QUANTIFYING THE IMPACT OF CONTROL LOOP PERFORMANCE, TIME DELAY, AND WHITE-NOISE OVER THE FINAL PRODUCT VARIABILITY

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Abstract: This article aims to propose three new indices that allow quantifying the influence of control loop performance, time delay, and white-noise over the total control loop variance. The signal is first decomposed in the deterministic and indeterministic parts, using an MA or AR model. Later, the deterministic part is divided into two parts: the feedback accessible and the inaccessible parts, as function of time delay. To estimate each influence, no invasive tests are required, only control loop routine operating data and process time delay, allowing the industrial application of the proposed indices in real time. The methodology was applied in a hypothetical case study, providing good results.

Keywords: Performance monitoring; Performance indices; Noise characterization; Minimum variance control; PID control

1 INTRODUCTION

Control performance assessment tools are important to maintain the plant in a high efficient operating point. The indices based on the work of Harris (1989) are widely used to assess control loop performance. The Harris index (η) can be defined as the ratio between process minimal variance (σ_{MV}^2) and actual variance (σ_y^2), i.e.

$$\eta(d) = \frac{\sigma_{MV}{}^2}{\sigma_y{}^2} . \qquad (1)$$

The best controller is achieved when η is close to the unity, i.e. the control loop variance is close to the minimum variance. The worst case occurs when the ratio is equal to zero (i.e. $\sigma_y^2 \to \infty$). To estimate σ_{MV}^2 only routine operating data and the time delay (d) are need and no extra experiments are required.

The minimal variance (σ_{MV}^2) can be high because of large pure time delay and/or large process/instrument white-noise. In both case, it is quite common to have process variance (σ_y^2) close to σ_{MV}^2 making the Harris index close to 1. Note that in this situation, the Harris index is not a good measurement for closed loop performance monitoring, since values close to 1 are usually interpreted as good performance, what in

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these cases it is not a true conclusion. Moreover, in these situations, the Harris Index are almost insensitivity to loop tuning, making it completely not useful for performance monitoring.

In this paper, three new indices are proposed. They allow quantifying the controller performance, time delay, and white-noise influence over the total variability. The new set of indices is a valuable tool in control loop performance assessment to diagnose and remove the component that affects the process variability.

The three new indices are based upon the minimal variance concept (Harris, 1989, Huang and Shah, 1999) and their calculation needs only routine operating data and the process time delay.

The methodology can be summarized as follows: first, the control loop signal is decomposed in the deterministic and nondeterministic parts, using an MA or AR model (Chatfield, 1989). Later, the deterministic part is divided into two parts: feedback accessible and inaccessible parts, as function of time delay. The schematic representation of proposed methodology is shown in Figure 1.

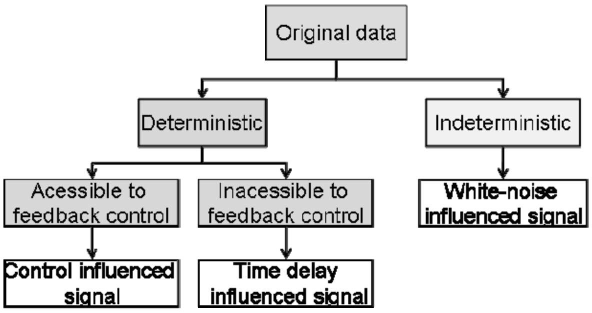

Fig. 1: Schematic representation for the three indices quantification algorithm.

The article is segmented as follows. Section 2 introduces the three new indices that decompose the process variability into white-noise, time delay and loop performance contributions. Section 3 shows the methodology to evaluate each proposed index. In section 4, the proposed method is applied in a case study. The paper ends with concluding remarks in section 5.

2 METHODOLOGY

This section introduces three new indices to decompose the influence of control loop tuning, time delay, and white-noise over total output variance. The methodology to calculate each one is based on routine operating data.

We can decompose the total signal variance (TSV) in the following three components:

- e_t is the time delay part
- g_t is the control performance component, which is affected by tuning
- w_t is the white-noise component of output signal (y_t).

and the correspondent variance can be computed as:

$$TSV = \sigma^2(e_t) + \sigma^2(g_t) + \sigma^2(w_t) \quad (2)$$

where σ^2 is the signal variance.

The first index, called *nosi*, quantifies the white-noise influence in the control loop. It is defined as the ratio between white-noise component variance and total signal variance.

$$nosi = \frac{\sigma^2(w_t)}{TSV} \quad (3)$$

None controller can remove this portion of process variability. Only an adjustment in the process or instrument can attenuate this component.

The second index, called *deli*, quantifies the time delay influence in the control loop. It is defined as the ratio between time delay component variance and total signal variance.

$$deli = \frac{\sigma^2(e_t)}{TSV} \quad (4)$$

If the time delay causes a big impact in product variability, a control structure that compensate the time delay should be used. In this case, a cascade control, FeedForward techniques (Adam and Marchetti, 2004) or, at some extension, a Smith Predictor (Weidong et al., 1998) can be used to attenuate the time delay strong influence.

The third index, called *tuni*, quantifies the feedback control performance impact over the total variability. It is defined as the ratio between control performance component variance and total signal variance.

$$tuni = \frac{\sigma^2(g_t)}{TSV} \quad (5)$$

If this term causes a significant impact in product variability, the tuning parameters should be changed.

3 CALCULATION OF NOSI, DELI, AND TUNI

This section introduces the methodology to quantify the three proposed indices.

3.1 Quantifying the white-noise influence

The Wold's decomposition theorem (Chatfield, 1989) says that any linear stationary process can be expressed as a sum of two uncorrelated processes, one purely deterministic and other purely indeterministic.

$$y_t = y_{t,d} + y_{t,i} \qquad (6)$$

Where $y_{t,d}$ and $y_{t,i}$ are the deterministic and indeterministic portion of the signal, respectively.

The signal y_t is purely deterministic if their values can be forecast exactly, using past data. On the other hand, if the past data of the process is useless to predict future behavior, we can say that process behavior is purely indeterministic.

The values of $y_{t,d}$ can be quantified using a moving-average (MA) or an autoregressive (AR) model (Oppenheim et al., 1999). The model order can be determined using the methodology shown in Chatfield (1989).

We can quantify the nondeterministic ($y_{t,i}$) portion of the signal by the difference between the original signal and their predicted values $y_{t,p}$.

$$y_{t,i} = y_t - y_{t,d} \qquad (7)$$

The white-noise component (w_t) can be approximated by the indeterministic portion of y_t ($y_{t,i}$).

3.2 Quantifying the time delay and control performance processes

In the previous section, we show the deterministic portion can be approximated by an MA model. The MA model can be written as:

$$y_{t,p} = f_0 w_t + f_1 w_{t-1} + \cdots + f_{d-1} w_{t-(d-1)} + f_d w_{t-d} + f_{d+1} w_{t-(d+1)} + \cdots + f_n w_{t-n} \qquad (8)$$

where f are the model parameters, w are the values of nondeterministic portion of the signal, d is the time delay, and n is the order of MA model.

The deterministic process can be split in two contributions. The first part, the portion that is inaccessible to feedback control can be modeled by the first d terms of MA model. This portion of the signal variance is consequence of time delay (e_t).

$$e_t = f_0 w_t + f_1 w_{t-1} + \cdots + f_{d-1} w_{t-(d-1)} \qquad (9)$$

The second portion of deterministic signal (g_t) is accessible to the feedback control and can be described as:

$$g_t = f_d w_{t-d} + f_{d+1} w_{t-(d+1)} + \cdots + f_n w_{t-n} \qquad (10)$$

Based on variances of each signal component, given by equations 7, 9, and 10, the three proposed indices (*nosi*, *deli*, and *tuni*) can be calculated.

3.3 Computation of nosi, deli, and tuni step-by-step

This section summarizes the methodology to estimate the three proposed indices: *nosi*, *deli*, and *tuni*. The scheme shown in Figure 1 summarizes the procedure to quantify the three proposed indices. The steps to quantify each index are described below:

1. Decompose the signal y_t in the deterministic and indeterministic parts using a moving average model (MA). The MA model order must be chosen adequately (Chatfield, 1989). The model parameters can be estimated using least squares.
2. Determine the white-noise portion of the signal (w_t), given by the difference between the deterministic part ($y_{t,d}$) and the original signal (y_t)
3. Calculate the *nosi* index (eq. 3).
4. Split the MA model in two models: one (MA_1) with the first *d* terms of the original MA model, and the second with the remaining terms (MA_2). Remember that *d* is the process time delay.
5. Determine the time delay signal component (e_t) using model MA_1. The model input is the white-noise (w_t).
6. Calculate the *deli* index (eq. 4)
7. Determine the control performance signal component (g_t) using model MA_2.
8. Calculate the *tuni* index (eq. 5).

4 CASE STUDIES

Values of three proposed indices have been examined using several simulation models. One of the models used is a first order with a first order disturbance, as shown in the following scheme (Figure 2).

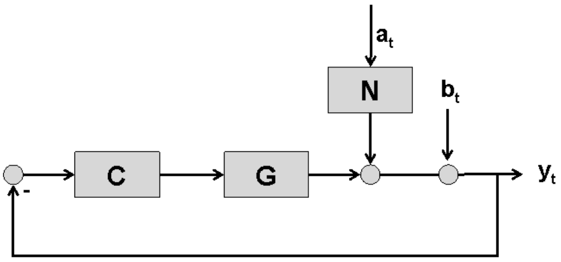

Fig. 2: Schematic representation of the system

Where a_t and b_t are signals with zero mean and amplitude A and B, respectively. C is the feedback PI type controller, G the plant, and N the disturbance. Table 1 shows the parameters used in this case study.

Table 1: Plant parameters of case study

Parameter	Value
C	Kp = 1.5, Ti = 55
G	$\frac{1}{50s+1}e^{-5s}$
N	$\frac{1}{30s+1}$
A	1
B	10^{-4}

The sample time used in the simulation is one time unit.

4.1 Case-study I – Signal decomposition

In the first scenario, the white-noise influence is determined. In this case, the load disturbance source is sinuswise. Applying the proposed methodology, the white-noise component is isolated. The comparison between the original white-noise variance and the estimated white-noise component is shown in Table 2.

Table 2: comparison between the original white-noise variance and the estimated white-noise

Original variance	Estimated Variance	Difference
0.0010	0.0011	0.0001

In the second test, the time-delay part is isolated. We have compared the predicted time delay variance with the difference between the system variances with original time delay and zero time delay. The variance comparison is shown in Table 3.

Table 3: comparison between the original and estimated time delay variance

Original variance	Estimated Variance	Difference
0.0007	0.0011	0.0004

Based on Tables 2 and 3, we can corroborate the proposed methodology is appropriate for isolate white-noise, time-delay, and control performance influence in control loop variance.

4.2 Case-study II – Computation of nosi, deli, and tuni indices

The current case study computes the three indices for scenarios with variable:

- time delay;
- white-noise;
- control performance.

The hypothetical plant used in this case study is shown in Figure 2.

In the first test, the influence of white-noise will be quantified. Table 4 shows the influence between noise amplitude (B) and each one of three indices. Figure 3 shows the relationship between each index and white-noise magnitude (B).

Table 4: influence between noise amplitude (B) and each one of three indices

B	*nosi*	*deli*	*tuni*
10^{-6}	0.08	0.33	0.59
10^{-5}	0.09	0.33	0.58
10^{-4}	0.17	0.31	0.52
10^{-3}	0.57	0.18	0.25
10^{-2}	0.94	0.03	0.03

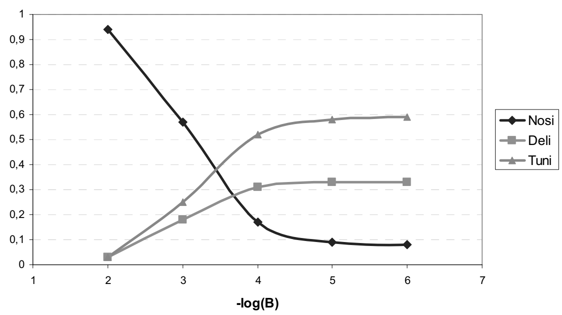

Fig. 3: White-noise influence over the three proposed index. See that horizontal scale is logarithmic.

Based on Table 4 and Figure 3, we can verify the white-noise amplitude increase was captured by the index that quantifies this influence: *nosi*. The two other indices decreased their importance as the influence of *nosi* increased.

In the second test, the time delay influence is quantified. Table 5 shows the relation between the time delay and the three proposed indices. Figure 4 shows the relationship between each index and time delay.

Table 5: influence between time delay (θ) and each one of three indices

θ	*nosi*	*deli*	*tuni*
5	0.17	0.31	0.52
10	0.15	0.55	0.30
20	0.11	0.70	0.19
30	0.08	0.72	0.20
50	0.05	0.78	0.17

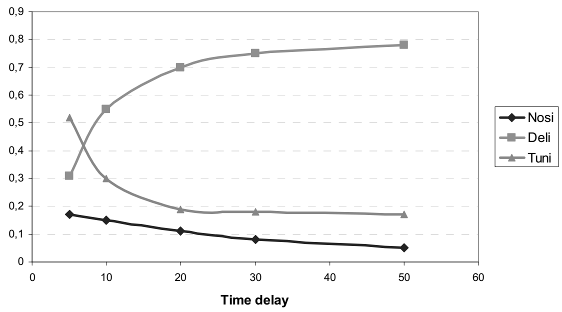

Fig. 4: Time delay influence over the three proposed index

Table 5 and Figure 4 show that *deli* index captured the time delay increase influence in the total output variance.

The influence of control performance is analyzed in Table 6 and Figure 5. They show the influence of controller gain (K_P) in each index.

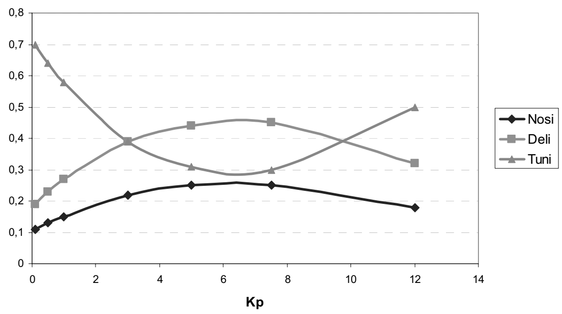

Fig. 5: Controller gain influence over the three proposed index

Table 6: influence between controller gain (K_P) and each one of three indices

K_P	*nosi*	*deli*	*tuni*
0.1	0.11	0.19	0.70
0.5	0.13	0.23	0.64
1	0.15	0.27	0.58
3	0.22	0.39	0.39
5	0.25	0.44	0.31
7.5	0.25	0.45	0.30
12	0.18	0.32	0.50

Table 6 and Figure 5 show that initially the increase in the control loop performance decreased the influence of tuning parameters in total variability. Only when the loop has very fast tuning ($K_p = 12$) the performance influence increases, because of closed-loop underdamped behavior.

5 CONCLUSIONS

In this article, a new set of indices – nosi, deli, and tuni - was proposed to quantify the influence of white-noise, time delay, and control loop performance in the total loop variance, respectively. These indices help the Harris index the diagnosis of each component in total control output variance.

Initially, the signal is decomposed into indeterministic and deterministic parts. Based on first portion, the white-noise index (noise) is quantified. The second portion is then decomposed into time delay and control performance components, then the respective indices are calculated (deli and tuni).

To calculate the given indices only routine operating data and plant time delay are required. Thus, their application in the industrial field is possible.

The proposed methodology was applied in a simulation case study. The proposed indices allowed quantifying the desired influence, under several scenarios, providing very good results.

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