SWITCHING PREDICTIVE CONTROL OF INPUT-SATURATED PLANTS UNDER PERSISTENT DISTURBANCES

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Abstract

Predictive switching logic schemes are considered whereby a feedback-gain is switched-on at any time from a bank of candidate feedback-gains so as to control a discrete-time input-saturated LTI system possibly subject to *persistent* bounded disturbances of unknown arbitrary magnitude. It is constructively shown that such schemes do exist which ensure, along with good tracking performance, global asymptotic and semi-global exponential stability in the noiseless case, as well as finite l_∞-induced gain to the disturbance-to-state map, whenever the structure of the disturbed plant can make such properties conceptually achievable.

1 Introduction

In recent years, control of input-saturated dynamical systems – a subject of ever-lasting fundamental interest in control engineering – has attracted significant research efforts.
A significant source of contributions to the subject has been originated within model-based predictive control (MBPC) [12, 11, 10]. MBPC, though inherently tailored to handle input and/or state-related constraints, has been so far hampered in having a significant impact on applications other than slow-process control, mainly because of its high computational load. Moreover, even if MBPC of input-saturated plants was developed so as to provide in the nominal case global high-performing regulation [2, 1], and, in robustified forms, can handle both model uncertainties [5, 8, 3] and vanishing disturbances [4], its extension to the case of persistent disturbances [15] appears to be so computationally demanding that it can be yet considered in practice an unsolved problem.
This paper aims at providing a computationally affordable solution to the regulation problem of discrete-time input-saturated linear time-invariant (LTI) systems subject to persistent bounded disturbances of unknown arbitrary magnitude.

2 Problem Formulation and Paper Overview

The paper deals with the regulation problem of a discrete-time input-saturated LTI plant

$$\begin{cases} x^+ = \Phi x + G\sigma(u) + \xi \\ \hat{x} = x + \tilde{x} \end{cases} \tag{1}$$

where: $x \in \mathbb{R}^n$ is the plant state; $x^+ := x(t+1)$ if $x = x(t)$; $t \in \mathbb{Z}_+ := \{0, 1, \ldots\}$; $u = [u_1, \cdots, u_m]' \in \mathbb{R}^m$; the prime denotes transpose;

$$\sigma(u) = \begin{cases} u & , \quad u_i \in [-\underline{u}_i, \overline{u}_i] \quad , \quad \forall i \in \underline{m} \\ n(u) & , \quad \text{elsewhere} \end{cases} \tag{2}$$

$\underline{m} := \{1, 2, \ldots, m\}$; $\underline{u}_i, \overline{u}_i > 0$; and $n(u) : \mathbb{R}^m \to \mathbb{R}^m$, an arbitrary bounded nonlinear function of the control vector u. In (1), both ξ and $\tilde{x}$ are bounded and possibly persistent disturbances of unknown arbitrary magnitude. The problem is to find, based on $\hat{x}$, a partial state-information vector, feedback controls

$$u = f(\hat{x}) \tag{3}$$

so as to ensure, under suitable conditions, exponential stability in the noiseless case as well as finite l_∞-induced gain of the disturbance-to-state map from ξ and $\tilde{x}$ to x embedded in (1)-(3). The adopted approach consists of selecting a discrete family $\mathcal{F} = \{F_h\}_{h=1}^{\infty}$ of linear state-feedback gains F_h and a switching logic

$$h(t) = \ell(\hat{x}(t), h(t-1)) \tag{4}$$

in such a way that the regulated plant

$$x^+ = \Phi x + G\sigma(F_{h(t)}\hat{x}) + \xi \tag{5}$$

have the stated stability properties. For recent results on switching among stabilizing controllers see [6].
The paper is organized as follows. Sect. 3 describes the specific type of feedback-gain matrices that are adopted to realize possible control actions. Sect. 4 shows that, thanks to the type of candidate feedback-gain matrices, stability under *arbitrary* switching control can be ensured to an LTI system with no input saturations by only imposing a simple but essential admissibility condition on the switching sequences. Sect. 5 exploits the stability results in Sect. 4 so as to extend them, via the adoption of appropriate switching logic supervisors, to

LTI systems subject to input saturations and persistent bounded disturbances of unknown arbitrary magnitude. Sect. 6 reports simulation examples, while Sect. 7 ends the paper with conclusive remarks. Because of space limitations, proofs are omitted.

3 Candidate Receding Horizon Feedback-Gains

Consider temporarily the noiseless linear variant of (1)

$$x^+ = \Phi x + Gu \tag{6}$$

Assume

$$\Sigma = (\Phi, G) \qquad \text{reachable} \tag{7}$$

Note that (7) entails no loss of generality in that, if Σ is stabilizable but not reachable, all subsequent developments apply to the reachable subsystem of the plant obtained via a Gilbert-Kalman reachability decomposition. Because of properties that are motivated next in some detail, the candidate feedback-gains are chosen as follows

$$F_h := -\Psi_u^{-1} G' (\Phi^{h-1})' \mathcal{G}_h^{-1} \Phi^h \tag{8}$$

where: h is a positive integer, $h \geq \nu$, ν the reachability index of Σ; $\Psi_u = \Psi_u' > 0$; and $\mathcal{G}_h$ is the h-order reachability Gramian

$$\mathcal{G}_h := \sum_{k=1}^{h} \Phi^{k-1} G \Psi_u^{-1} G' (\Phi^{k-1})' \tag{9}$$

An explanation on where (8) stems from is in order. To this end, we consider the following control problem $\mathcal{P}_h$.

Zero terminal-state minimum energy control problem $\mathcal{P}_h$. Let x be the state of (6) at time 0, and $\mathcal{U}_h(x)$ the set of all input sequences $u(\cdot) = \{u(k)\}_{k=0}^{h-1}$ which drive the system state x to the zero-state 0_X in h steps

$$\mathcal{U}_h(x) := \left\{ u(\cdot) \in (\mathbb{R}^m)^h : \varphi(h, x, u(\cdot)) = 0_X \right\} \tag{10}$$

where $\varphi(h, x, u(\cdot)) = \Phi^h x + \sum_{k=0}^{h-1} \Phi^{h-1-k} G u(k)$. Note that $\mathcal{U}_h(x) \neq \emptyset, \forall x \in \mathbb{R}^n$, if $h \geq \nu$. $\mathcal{P}_h$ consists of finding the element $u_h(\cdot|x)$ in $\mathcal{U}_h(x)$ of minimum energy

$$\|u(\cdot)\|^2 := \sum_{k=0}^{h-1} |u(k)|^2_{\Psi_u} \tag{11}$$

where $|u|$ denotes Euclidean norm, $|u|^2_{\Psi_u} := u' \Psi_u u$ and $\Psi_u = \Psi_u' > 0$. For $h \geq \nu$, such an element $u_h(\cdot|x)$, if written in vector form, is as follows

$$\begin{aligned} \underline{u}_h(x) &:= [u_h'(0|x), \ldots, u_h'(h-1|x)]' \\ &= [\mathcal{F}_h'(0) \cdots \mathcal{F}_h'(h-1)]' x \\ &= \mathcal{F}_h x & (12) \\ \mathcal{F}_h &:= -\widehat{\Psi}_u^{-1} \mathcal{R}_h' \mathcal{G}_h^{-1} \Phi^h & (13) \end{aligned}$$

where $\widehat{\Psi}_u := \text{Diag}\{\Psi_u, \ldots, \Psi_u\}$ -(h-times) and

$$\mathcal{R}_h := \left[\Phi^{h-1} G | \ldots | \Phi G \, | G\right] \tag{14}$$

The integer h is referred to as the *control horizon* associated to $\mathcal{P}_h$. ■

Note that F_h in (8) is given in terms of $\mathcal{F}_h$ as follows

$$F_h = \begin{bmatrix} I_m & 0_{m \times m(h-1)} \end{bmatrix} \mathcal{F}_h \tag{15}$$

Hence, F_h is recognized to be the feedback-gain matrix of the receding horizon regulator [9] associated to problem $\mathcal{P}_h$.

From (12) and (13) it follows that

$$\|u_h(\cdot|x)\| = \|x\|_h := |x|_{P(h)} \tag{16}$$

where

$$\begin{aligned} P(h) &= (\Phi^h)' \mathcal{G}_h^{-1} \Phi^h \\ &= P'(h) \geq 0 & (17) \end{aligned}$$

It can be seen [12] that, if $P(\nu)$ is as in (17) for $h = \nu$, then for $h \geq \nu$, $P(h+1)$ satisfies the Riccati difference equation

$$\begin{aligned} P(h+1) \quad = \quad & \Phi' P(h) \Phi - \\ & \Phi' P(h) G \left(\Psi_u + G' P(h) G\right)^{-1} G' P(h) \Phi \end{aligned} \tag{18}$$

and

$$F_{h+1} = -[\Psi_u + G' P(h) G]^{-1} G' P(h) \Phi \tag{19}$$

In addition,

$$P(h+1) \leq P(h) \tag{20}$$

From (17) and (20), it follows that the following limit exists

$$\lim_{h \to \infty} P(h) =: P(\infty) \geq 0 \tag{21}$$

Further, if (6) is ANCBI (asymptotically null-controllable with bounded input), *viz.* [16], (6) is stabilizable and has no exponentially unstable eigenvalue,

$$P(\infty) = 0_{n \times n} \tag{22}$$

Note that if

$$\Omega(\delta) := \{u \in \mathbb{R}^m : u_i \in [-\underline{u}_i + \delta, \bar{u}_i - \delta], \forall i \in \underline{m}\} \tag{23}$$

$0 \leq \delta < \min\{\underline{u}_i, \bar{u}_i, \, i \in \underline{m}\}$, from (22) it follows that it is always possible for an ANCBI system to find a large enough horizon h so as to satisfy

$$u_h(\cdot|x) \in \Omega^h(\delta) \tag{24}$$

for every $x \in \mathbb{R}^n$ and $\bar{u} > 0$.

In the sequel, our attention will be focused on the family of state-feedback gain-matrices

$$\mathcal{F} := \{F_h\}_{h=\underline{h}}^{\infty} \quad , \quad \underline{h} \geq \nu \tag{25}$$

along with the system (6) under a time-varying state-feedback control $u(t) = F_{h(t)} x(t)$, $F_{h(t)} \in \mathcal{F}$,

$$\begin{aligned} x(t+1) &= \Phi_{h(t)} x(t) \\ \Phi_{h(t)} &:= \Phi + G F_{h(t)} \end{aligned} \tag{26}$$

4 Exponential stability under arbitrary admissible switching

A control horizon sequence $\{h(t)\}_{t\in\mathbb{Z}_+}$, $h(t)\in\mathbb{Z}_+$, $h(t)\geq\nu$, is called *admissible* if

$$h(t+1)\geq h(t)-1 \tag{27}$$

Let $\mathcal{S}$ denote the set of all such admissible sequences

$$\begin{aligned}\mathcal{S} \quad := \quad \{\{h(t)\}_{t\in\mathbb{Z}_+}: \quad & h(t)\in\mathbb{Z}_+,\\ h(t)\geq\nu, \quad & h(t+1)\geq h(t)-1\}\end{aligned} \tag{28}$$

and $(\Sigma,\mathcal{S})$ the system (26) under an arbitrary admissible switching sequence in $\mathcal{S}$.

Lemma 1 *Along all possible trajectories of $(\Sigma,\mathcal{S})$, the following property holds*

$$\|x(t+1)\|^2_{h(t+1)}\leq\|x(t)\|^2_{h(t)}-|u(t)|^2_{\Psi_u} \tag{29}$$

where $\|x\|_h$ is as in (16), $u(t)=F_{h(t)}x(t)$ and $x(t+1)=\Phi_{h(t)}x(t)$. ■

Lemma 2 *Along all possible trajectories of $(\Sigma,\mathcal{S})$, one has*

$$\|x(t+h(t))\|_{h(t+h(t))}<\|x(t)\|_{h(t)} \tag{30}$$

for all $x(t)$ such that $\|x(t)\|_{h(t)}>0$. ■

The theorem that follows is the main result of this section and fundamental for our subsequent developments.

Theorem 1 *Consider the control system $(\Sigma,\mathcal{S})$ composed of the LTI reachable plant (6) under a time-varying state-feedback control $u(t)=F_{h(t)}x(t)$ realized by arbitrary admissible switching sequences in $\mathcal{S}$ as in (28). Then, provided that $\underline{h}\geq n$ and $\mathcal{S}$ be a finite family, viz.*

$$\mathcal{S}:=\{\underline{h},\underline{h}+1,\ldots,\overline{h}-1,\overline{h}\} \tag{31}$$

with $\underline{h}\leq\overline{h}<\infty$, $(\Sigma,\mathcal{S})$ is exponentially stable

$$|x(t)|=M\lambda^t|x(0)| \tag{32}$$

with $0<M<\infty$, and decaying rate λ depending on $\bar{h}$ and $\underline{h}$.

■

Remark 1 - It is to be pointed out that (31) encompasses the case of a fixed regulation horizon h, $\underline{h}=\overline{h}=h\geq n$, for which, to the best of the author's knowledge, exponential stability of (26) for an arbitrary h has remained so far an open problem [12, 13]. ■

Remark 2 - The admissibility condition (27) turns out to be not only sufficient for the stability property stated in Theorem 1, but, in a wide sense, also necessary. In fact, there are cases wherein, if (27) is violated, stability is lost even if $\mathcal{F}$ remains finite. ■

5 Hysteresis switching regulation

Noiseless case - Consider the noiseless variant of (1)

$$x^+=\Phi x+G\sigma(u) \tag{33}$$

along with (2) and (7). Given the system state x, $u_h(\cdot|x)$ will denote, as in Sect. 3, the minimum energy input sequence in $\mathcal{U}_h(x)$. Let $x(t)$ denote the system state at time t. Consider the following horizon switching logic ($\underline{h}\geq n$)

$$h(t)=\begin{cases}h(t-1)-1, & \text{if } \begin{array}{l}h(t-1)-1\geq\underline{h} \text{ and}\\ u_{h(t-1)-1}(\cdot|x(t))\in\Omega^{h(t-1)-1}\end{array}\\ \hat{h}(t), & \text{otherwise.}\end{cases}$$

$$\hat{h}(t):=\min\left\{h\in\mathbb{Z}_+: h\geq h(t-1), u_h(\cdot|x(t))\in\Omega^h\right\} \tag{34}$$

with $t=1,2,\ldots$, $h(0)=\hat{h}(0)$ with $h(-1)=\underline{h}$, and $\Omega:=\Omega(\delta=0)$. Then, the following theorem follows.

Theorem 2 *Consider the noiseless input-saturated system (33), (2) and (7) with $u(t)=F_{h(t)}x(t)$ subject to the feedback switching logic (34). Assume that the initial system state $x(0)$ at time 0 be such that $h(0)$ exist finite. Then, logic (34) yields the admissible switching sequence $h(t+1)=h(t)-1$, $t=0,\ldots,h(0)-\underline{h}$, $h(t)=\underline{h}$, $t\geq h(0)-\underline{h}$, the resulting switched system $x(t+1)=\Phi_{h(t)}x(t)$ satisfies the input saturation constraints, and is exponentially stable. In particular, if Σ is ANCBI, the resulting switched system is globally asymptotically stable and semi-globally exponentially stable.* ■

Noisy case - Consider the following input-saturated noisy plant

$$z^+=\begin{bmatrix}A & \tilde{A}\\ 0 & \mathcal{A}\end{bmatrix}\begin{bmatrix}z_s\\ z_q\end{bmatrix}+\begin{bmatrix}B\\ \mathcal{B}\end{bmatrix}\sigma(u)+\begin{bmatrix}\tilde{B}\\ 0\end{bmatrix}\omega \tag{35}$$

where $z=[z_s'\ z_q']'$, $dim z_s=dim A$, $\omega=\omega(t)$ is a bounded disturbance, A a stability matrix, and $\mathcal{A}$ has all its eigenvalues of unit modulus. Assume that the linear (unsaturated) variant of (35) be reachable by the input $\sigma(u)=u$. As can be shown, there exists a change of basis for the state z such that for $x=P^{-1}z=[s'\ q']'$, $n_s:=dim s=dim A$,

$$\begin{cases}s^+=Ss+G_s\sigma(u)+\xi\\ q^+=Qq+G_q\sigma(u)\end{cases} \tag{36}$$

where: ξ denotes the effect on the x-state of the disturbance ω; $Sp(S)=Sp(A)$ and $Sp(Q)=Sp(\mathcal{A})$, if $Sp(A)$ denotes the set of the eigenvalues of A.
It is known [7, 14] that (36), or equivalently (35), has the most general structure of an input-saturated LTI system for which it makes sense to consider stability and boundedness under arbitrary l_∞-disturbances.
The argument that follows is used so as to make it plausible the conjecture that $h(t)$, chosen by (a suitably modified version of) (34), cannot get unbounded. If this is the case, according to Theorem 1, the contribution of any past noisy sample vanishes

exponentially fast, and hence $x(t)$ stays bounded.
Consider first that, by the switching logic (34), $h(0) < \infty$ for any $x(0) \in \mathbb{R}^n$, and $\{h(t)\}_{t=0}^{\infty}$ is in $\mathcal{S}$. Next, assume, by contradiction, that $h(\cdot)$ grows unbounded. This implies that $|x(\cdot)|$ does the same. As $|s(\cdot)|$ is bounded because S is stable and $\sigma(u)$ and ξ are both bounded, there are times t large enough at which $|x(t)|^2 = |s(t)|^2 + |q(t)|^2 \simeq |q(t)|^2$. Under these circumstances, $h(t)$ is essentially chosen according to the restricted noiseless system with state $q(t)$. Hence, according to Theorem 2, at subsequent times, $h(t+k) = h(t) - k$ till $|q(t+k)|$ is reduced so much that the effect of $|s(t+k)|$ becomes significant again for the selection of subsequent horizons. Thereafter, $h(\cdot)$ may start to increase till the condition $|x(t)|^2 \simeq |q(t)|^2$ is possibly restored. Consequently, the regulation horizon begins again to decrease by one at each subsequent time. In words, a "*horizon resetting*" mechanism is inherently enforced. The conjecture is that such a mechanism prevents $h(\cdot)$, and hence the plant state, from becoming unbounded.
However, in order to prove that the horizon resetting property holds, it is required to replace the switching logic (34) with its variant (37) equipped with a "*hysteresis*" facility.

Theorem 3 *Consider the input-saturated noisy plant (35) with the stated properties and subject to the following hysteresis switching logic ($\underline{h} \geq n$)*

$$h(t) = \begin{cases} h(t-1) - 1, & \text{if } \begin{array}{l} h(t-1) - 1 \geq \underline{h} \text{ and} \\ u_{h(t-1)-1}(\cdot|z(t)) \in \Omega^{h(t-1)-1} \end{array} \\ \hat{h}(t), & \text{otherwise.} \end{cases} \tag{37}$$

$$\hat{h}(t) := \min\left\{h \in \mathbb{Z}_+ : h \geq h(t-1), u_h(\cdot|z(t)) \in \Omega^h(\delta)\right\}$$

where $t = 1, \cdots$, and $h(0) = \hat{h}(0)$ with $h(-1) = \underline{h}$. Then, the resulting closed-loop hysteresis switched system (35) with control $u(t) = F_{h(t)} z(t)$ is bounded-noise bounded-state l_∞-stable irrespective of the initial state $x(0) \in \mathbb{R}^n$. ■

Partial state information - If in the hysteresis switching logic (37) the true plant state $z(t)$ is replaced by the vector

$$z(t) + [\zeta'(t)\, 0']' \tag{38}$$

where $\zeta(t) \in \mathbb{R}^{n_s}$ is a bounded sensor-noise acting on the stable component of the z-state, it is immediate to see that the conclusions of Theorem 3 hold true. This implies that the statement of Theorem 3 can be extended to a plant of the form (35) under the hysteresis switching logic (37) based on a partial state information,

$$\hat{z}(t) = [\hat{z'}_s(t)\, z'_q(t)]' \tag{39}$$

where $\hat{z_s}(t)$ is a filtered-estimate of $\hat{z}_s(t)$ based on observations

$$y(t) = [\gamma'(t)\, z'_q(t)]' \tag{40}$$

with $\gamma(t) = Hz_s(t) + n(t) \in \mathbb{R}^p$ with $n(\cdot)$ a bounded sensor-noise.

6 Examples

Example 1 Consider the ANCBI system

$$\begin{cases} x^+ = \begin{bmatrix} 0.9975 & 0.0999 & 0.0025 & 0.0001 \\ -0.0499 & 0.9975 & 0.0499 & 0.0025 \\ 0.0025 & 0.0001 & 0.9975 & 0.0999 \\ 0.0499 & 0.0025 & 0.0499 & 0.9975 \end{bmatrix} x + \begin{bmatrix} 0.0050 \\ 0.0999 \\ 0.0000 \\ 0.0001 \end{bmatrix} u \\ y = [0 \quad 0 \quad 1 \quad 0]\, x \end{cases} \tag{41}$$

This is the discrete-time version (0-order hold input and sampling-time equal to 0.1 s) of a mechanical frictionless system composed of two carts of equal 1 Kg mass coupled by a link of stiffness equal to 0.5 N/m. The problem is to control the position y of one cart by using a force u, expressed in N units, applied to the other cart along the horizontal direction of motion.
System (41) can be used to show that, in general, the admissibility condition (27) cannot be waived. In fact, if one controls (41) by the use of a switched sequence $\{F_{h(t)}\}_{t \in \mathbb{Z}_+}$ of feedback-gains $F_{h(t)}$, with $h(\cdot)$ an uncorrelated random sequence uniformly distributed over the integers between $\underline{h} = 4$ and $\bar{h} = 50$, the controlled system quickly becomes unstable, and its state diverges. It is to be underlined that, without enforcing the admissibility condition (27), stability losses can be experienced also when control horizon sequences of deterministic type with large enough negative changes are used.
It can be shown that, using a time-invariant horizon h not large enough, the control action $u(t) = F_h x(t)$ makes the closed-loop system unstable in the presence of input saturation. *E.g.*, the control $F_h x(t)$ with $h \equiv 25$ when the output reference is the unit step and the input $\sigma(u)$ to the plant saturates outside $[-5, 5]$ makes the output y unbounded.

Finally, the performance achieved for plant (41) by the supervisory horizon switching logic (34) is exhibited in Fig. 1, where $\underline{h} = n = 4$, the output reference is a square wave between 0 and 1, and the plant input saturates outside $[-5, 5]$. One can see that stability and set-point tracking are achieved, and the resulting time-variations of the switched control-horizon agree with the statement of Theorem 2.

Example 2 Consider the system

$$z^+ = \begin{bmatrix} 0.9 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \sigma(u) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \nu \tag{42}$$

where $z = [z_1\ z_2\ z_3]' \in \mathbb{R}^3$; z_1 is a stable state; z_2,z_3 are the ANCBI part of the state; and $\nu \in \mathbb{R}$ is a bounded disturbance. One reason for considering this system is to verify the horizon resetting property qualitatively discussed just before Theorem 3 and established in its proof. To do this, we refer to the system (42) which is algebrically equivalent to a system of the form (36). In particular, the Jordan normal form of (42) is as follows

$$x^+ = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0.9 \\ 1 \\ 1 \end{bmatrix} \sigma(u) + \begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix} \nu \tag{43}$$

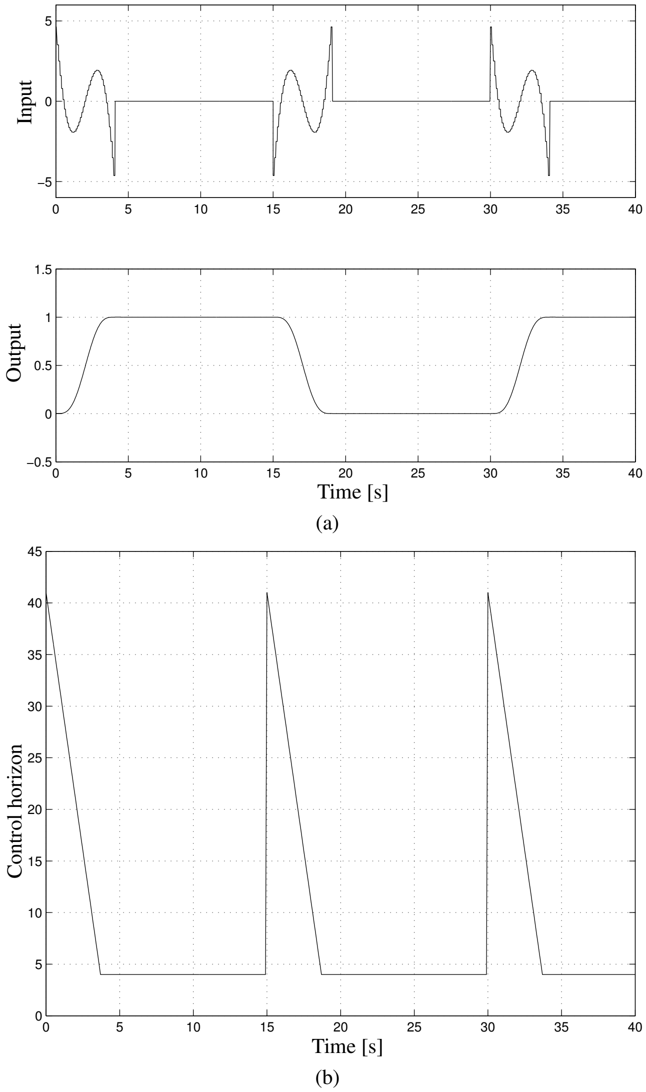

Figure 1: (a) Input (upper), and output (lower) obtained by the switching logic (34); (b) Control horizon selected by logic (34).

Here, the aim is to regulate z, or equivalently x, to $[0\ 0\ 0]'$. The simulations in Fig. 2 refer to a disturbance ν uniformly distributed on $[-100, 100]$. In order to enforce the horizon resetting mechanism, an initial state $x(0)$ is used such that

$$|s(0)|^2 = |x_1(0)|^2 \ll |x_2(0)|^2 + |x_3(0)|^2 = |q(0)|^2 \quad (44)$$

From Fig. 2 one sees that, in agreement with the horizon resetting property, the control horizon h decreases, irrespective of the disturbance, by one at each time-step, as long as $|s(t)|^2 \ll |q(t)|^2$. Hence, the control horizon stays bounded and, consequently, the same for the state.

7 Conclusions

The paper provides, relatively to alternative approaches, a computationally affordable solution to the regulation problem of discrete-time input-saturated LTI systems subject to persistent bounded disturbances of unknown arbitrary magnitude. The

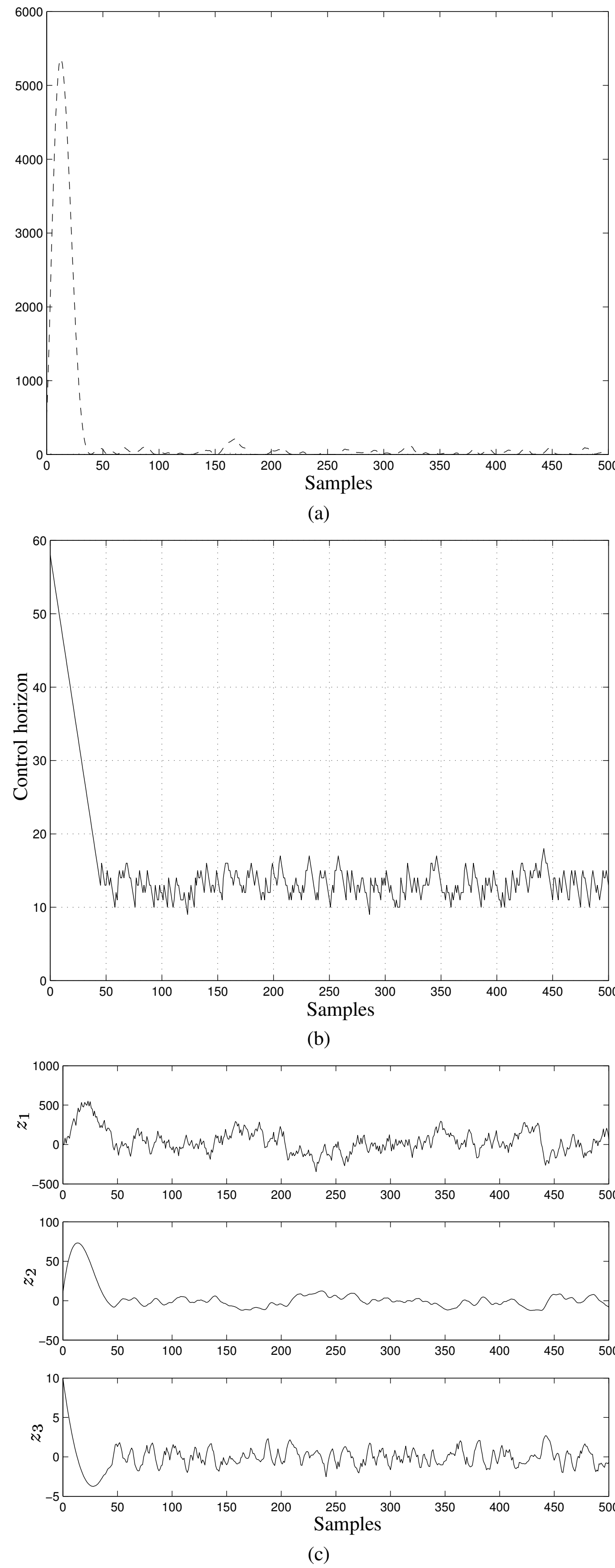

Figure 2: (a) $|x_2|^2 + |x_3|^2$ while $|x_1|^2 \simeq 0$; (b) Control horizon; (c) State z.

proposed solution enjoys the following features: It consists of a supervisory switching control logic whereby a feedback-gain, selected at any time from a family of pre-designed candidate feedback-gains, is switched-on in feedback to the plant according to the information, either complete or partial, on the current plant state; No disturbance upper-bound need to be known; The controller selection is made in accordance with a predictive control philosophy, and each candidate feedback-gain is tuned on to a different horizon in a receding-horizon control sense; The supervisory switching logic is flexible enough so as to enable the designer to simplify the scheme by trading off performance vs. memory and/or computational complexity while retaining the required stability properties. It is proved that the adopted switching logic ensures global asymptotic and semi-global exponential stability in the ideal noiseless case, and finite l_∞ -induced gain to the disturbance-to-state map, whenever the structure of the disturbed plant can make such properties conceptually achievable.

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References

[1] Angeli D., A. Casavola & E. Mosca (2000). Predictive PI-control under positional and incremental input saturations. *Automatica*, Vol. 36, pp. 1505–1516.

[2] Casavola A., M. Giannelli & E. Mosca (1999). Global predictive regulation of null-controllable input-satureted linear systems. *IEEE Transactions on Automatic Control*, Vol. 36, pp. 2226–2230.

[3] Casavola A., M. Giannelli & E. Mosca (2000). Min-max predictive control strategies for input-saturated polytopic uncertain systems. *Automatica*, Vol. 36, pp. 125-133.

[4] Casavola A. & E. Mosca (2001). Global switching regulation of input-saturated discrete-time linear systems with arbitrary l_2 state disturbances. *IEEE Transactions on Automatic Control*, Vol. 46, pp. 915–919.

[5] De Nicolao G., L. Magni & R. Scattolini (1996). On the robustness of receding horizon control with terminal constraints. *IEEE Transactions on Automatic Control*, Vol. 41, pp. 451–453.

[6] Hespanha J.P. & A.S. Morse (2002). Switching between stabilizing controllers. *Automatica*, Vol. 38, pp. 1905–1917.

[7] Hou P., A. Saberi, Z. Lin & P. Sannuti (1998). Simultaneously external and internal stabilization for continuos and discrete-time critically unstable systems with saturating actuators. *Automatica*, Vol. 34, pp. 1547–1557.

[8] Kothare M.V., Balakrishnan & M. Morari (1996). Robust constrained model predictive control using linear matrix inequalties. *Automatica*, Vol. 32, pp. 1361–1379.

[9] Kwon W.H. & A.E. Pearson (1975). On the stabilization of a discrete constant linear system. *IEEE Transactions on Automatic Control*, Vol. 20, pp. 800-801.

[10] Mayne D.Q. (2001). Control of constrained dynamic systems. *European J. of Control*, Vol. 7, pp. 87–99.

[11] Mayne D.Q., J.B. Rawlings, C.V. Rao & P.O.M. Scokaert (2000). Constrained model predictive control. *Automatica*, Vol. 36, pp. 789–814.

[12] Mosca E.(1995). *Optimal, Predictive, and Adaptive Control*. Prentice Hall, Englewood Cliffs, NJ.

[13] Rugh W.J. (1996). *Linear System Theory*. Prentice Hall, Englewood Cliffs, NJ.

[14] Saberi A., P. Hou & A.A. Stoorvogel (2000). On simultaneous global external and global internal stabilization of critically unstable linear systems with saturating actuators. *IEEE Transactions on Automatic Control*, Vol. 45, pp. 1042–1052.

[15] Scokaert P.O.M. & D.Q. Mayne (1998). Min-max feedback model predictive control for constrained linear systems. *IEEE Transactions on Automatic Control*, Vol. 43, pp. 648–654.

[16] Sussmann H.J., E.D. Sontag & Y. Yang (1994). A general result on the stabilization of linear systems using bounded control. *IEEEs Transactions on Automatic Control*, Vol. 39, pp. 2411–2424.