous schema consists of the following steps: (1)Transform the particular control scheme into a scheme as in Figure 2. (2) Define the function $X(s) = \alpha K(s) + \beta = X_1(H)X_2(P,G)$, i.e define a function $X(s)$ so that it may be split in two factors, one factor, X_1, depending on the degree of freedom added to the system for the anti-windup mechanism, and other, X_2, which exclusively will depend on a previous design carried out without any consideration of the presence of saturation. (3) From $X_2(P,G)$ a nl-template is defined , and with $X_1(H)$ this nl-template is shifted in the NP in order to assure that the nominal $X_2(P^0,G)$ lies within allowed zones defined by the stability result in Lemmas 2.1 and 2.2. In table 1 particular values for terms X_2 and X_1 for the schema introduced in this section are shown. Note that in the case of Horowitz scheme the term G_l is equal to L^0/P^0 (where L^0 is the nominal open loop transfer function shaped in the loop shaping stage in QFT). For CAW scheme, cases 1 and 2, it is not possible to find a term X_1 that only depends on the new added degree of freedom. In these particular cases $X(s) = \alpha K(s) + \beta = X_1(H,G)X_2(P,G)$ is satisfied. In table 2 values of X_1 and X_2 in these particular cases are shown.

4. Conclusions

In this paper the I/O stability problem for LTI uncertain systems with an amplitude or rate saturation element is considered. A 3DoF control scheme is proposed to cope with the stability problem. The scheme is based on the classic two steps antiwindup paradigm. In the first step the compensator $\{F, G\}$ is designed using QFT to take the uncertainty of the plant into account. In the second step the compensator H is designed in order to achieve a closed loop stable system in non-linear operation mode for all values of uncertain parameters in the plant. This scheme has been previously developed only for amplitude saturation and the control scheme in Figure 1 in [2]. From a computational point of view, the nl-templates computation is not required (templates for the non-linear case) if a single transformation of corresponding l-templates (templates for the linear case), used to design $\{F, G\}$, is introduced. Finally, the proposed scheme is formulated as a general framework including traditional anti-windup schemes. These traditional schemes are generally applicable, with a great heuristic component, to systems without uncertainty. The design method proposed in this paper provides an accurate graphic design procedure that guarantees the closed loop global stability in presence of saturation type non-linearities and with uncertainty in plant. Examples may be found in [2].

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