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Closed Loop NO_x Control by Discrete Time Sliding Mode

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***Abstract*— Nitrogen oxides (NO_x) are besides particulate matter (PM), especially for diesel engines, the most relevant emission components. Against this background and the fact that passenger car emission legislation is permanently becoming stricter, there is still a growing interest in minimizing these pollutants. Beside the highly nonlinear behavior of engines also the tradeoff between PM and NO_x is a challenging task for the evaluation of a control scheme. In this work the NO_x emissions are addressed and a discrete time sliding mode controller (DSMC) is proposed to control the exhaust gas recirculation (EGR) valve in order to provide tracking of a time varying NO_x reference while disturbances are rejected. The method presented in this report avoids the common use of intermediate variables like intake fresh air or inlet manifold pressure and shows proper results in both simulation and measurements with a Euro 5 engine.**

I. INTRODUCTION

Due to more and more restricted legislation during the last years the efforts in reducing the two most significant pollutants of Diesel engines, namely the Nitrogen oxides (NO_x) and the particulate matter (PM) have been increased. While diesel particulate filters (DPF) are very effective in reducing the tailpipe PM emissions, reducing the NO_x emissions is a more challenging task.

Besides exhaust after treatment generally also a direct approach to keep the raw emissions in a proper range is useful to reach this goal. For emission oriented control already a vast literature (e.g. [1]) exists. For most control design methods first a model has to be estimated (see e.g. [2]) for a comprehensive overview of engine models), because the NO_x formation is a complex topic (cf. [3]) and depends on many factors. Therefore a relatively simple model with a limited amount of parameters is used (in this work mainly ECU (Engine-Control-Unit) standard information is assumed to be available). The occurring model imprecision is compensated by a robust design approach. In the current proposal a discrete time sliding mode controller based on a polynomial NO_x model is developed.

Contributions related to this work have been proposed in different contexts. A sliding mode approach on air path control (air-fuel ratio and exhaust gas recirculation flow fraction) was done by [4] and in simulation by [5] (compressor and exhaust gas recirculation flow rates).

Also [6] outlined different NO_x description methods, including a polynomial NARX approach. The extension of the sliding mode method on discrete time systems was examined for linear systems from [7], [8]. For non-linear systems, at least in continuous time, [9], [10] and [11] deal with this control approaches. As it turned out, the test bench engine also operates in regions where some system dynamics show a non-minimal phase behavior, hence also studies on this topic from [12], [13] are of interest.

In this work a method is proposed which connects the sliding mode principle for non-linear continuous systems with the NARX model structure via the Euler methods. The main focus of this contribution lies on modeling and control of the NO_x emissions with respect to tracking quality and disturbance rejection. The rest of the report is organized as follows. First, the modeling framework will be presented. Second, the sliding mode control design approach is introduced and extended to non-linear discrete time systems. Due to the partially non-minimum phase response an additional control law is defined for unfeasible regions. Afterwards the results of the control approach on the test bench are shown and discussed and finally the whole approach is concluded in the last section.

II. SYSTEM SETUP

The measurements for model identification, verification as well as all tests of the control scheme were carried out on a dynamical passenger car engine test bench at the Johannes Kepler University in Linz, Austria. The considered engine was a 4 cylinder 2.0 liter diesel production type engine which fulfills the EURO 5 legislation standard. This Diesel engine is equipped with a common rail injection system, an exhaust gas recirculation and a variable geometry turbine. The NO_x measurement was done by a production on board sensor (SMART NO_x) and due to the slow response of it also validated with the Cambustion fNOx400. The control signals were applied via an ECU bypass and the use of a dSpace rapid prototyping system. All data acquisition was done with the dSpace system, too.

III. MODELLING APPROACH

In this work, results using a polynomial NO_x modeling approach will be discussed, as they are simple to derive and the model quality is adequate if the inputs and identification data set are chosen correctly. Suitable inputs were selected based on former works [14] and in order to generate global valid models the inputs (Wf, N, XEGR, XVGT) are generated by a D-optimal DoE (Design-Of-Experiment) approach (see [14]). In this work only signals available in the ECU, despite the Cambustion fNOx400 signal indispensable to evaluate the performance, have been used. The used model structure shown in Fig. 1 consists of 3 parts the dynamic

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modeling of the mass air flow (MAF) and intake manifold absolute pressure (MAP) and the modeling of the (NO_x) emissions based on MAF and MAP. In this structure the injected fuel amount (Wf) and the engine speed (N) act as measured disturbances and the exhaust gas recirculation valve position (XEGR) as well as the turbo guide vane position (XVGT) are the manipulated actors for the control scheme. The manipulated variables XEGR and XVGT have a strong influence on MAF and MAP and this effect is coupled with the NO_x emissions.

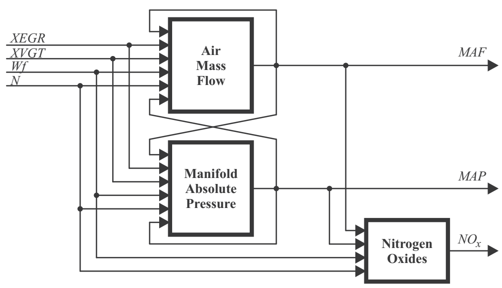

Fig. 1. Air path and NO_x model structure

Based on the ARX error model the full factorial polynomial l^{th} degree NARX model with r inputs and dynamic order n_y and $n_{u,r}$ has the form:

$$\begin{aligned} y(k) = f^l\,(&y(k-1),\ldots,y(k-n_y),\ldots \\ &\ldots,u_1(k-1),\ldots,u_1(k-n_{u,1}),\ldots \\ &\ldots,u_r(k-1),\ldots,u_r(k-n_{u,r});\theta)+e(k) \end{aligned} \tag{1}$$

For the dynamic models (MAF and MAP) it turned out that a time lag of $n_y = n_{u,r} = 2$ and regressors with a polynomial degree up to 2 are convenient to obtain adequate models. The static behavior of the NO_x is also modeled by a polynomial approach of 2nd order. Due to the fact that an input affine structure is necessary to derive the equivalent control signal for the sliding mode controller, the following non-linear model output formulation is obtained.

$$\begin{aligned} y(k) &= f_1\,(y(k-1),y(k-2),\ldots \\ &\quad\ldots,u_{md}(k-1),u_{md}(k-2),u_{mv}(k-2);\theta_1)+\ldots \\ &\quad\ldots+f_2\,(y(k-1),u_{md}(k-1),u_{mv}(k-1);\theta_2)+e(k) \\ &= \begin{bmatrix} \varphi_1^T(k) & \varphi_2^T(k) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + e(k) \\ &= \varphi^T(k)\theta + e(k) \end{aligned} \tag{2}$$

In (2) f_2 only contains linear terms of the manipulated variables u_{mv} at $k-1$, the measured disturbances and the output ($\varphi_2^T(k) = u_{mv}(k-1)\cdot[1 \quad y(k-1) \quad u_{md}(k-1)]$). All other polynomial regressors in y and u_{md} and their combination with $u_{mv}(k-2)$ are condensed in f_1. This model structure does not contain quadratic terms in the manipulated variables (e.g. $u_{mv}^2(k-1)$) to avoid solving complex equations and guarantee a solution for (14) and (15).Since this model structure is linear in parameters it can be represented by the product of a parameter vector θ with a data vector φ. According to the assumption of an error e uncorrelated with f_1 and f_2, the standard method of least squares can be used to get an estimation $\hat{\theta}$ for the parameter vector. The model output signals are collected in the data vector Y and the information matrix Φ contains all necessary regressors combinations (see (3)).

$$\begin{aligned} \hat{\theta} &= \arg\min_{\theta} \sum_{k=1}^{N} \left(y(k) - \varphi^T(k)\theta\right)^2 \\ &= \arg\min_{\theta} [Y-\Phi\theta]^T\,[Y-\Phi\theta] \\ &= \left(\Phi^T\Phi\right)^{-1}\Phi^T Y \end{aligned} \tag{3}$$

To achieve comparable model parameters all measured signals are scaled by their min- and maximum values to the range -1 to +1. Based on this transformation it is possible to replace the autoregressive terms in the MAF and MAP model by

$$x^2(k-i) \rightarrow \tanh^2(x(k-i))\,\forall i \in \{1,2\}$$

which matches the quadratic part in the set $[-1,\ldots,+1]$ and relaxes the influence outside of them leads to a stable system in simulation mode. To reduce the model complexity and the number of required regressors an F-Test [14] was applied.

A. Identification

For the final model, a sample time of $Ts = 20ms$ was chosen. On one hand this choice is limited by the complexity of the resulting model as well as on the computa

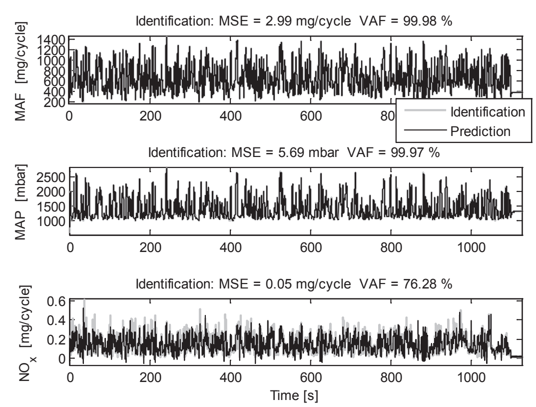

Fig. 2. NARX identification with predicted and measured output signals (MSE - mean square error, VAF - variance accounted for)

In total 88 parameters were estimated for two dynamic models (MAF, MAP: each 38 parameters) and the static relationship for NO_x (12 parameters), see (4).

$$\begin{aligned} y_j(k) &= \sum_i \varphi_{i,j}(k)\cdot\theta_{i,j} + e_j(k) \\ &\forall j \in \{1,2,3\}, i \in \{1,\ldots,12;1,\ldots,38;1,\ldots,38\}_j \end{aligned} \tag{4}$$

Fig. 3 shows the coefficients of the identified NARX model, the linear part in subplot a) and the non-linear terms in subplot b), c) and d). It turned out, that the linear system part is dominated by the autoregressive terms. Additionally also the couplings between MAP and MAF are significant for the MAF model. The non-linear part strongly depends on the quadratic system states (MAF and MAP) and the cross connections between the states and system inputs. Combinations and higher order terms regarding the system inputs have only a small influences (e.g.: $XEGR(k-2) \times XEGR(k-2)$ or $Wf(k-1) \times N(k-1)$). Quadratic regressors in $XEGR(k-1)$ and $XVGT(k-1)$, as well as couplings between them were removed to achieve an input affine model structure which is necessary for the controller design (see (7)).

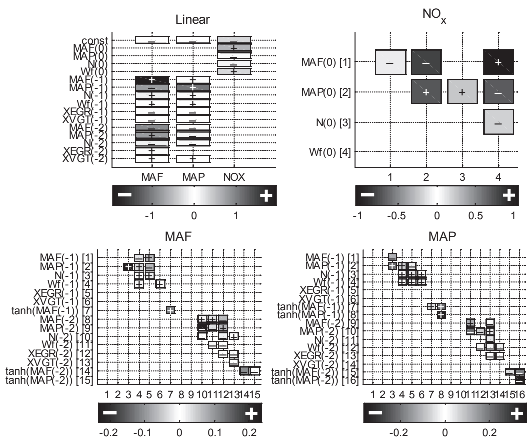

Fig. 3. NARX model coefficients a) left hand side upper plot b) right hand side upper plot, c) left hand side lower plot, d)right hand side lower plot

B. Verification

As verification data set, the first part of the FTP75 was chosen. Consequently Fig. 4 shows the VAF and MSE for MAF, MAP and the NO_x emissions. While the MAF and MAP model shows accurate results the NO_x model has some deviations in the dynamic and static behavior. Nevertheless, the fundamental dynamics can be modeled sufficiently, although only 4 inputs (MAF, MAP, N and Wf) are used.

C. State Observer

Due to the fact that the system description in (1) is not convenient for an extended Kalman filter (EKF, cf. [15]) a transformation in a canonical form is necessary. This can be

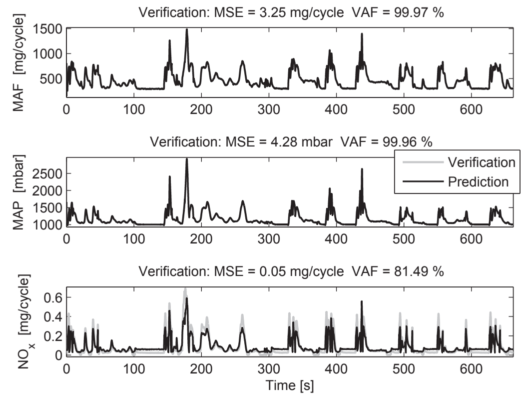

Fig. 4. Outputs for NARX model verification - Part of FTP75

done by adding additional states for the time shifted signals.

$$
\begin{aligned}
u_r(k-2) &= u_r(k-1) \\
&\vdots \\
u_1(k-2) &= u_1(k-1) \\
y(k-2) &= y(k-1) \\
y(k-1) &= y(k) \\
y(k) &= f^l\,(y(k-1),\ldots,y(k-n_y),\ldots \\
&\quad \ldots,u_1(k-1),\ldots,u_1(k-n_{u,1}),\ldots \\
&\quad \ldots,u_r(k-1),\ldots,u_r(k-n_{u,r});\theta)
\end{aligned} \tag{5}
$$

This formulation is only one possible transformation. In (5) the system has r inputs (instead of $r \times n_u + n_y$) and all non-linear terms are collected in the last equation.

IV. DISCRETE TIME SLIDING MODE CONTROL

The main idea behind the sliding mode theory is to use a feedback part with high gains. From the point of system stability this is only possible if the relative degree of the system is $r = 1$. Therefore a additional model output - the switching function s - is used to guarantee stability for systems with $r > 1$. The control task can be formed as:

- Define a switching function s with a desired dynamic which is of lower order than the controlled system.
- Define a control law so that the system steers toward the sliding surface and reaches it in finite time (reaching phase) and remains there.
- Once the system has met the sliding surface it should stay on the sliding surface (sliding mode), or is in a proper range, and reach the equilibrium (steady state phase) asymptotically.

A big advantage of that concept is that the dynamics during the sliding mode is independent from the system behavior which leads to a robust control scheme. In this paper a method was required which is applicable for non-linear and discrete time systems. The equivalent control method and the direct reaching law approach from [10] address this problem for continuous time systems. This theory was successfully extended to discrete time systems by making use of the Euler method (see Table I).

A. Equivalent Control Method

Initially, it is assumed that there is an existing switching function with a sliding surface. As mentioned above, the dynamics during the sliding mode are not dependent on the plant itself and therefore it is necessary to keep the system on the sliding surface. The equation

$$\dot{s} = s = 0 \tag{6}$$

satisfies this condition. For a non-linear input affine system

$$\dot{x} = f(x,t) + g(x,t)u \tag{7}$$

with the aid of (6) the feed forward signal u_{eq} leads with

$$\dot{s} = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x}\dot{x} = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x}\left(f(x,t) + g(x,t)u\right) \equiv 0 \tag{8}$$

to

$$u_{eq} = -\left[\frac{\partial s}{\partial x} g(x,t)\right]^{-1} \left(\frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} f(x,t)\right) . \tag{9}$$

Notice that $\frac{\partial s}{\partial x} g(x,t)$ must not be singular and the switching function s needs to have a relative degree $r = 1$ to fulfill (6). Furthermore, it is common to design a time invariant switching function ($\frac{\partial s}{\partial t} = 0$).

B. Direct Reaching Law Approach

To characterize the system behavior during the reaching phase a possible choice is the so called direct approach from [10]:

$$\dot{s} = -Q\,\mathrm{sign}(s) - Ks \tag{10}$$

The term $-Q\,\mathrm{sign}(s)$ generates a constant rate of change depending on the system is near or far away from the sliding surface. An elegant method to increase this factor is by adding a proportional rate term. This term forces the state much faster to the sliding surface when s is large and for a small s the influence of Ks is comparatively low. Due to the sign-function this term is often called discontinuous part.

C. Sliding Surface

In the beginnings of VSC (Variable-Structure-Control, cf. [9]) the design of the switching manifolds for systems of 2^{nd} order was often done by graphical tools in the phase plane. For MIMO systems or systems with a degree higher than 2^{nd} order this method is not feasible and pole placement, quadratic optimization or Lyapunov's second method [10] have to be employed. A common approach to specify the switching function is

$$s = \left(\lambda + \frac{d}{dt}\right)^{r-1} e \tag{11}$$

with the error $e = y - y_{Ref}$ and the relative degree r. This leads in combination with the direct reaching law (10) in the MIMO case to a free order switching scheme which appears to be the most efficient from a practical point of view [10]. The tuning parameter λ is a strictly positive constant to achieve an asymptotic sliding mode dynamic.

D. Sliding Mode Control on Discrete Time Systems

The system behavior of the diesel engine between XEGR input and NO_x output has a relative degree larger than 2. Nevertheless, as overall approximation a relative degree of $r = 2$ was assumed which relaxes the design of the switching function, the requirements on the trajectory and the transformation with the Euler methods. Equation (11) leads in case of NO_x tracking with $r = 2$ and the Euler methods from Table I to

$$\begin{aligned} s(k) &= \lambda e_{NO_x}(k) + \dot{e}_{NO_x}(k) \\ &= \lambda\left(NO_x(k) - NO_{x,Ref}(k)\right) + \left(\dot{NO}_x(k) - \dot{NO}_{x,Ref}(k)\right) \end{aligned} \tag{12}$$

As mentioned above, the system model has to be input affine as shown in (13) where all non-linear terms as well as combinations with $u_{EGR}(k-1)$ and constants are collected in $f_{NO_x}(k)$. Only coefficients according to the linear part in (4) are summarized in $g_{NO_{x,EGR}}(k)$.

$$NO_x(k+1) = f_{NO_x}(k) + g_{NO_x,EGR}(k) \cdot u_{EGR}(k) \tag{13}$$

Equation (6) in discrete time $\dot{s}(k) = 0$ combined with the system dynamic (13) and the switching function (12) yields under usage of the Euler methods from Table I to the equivalent control (14).

$$\begin{aligned} u_{EGR,eq}(k) = &\frac{1}{g_{NO_x,Ref}(k)\left[\lambda + 1/Ts\right]}\left(NO_x(k)/Ts - \dots\right. \\ &\dots - f_{NO_x}(k)\left[\lambda + 1/Ts\right] + s(k) + \dots \\ &\dots + \lambda \cdot NO_{x,Ref}(k) + \left[\lambda \cdot Ts + 1\right]\dot{NO}_{x,Ref}(k) + \dots \\ &\left.\dots + Ts \cdot \ddot{NO}_{x,Ref}(k)\right) \end{aligned} \tag{14}$$

The discontinuous part of the controller can be found by the use of the reaching law equation (10) and (12) to (14):

$$u_{EGR,disc}(k) = -\frac{Ts \cdot \left(Q\,\mathrm{sign}\left(s(k)\right) + Ks(k)\right)}{g_{NO_x,EGR}(k)\left[\lambda + 1/Ts\right]} \tag{15}$$

The final DSMC is the sum of the equivalent control (14) and the discontinuous term (15)

$$u_{DSMC}(k) = u_{EGR,eq}(k) + u_{EGR,disc}(k) \tag{16}$$

whereby Q defines the switching gain, K the proportional term and λ specifies the characteristic during the sliding mode.

Note that by using the Euler methods from Table I the 2^{nd} derivative of the NO_x reference has to be bounded.

TABLE I
EULER APPROXIMATIONS

Forward Euler methods for:
$\dot{s} \cong \frac{s(k+1)-s(k)}{Ts}$
$NO_{x,Ref}(k+1) \cong \dot{NO}_{x,Ref}(k) \cdot Ts + NO_{x,Ref}(k)$
$\dot{NO}_{x,Ref}(k+1) \cong \ddot{NO}_{x,Ref}(k) \cdot Ts + \dot{NO}_{x,Ref}(k)$
Backward Euler methods for:
$\dot{NO}_x(k) \cong \frac{NO_x(k)-NO_x(k-1)}{Ts}$

E. Chattering

For a switching function near $s = 0$ the discontinuous part (15) results a high frequency switching signal. Since it is not possible to switch the control infinitely fast, chattering occurs in the sliding mode and the steady state phase. Chattering is almost always undesired and there were several works focusing on this topic [9]. One possibility to reduce the chattering and to protect the actors from physical damage is the use of anti chattering techniques. Here, a common approach is to replace the sign-function by a saturation relation (17).

$$\text{sign}(s) \to \text{sat}(s,L) = \begin{cases} \frac{s}{L} & \text{if } |s| \leq L \\ \text{sign}(s) & \text{otherwise} \end{cases} \tag{17}$$

V. TESTBENCH MEASUREMENTS

In this section the experimental results obtained by applying the proposed sliding mode controller are shown. Based on the NOx measurements recorded by the fNOx400 which was used as input for the extended Kalman filter (EKF) the results depicted in Fig. 5 were archived. Additionally in the subplot of the figure also the switching function s is illustrated.

Here 2 different scenarios were evaluated. During the first part only disturbance rejection was applied and in a second phase, the reference is abruptly changed but the control is able to follow and adapt to the desired emission levels.

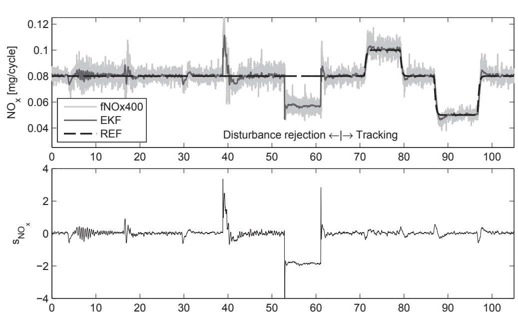

Fig. 5. Disturbance rejection and tracking done by Discrete time Sliding Mode Control (DSMC)

From the control point of view the dead times introduced by the communication via Bypass and CAN also have a certain influence on the performance of the feedback part. Furthermore the EGR valve control, which is done by a subordinate cascaded PI controller, leads to a dead time of approximately 150*ms* (see Fig. 7 and Fig. 9) where the DSMC signal is the desired manipulated variable and the ECU signal reflects the PI controller output value. Currently this dead time is the limiting factor for the DSMC feedback performance. Nevertheless, it turned out that the proposed DSMC method achieves adequate results.

Another limiting factor which should be taken into account if the desired NO_x values are not reachable because it is not possible to achieve the required EGR flow - even though the EGR valve is fully open - an extension of the control scheme is necessary. A possibility to achieve the desired NO_x values to a certain extend is the used of XVGT position. The XVGT reaction can be simplified described by the following: In the intake manifold the turbo guide vane position has mainly an influence on the manifold air pressure and in the exhaust manifold it is acting as well as the EGR valve like a throttle. Thus a high VGT valve position raises the back pressure and the pressure drop over the EGR valve, which further increases the EGR mass flow. Due to non-minimum phase system characteristics in XVGT to NO_x there is no direct access to an equivalent control law to keep the system on the sliding surface also this would yield to an instable system through (9). In the literature some contributions can be found which deal with sliding mode in case of non-minimal phase systems for continuous non-linear systems (cf. [12], [13]). Therefore an additional control law (18) was applied. $XVGT_{OP}$ defines the guide vane position in the operating point (e.g. ECU or manual controlled) and UB as well as LB are the upper and lower bound of the interpolation region.

$$XVGT = \begin{cases} XVGT_{OP} & XEGR < LB \\ 100\% & XEGR > UB \\ XVGT_{OP} + \Delta Y \cdot \Delta X & \text{otherwise} \end{cases} \tag{18}$$

with

$$\Delta Y = \frac{100\% - XVGT_{OP}}{UB - LB} \text{ and } \Delta X = LB + XEGR$$

Fig.6 shows the DSMC result in tracking mode with the controller output in Fig.7 and suggests that positive set point changes are tracked sufficiently. In some regions the feed forward part causes an overshoot (see Fig.6 at $t = 7s$). In this case an improvement of the feed forward part will lead to better results. In Fig. 7 the whole controller signal and the ECU output signal from the PI controller are proposed together. It should be noted that in regions where the EGR is fully the PI controller signal becomes negative. Due to that it is possible to keep the valve with a negative force in the fully closed position.

Fig. 8 shows the tracking of a negative set point change with a manually optimized feed forward signal (see Fig. 9). To provide an optimal feed forward part a model with additional influence parameters is expected to enhance the model quality (cf. [6], e.g.: p_{Rail}, φ_{MI} and $O_{2,Exh}$).

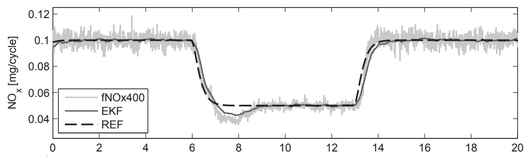

Fig. 6. NO$_x$ tracking with sliding mode method - $Wf = 10mg/cycle$ and $N = 1700rpm$

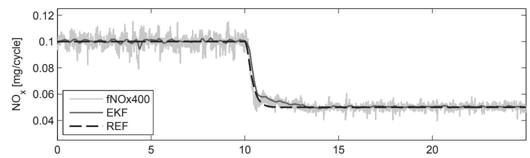

Fig. 8. NO$_x$ tracking and switching function with sliding mode method and optimized feed forward part - $Wf = 10mg/cycle$ and $N = 1700rpm$

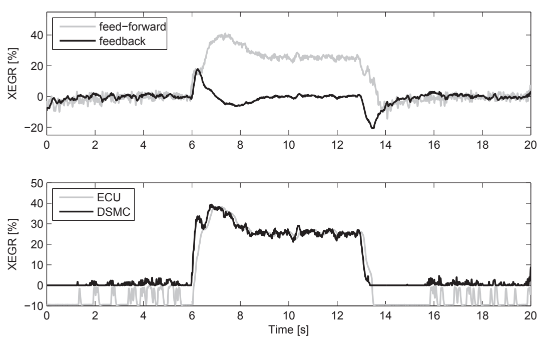

Fig. 7. Manipulated XEGR signal from the sliding mode controller

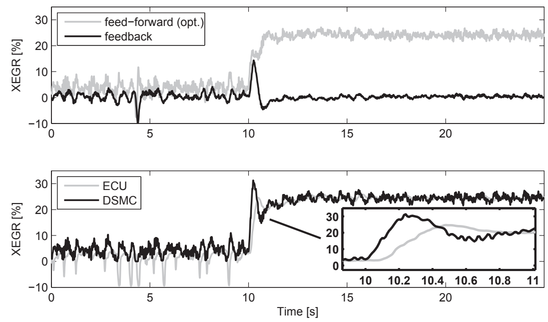

Fig. 9. EGR valve control signal with optimized feed forward part

VI. CONCLUSIONS

A discrete time sliding mode approach for NO$_x$ emissions was successfully applied to a diesel production engine with the manipulated variable XEGR and a secondary control law for XVGT to reach the NO$_x$ demanded in critical regions. Based on the continuous sliding mode theory for non-linear models it was possible to apply this approach via the Euler methods to discrete time systems. In case of MIMO systems the challenge of further works is to design multidimensional switching functions and to extend the equivalent control method to non-minimal phase systems. Nevertheless the proposed method is able to fulfill time varying emission requirements and/or rejecting disturbances, in a robust way, under the condition of feasibility.

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