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Use of Time-Varying Oil Price in Short-term Production Optimization for a Reservoir

Mansoureh Jesmani * **Bjarne Foss** **

Department of Engineering Cybernetics, NTNU and the Center for Integrated Operations in the Petroleum Industry, IO-Center NTNU, Trondheim, Norway
* *e-mail: mansoureh.jesmani@ntnu.no*
** *e-mail: bjarne.foss@ntnu.no*

Abstract: Oil price significantly affects the economic performance of an oil field. In this study, Model Predictive Control (MPC) is used as a reference tracking method to provide corrective control action by meeting an objective, which balances short term and long term effects, based on an updated oil price. The long-term optimized production profile is based on a predicted oil price while the short-term objective is optimized with respect to the predicted short-term oil price by applying MPC. The fluid flow equations must be solved many times to find an optimal input for a short-term horizon; therefore, instead of a large-scale nonlinear reservoir model, local linear models are used to decrease the computation time for short-term optimization. The developed method is tested with several oil price scenarios on a synthetic case. The efficiency of the developed method is confirmed by comparing its economic performance to that using open loop long-term optimization.

Keywords: Short-term production optimization, Balanced objective function, Time-varying oil price, Multi-level control hierarchy, Model predictive control.

1. INTRODUCTION

Optimizing oil production is important to ensure good economic performance of an oil field. Many parameters such as heterogeneities in reservoir properties do affect such an economic objective. The situation is further complicated by the uncertainty in the parameters. The variation in oil price is a major factor in oil field management. For instance, an oil price increase may result in an operational strategy with increased production, possibly harming long-term oil recovery (van Essen et al. (2011)). A balanced objective function that incorporates both long-term and short-term objectives, and which is determined by the oil price is therefore of interest and can help to avoid these problems.

Handling constraints consisting of bounds on injection rates and bottom hole pressures (bhp) is an inseparable part of production optimization of an oil field. MPC is widely used as a robust controller with constraint handling capabilities in the process industries (Qin and Badgewell (2003)), but has only been used to a limited extent in oil and gas production. In this study we apply MPC to production optimization of an oil field for two more reasons. First, MPC enables feedback so as to incorporate recent short-term predictions of the oil price into the scheme. Such predictions are significantly more accurate than long-term predictions. Second, MPC results in robust and efficient solutions.

In short-term production optimization, as in all optimization problems, the fluid flow equations must be solved many times to obtain an optimal solution. Reduced-order modeling procedures are therefore useful when the simulation model requires long run times (van Essen et al. (2010)). This paper thus uses local linear models to decrease computation time.

In this paper, we first develop a set of local linear models. A typical multi-level control hierarchy is subsequently defined for an oil field, and the developed long-term and short-term optimization methods are presented in the following section. In Section 4 linear MPC is briefly described. Finally, a synthetic simulation is conducted for 10 different oil price scenarios to verify the accuracy and efficiency of the developed method; the paper ends with conclusions.

2. RESERVOIR LOCAL LINEAR MODELING

In a typical commercial reservoir simulator, the well conditions are used as inputs and there are various outputs, such as the production rates of each phase for a given reservoir. Thus, the simulator maps the control actions to an output vector. (1) shows a map for a two-phase reservoir:

$$\begin{bmatrix} q_o \\ q_w \end{bmatrix} = M(\begin{bmatrix} q_i \\ p_{bh} \end{bmatrix}), \tag{1}$$

where q_i and p_{bh} denote the vectors corresponding to the injection rates and the bhp of the wells, respectively. The terms q_o and q_w denote the oil and water production rates, respectively.

Here, we develop a set of local linear models, which are mathematically defined functions that describe the dynamic relationship between the inputs and the outputs for short prediction horizons and uses autoregressive models with exogenous inputs (ARX models) in the identification algorithm. In this paper, a set of ARX models is used for reference tracking of the long-term optimization to relate production rates to the injection rates and bhp of the wells. Each local linear model is computed around a long-term operational trajectory, i.e., the control inputs and the states that are defined for a long-term scenario. Consequently, each local model is valid on a short-term horizon in some neighborhood of the long-term trajectory. Each local linear model is formulated as follows:

$$A(z)\ y = B(z)\ u \tag{2}$$

where

$$y = \begin{bmatrix} q_o \\ q_w \end{bmatrix}, u = \begin{bmatrix} q_i \\ p_{bh} \end{bmatrix} \tag{3}$$

where $A(z)$ denotes an $n \times n$ diagonal matrix, and the number of outputs, y, is denoted by n, the number of control actions by m, and $B(z)$ is an $n \times m$ matrix. The delay operator is denoted by z, while the elements of $A(z)$ and $B(z)$ are defined as follows:

$$A_{ii}(z) = 1 + a_{i1}z^{-1} + \ldots + a_{n_{a_i}} z^{-n_{a_i}} \tag{4}$$

$$B_{ij}(z) = b_{i1} + b_{i2}z^{-1} + \ldots + b_{n_{b_{ij}}} z^{-n_{b_{ij}}} \tag{5}$$

$$i \in \{1, \ldots, n\}, \quad j \in \{1, \ldots, m\}$$

where n_{a_i} denotes the number of poles, and $n_{b_{ij}}$ defines the zeros.

Persistent excitation is essential for parameter convergence in model identification; that is, the system must be excited for the model to capture all of the relevant dynamics in an identified model (Gevers (2005)). In this study, PRBS signals are superimposed onto the long-term optimized production strategy, u^*.

Obviously, a local model may have a smaller validity range than typical variations in a long-term scenario. Therefore, several local models are combined in this study (Johansen and Foss (1993)). Consequently, the long-term horizon will be divided into a number of shorter ranges over which the long-term optimization solutions are approximately constant. One local model will be identified for each of these intervals.

3. MULTI-LEVEL CONTROL HIERARCHY FOR AN OIL FIELD

In this study, production optimization of oil and gas fields is performed using a hierarchical approach (Foss and Jensen (2011)). The production and injection targets are defined for a long-term horizon on the order of years and a short-term horizon with the range of weeks as illustrated by level 2 and 3 in Figure 1.

3.1 Long-term Production Optimization

The well settings, i.e., the bhp and the flow rates are determined by maximizing an economic objective function.

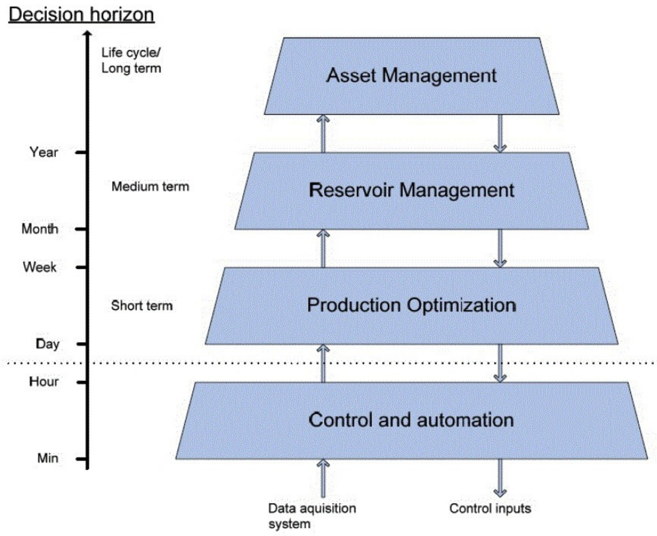

Fig. 1. Multi-level control hierarchy (Foss and Jensen (2011)).

The discrete form of the net present value (NPV) is widely used and is defined by (6):

$$J = \sum_{k=1}^{K_T} \frac{r_o q_{o,k} - r_w q_{w,k} - r_i q_{i,k}}{(1+b)^{\frac{t_k}{\tau_t}}} \Delta t_k \tag{6}$$

where r_o denotes the fixed oil price, and r_w and r_i are the water production and the water injection costs, respectively, all of which are assumed to be constant. To account for depreciation, the discount rate, b, is added for a certain reference time, τ_t. The final time step is K_T, and Δt_k denotes the time interval at the k^{th} time step. The oil production, water production and water injection rates are denoted by $q_{o,k}$, $q_{w,k}$, and $q_{i,k}$, respectively, at the time step k.

The optimization problem can be formulated using a constant oil price as follows:

$$u^* = \arg\max_u J \tag{7}$$
$$s.t.\ f(x_{k+1}, x_k, u_k) = 0, \quad k = 1, \ldots, K_T$$
$$g(u_k) \le 0, \quad x_0 = \hat{x_0}$$

where f denotes the reservoir model. The initial conditions are represented by $\hat{x_0}$, and g are the constraints corresponding to the lower and upper bounds on the injection rates and the bhp. The optimal control actions that are used as a reference trajectory in short-term optimization are denoted by u^*.

Many methods are available for solving the problem (7). Gradient-based optimization is efficient to deal with large-scale systems, such as production optimization in oil reservoirs, provided the gradient can be computed efficiently. The adjoint method has been used in many papers for similar problems, see for example Jansen (2011). This method is sufficiently efficient since the gradient can be computed in only two simulation runs regardless of the number of optimization variables.

In this study, the adjoint method is used to determine the gradient of the objective function with respect to the control settings. We omit the detailed derivation of the reservoir model equations because of space limitations, further details on the reservoir equations and the adjoint model are available in Krogstad and Gulbransen (2011).

Let p^n denote the vector that consists of the grid block pressures and the unknown wellbore pressures at the time-step n. Similarly, let s^{n-1} denote the grid block saturations at the time step $n-1$. Enforcing a volume balance, i.e., setting the sum of all of the outward fluxes from each block equal to the source flux, yields to a positive semidefinite matrix in a linear system equation

$$\begin{bmatrix} B(s^{n-1}) & C & D \\ C^T & 0 & 0 \\ D^T & 0 & 0 \end{bmatrix} p^n = Bu^n \tag{8}$$

Here, the right-hand side is a function of the control input vector u^n at the time step n.

We discretize the saturation equation using a standard upstream weighted implicit finite volume method to yield the following equation:

$$s^n = s^{n-1} + g(\upsilon^{n-1}, s^{n-1}), \tag{9}$$

where υ denotes the outward face fluxes and the well perforation rates.

To derive the adjoint equations corresponding to (8) and (9), we introduce the Lagrange multipliers λ_u^n, λ_p^n, $\lambda_{\hat{\pi}}^n$, and λ_s^n for each time step, t^n, which correspond to the dual variables υ^n, p^n, $\hat{\pi}^n$, and s^n, respectively. The adjoint equations for the time step n are then given by

$$(I - \frac{\partial g(\upsilon^n, s^n)}{\partial s^n})\lambda_s^n = \lambda_s^{n+1} - \frac{\partial J^{n\,T}}{\partial s^n} - (\frac{\partial B(s^n)\upsilon^{n+1}}{\partial s^n})\lambda_u^{n+1} \tag{10}$$

$$\begin{bmatrix} B(s^{n-1}) & C & \hat{D} \\ C^T & 0 & 0 \\ \hat{D}^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_u^n \\ \lambda_p^n \\ \lambda_{\hat{\pi}}^n \end{bmatrix} = \begin{bmatrix} -\frac{\partial J^{n\,T}}{\partial \upsilon^n} - \frac{\partial g(\upsilon^n, s^n)^T}{\partial \upsilon^n}\lambda_s^n \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

Once the adjoint equations have been solved, the gradient of the objective function J at the time step t^n can be computed as follows:

$$\nabla_{u^n} J = \frac{\partial J^T}{\partial u^n} - {A_D^n}^T \lambda_\upsilon^n - {A_N^n}^T \lambda_{\hat{\pi}}^n \tag{12}$$

where the matrices A_D^n and A_N^n are defined below:

$$-D_D \pi_D^n = A_D^n u + b_D^n \tag{13}$$

$$\hat{\upsilon_N^n} = A_N^n u + b_N^n \tag{14}$$

This gradient can be used with any gradient-based algorithm to determine the new search direction and step length and, thereby, the new control actions. The process is repeated until some termination criterion, such as a vanishing gradient, is satisfied.

3.2 Production Optimization in Short-term Horizon

The life-cycle water flooding problem, which was defined in the previous section, can be solved by maximizing the NPV. However, short-term effects must in practice also be considered in reservoir management. As previously mentioned, short-term considerations may adversely impact the long-term NPV (Suwartadi et al. (2012)). In this paper, we therefore develop the following balanced objective function to track the long-term control actions while meeting the short-term objective:

$$J_{\text{bal}} = J_s + J_l \tag{15}$$

$$J_s = \sum_{k=1}^{N} -w_{s,k}(q_{o,k}^T q_1) \tag{16}$$

$$J_l = \sum_{k=1}^{N} w_{l,k}[(u_k - u_k^*)^T R_2 (u_k - u_k^*) + (y_k - y_k^*)^T Q_2 (y_k - y_k^*)] \tag{17}$$

$$R_2, Q_2 \succeq 0$$

where J_s and J_l represent the short- and long-term objectives, respectively, N denotes the short-term horizon, and q_1, R_2 and Q_2 are a constant vector and two constant matrices, respectively. The terms w_l and w_s are the weighting factors for the short- and long-term objectives. The vector of control actions is denoted by u and consists of the injection rates and the bottom hole pressures. The output vector of the phase production rates is denoted by y. These two vectors are defined in (3).

It is challenging to determine suitable weighting factors (van Essen et al. (2011)). In this paper, we develop a method based on the oil price to determine these weights. As previously stated, the long-term optimization problem is solved using a fixed oil price on the entire horizon, which is denoted by r_o^*. The method developed for determining the weights is based on the premise that if the oil price is higher than the fixed price, r_o^*, it is economically prudent to increase production and vice versa. Therefore, w_s and w_l can be defined by (18) - (20) below:

$$w_{s,k} = \begin{cases} -1 & \text{if } r_{o,k} \in s_1 \\ \sqrt[3]{\dfrac{r_{o,k} - r_o^*}{\Delta r_o^*}} & \text{if } r_{o,k} \in s_2 \\ 1 & \text{if } r_{o,k} \in s_3 \end{cases} \tag{18}$$

where

$$\begin{cases} s_1 : r_{o,k} < r_o^* - \Delta r_o^*, \\ s_2 : r_o^* - \Delta r_o^* \le r_{o,k} \le r_o^* + \Delta r_o^*, \\ s_3 : r_{o,k} > r_o^* + \Delta r_o^*. \end{cases} \tag{19}$$

$$\mid w_{s,k} \mid + w_{l,k} = 1. \tag{20}$$

where $r_{o,k}$ is the oil price at the k^{th} time step. The variable Δr_o^* depends on the operational strategy chosen by the managers. Figure 2 shows the weight of the short-term objective versus the oil price.

4. MODEL PREDICTIVE CONTROL

The optimization problem is formulated as follows:

$$\begin{aligned} \min \quad & J_{\text{bal}} \\ s.t. \quad & A(z)\begin{bmatrix} q_o(k) \\ q_w(k) \end{bmatrix} = B(z)\begin{bmatrix} q_i(k) \\ p_{bh}(k) \end{bmatrix}, \\ & \begin{bmatrix} q_o^l(k) \\ q_w^l(k) \end{bmatrix} \le \begin{bmatrix} q_o(k) \\ q_w(k) \end{bmatrix} \le \begin{bmatrix} q_o^u(k) \\ q_w^u(k) \end{bmatrix}, k = 1, \ldots, N \end{aligned} \tag{21}$$

where J_{bal} is defined by (15), and $A(z)$ and $B(z)$ are obtained by the local linear model identification. The lower

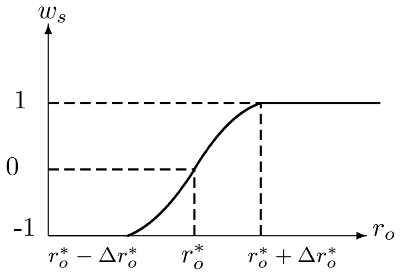

Fig. 2. Weight of short-term objective as a function of oil price.

and upper bound of control actions are denoted by l and u, respectively.

The theory of constrained linear MPC has been explained in many papers, see for example Garcia et al. (1989). The MPC algorithm is briefly explained below.

(1) Given a measurement of the output vector, the system state is estimated at the current control time, and the future control input is computed by solving an optimization problem over a prediction horizon.

(2) The first part of the control input is implemented.

(3) The system is reoptimized over a receding horizon at the next control time.

5. SIMULATION

In this example, we consider the 3D two-phase reservoir described in Lie et al. (2012). The model contains a production well and a couple of injector wells, and it is mildly nonlinear. Figure 3 illustrates the entire model configuration and Table 1 presents the simulation and optimization parameters. There are a total of 13240 active grid blocks.

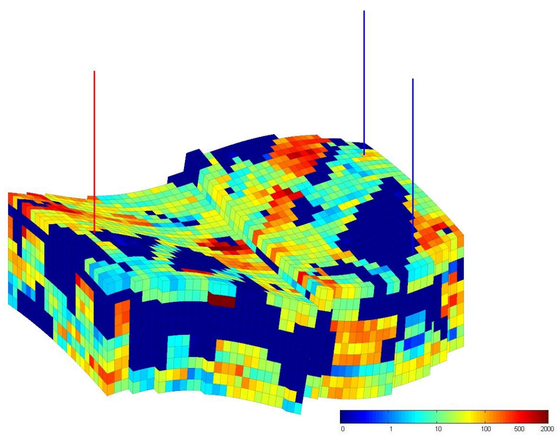

Fig. 3. Permeability distribution, where red indicates high permeability zones while blue shows low permeability zones, of the reservoir model. The red line indicates the production well location, the blue line on the right corresponds to injector well 1 and the blue line on the left corresponds to injector well 2.

Parameter	Value	Unit
r_o^*	630	USD/m^3
r_w^*	10	USD/m^3
r_i^*	10	USD/m^3
b	10	%
Δr_o^*	100	USD/m^3
μ_o	5×10^-3	$Pa.s$
μ_w	1×10^-3	$Pa.s$
ρ_o	859	Kg/m^3
ρ_w	1014	Kg/m^3
p_{int}	200	bar

Table 1. Simulation and Optimization Parameters.

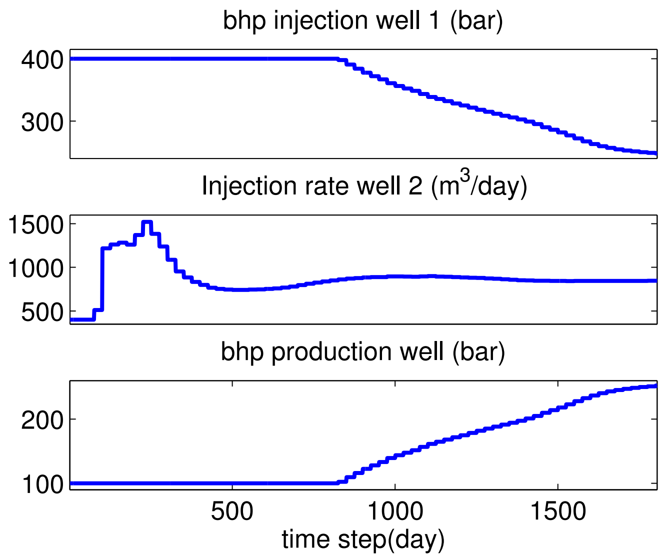

Fig. 4. Solution of the long-term optimization problem by applying the line search method, which uses adjoints to calculate the gradient.

The long-term horizon is 5 years. The control actions are the injection rate of the first injector well and the bhp of the second injector well and the production well. The lower and upper bounds on the bhp are set equal to 100 bar and 400 bar, respectively. The lower and upper bounds on the injection rate are set to 400 m^3/day and 5000 m^3/day, respectively. The control actions are piecewise constant and change every 25 days. Open loop production optimization is implemented by using *MATLAB Reservoir Simulation Toolbox* (MRST (2012)). Figure 4 illustrates the long-term optimization results. We observe that the bhp of injection well 1 starts from its upper bound, while the injection rate of well 2 is equal to its lower bound at the beginning. This stems from higher permeability in the neighborhood of the injection well 2. As expected, the bhp of production well initializes at its lower bound, which results in maximum production rate.

As mentioned earlier the model is nonlinear, hence, we identify a set of local linear models instead of one linear model. By inspection of the long-term solution we divide the 5 year period into 6 intervals at the following times: 80, 105, 250, 500 and 840. The order of the first model is 5 while the others are second order models. The model order choice is based on an assessment of a misfit measure. To identify the local models, the control actions are

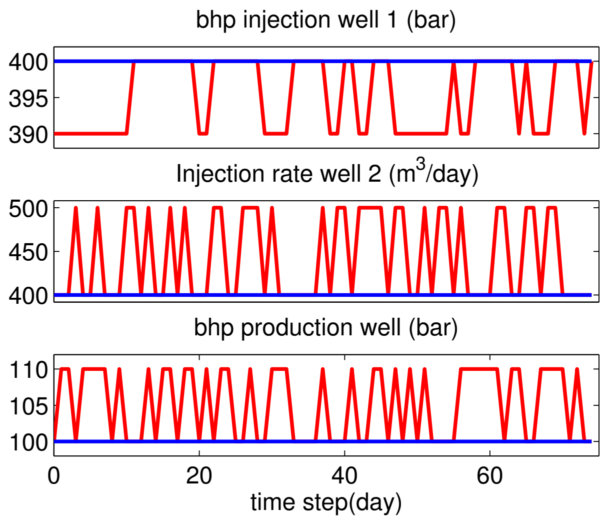

Fig. 5. First 75 days of exciting control actions (red line) and long-term optimization solution(blue line)

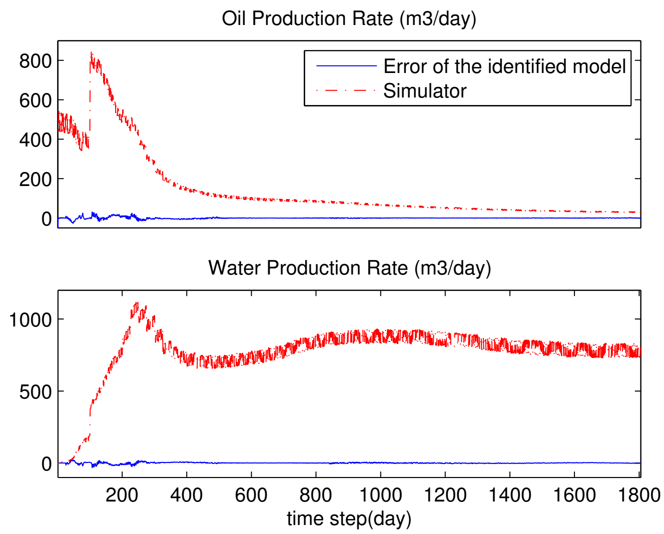

Fig. 6. Comparison between the liquid production rates from the simulator and the error of the identified model

constructed by superimposing PRBS signals onto the long-term optimization solution. The minimum switching time is 1 day. This is a reasonable choice even though a longer switching time, e.g. 3 days, would also work. The standard deviation in the PRBS signal for bhp of the first injector well and the production well is $5bar$ and 20% of the long-term solution for the injection rate of the second injector well. The PRBS signal may result in violations of the bound constraints on the problem. This problem is solved by moving the average value away from the constraint, as illustrated in Figure 5. The exciting control actions are applied to the simulator to identify the local models; Figure 5 shows the exciting control actions for the first 75 days. The accuracy of replacing the simulator by the local linear models in the short-term optimization procedure is verified by applying the exciting inputs to the simulator, and the output of the simulator and the error of the local linear models are compared in Figure 6.

The MPC has a prediction horizon of 25 days. The weights of the short- and long-term objectives are obtained using (18)- (20). The other variables are normalized according to their ranges. The MPC control actions are determined by changes around the nominal control action obtained from the long-term optimization solution.

The method is implemented for 10 different oil price scenarios where the oil price may change every 5 days, and it is tested on the high fidelity reservoir simulator. Figure 7 compares the NPVs for half of the scenarios for both methods, the open loop long-term optimization method and the method developed here. The developed method is superior to the open loop long-term optimization method for all of the different scenarios; therefore the developed method is concluded to be robust with respect to oil price variations. Applying the solution of the long-term optimization and the developed method to the model for 5 years yields average NPV values for the 10 scenarios of 1.37×10^8\$ and 1.43×10^8\$, respectively. Consequently, the average increase in profit equals 3.8%. Figure 8 shows the production rates for the first oil price scenario. The production rates track the references when the oil price is close to the predicted oil price for the long-term optimization problem while the rates vary from the long-term solutions for any deviation of the oil price from the long-term predicted prices.

Figure 9 illustrates the difference between the oil production rate of applying the developed method and the open loop optimization solution divided by the nominal rate for the first scenario ($\Delta q_o = (q_o - q_o^*)/q_o^*$) versus the difference between the oil price of the aforementioned scenario and the nominal oil price ($\Delta r_o = r_o - r_o^*$). The big picture is that the oil rate increases when the oil price increases and vice versa. However, the dependency is clearly not linear. Further, there are even circles in the second and fourth quadrant; meaning that production rate may decrease when the oil price is above the nominal value and vice versa. The circles that are near to the horizontal axis are related to situations where there are significant oil prices variations within the 25 day MPC prediction horizon.

6. CONCLUSIONS

A new production optimization method is presented where the time-varying oil price is the key variable for making decisions. Both long-term and short-term objectives are incorporated into a balanced objective function, the balance being determined by the oil price. The long-term and short-term objectives are optimized using a long-term and a short-term oil price prediction, respectively. MPC is applied as a reference tracking method where the reference is the long-term optimized solution. Local linear models are developed to decrease the computation time in the short-term optimization problem. The simulation results, using a high fidelity reservoir simulator, with different oil price scenarios verify that the method can indeed be useful when there is considerable short-term variations in the oil price. If better control performance is required regardless of the computational time, nonlinear MPC applied to the nonlinear model may be an option.

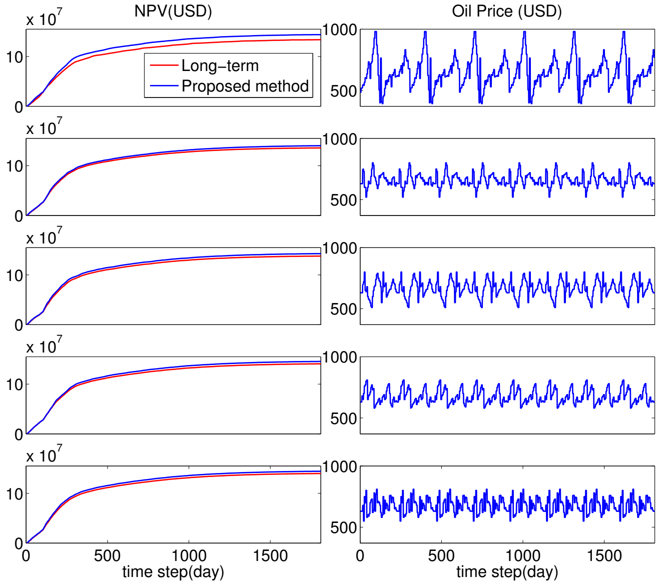

Fig. 7. Comparison between the NPV from the long-term optimization and the developed method for different oil price scenarios

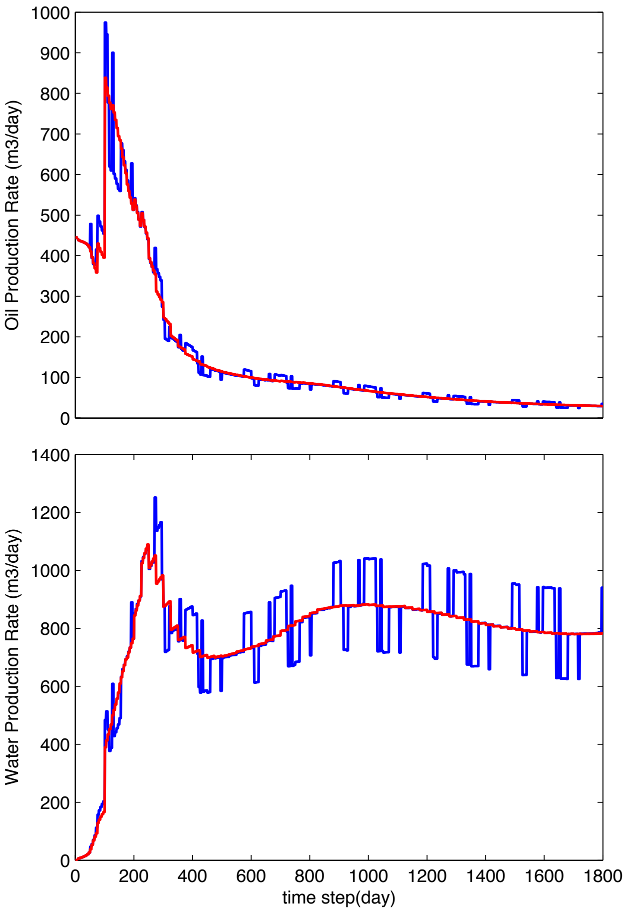

Fig. 8. Comparison between the production rate from the long-term optimization (red line) and the developed method (blue line) for the first oil price scenario

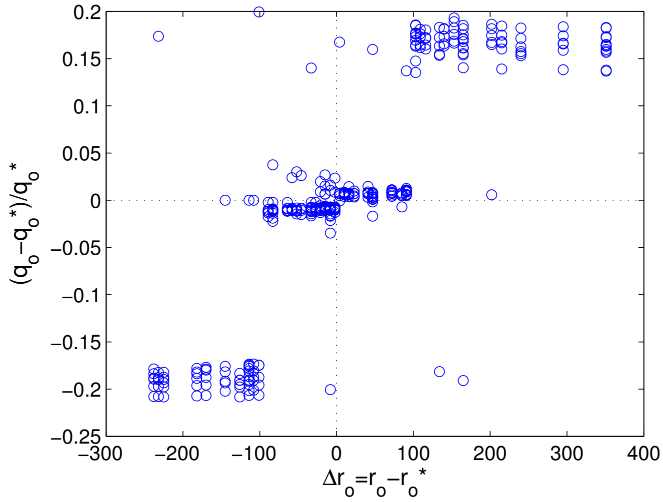

Fig. 9. Change in the oil production rate as a function of variation in the oil price

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