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DESIGN OF A SLIDING MODE NEUROCONTROLLER FOR A NUCLEAR RESEARCH REACTOR

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Abstract: This paper presents the application of a special technique which combines neural networks and sliding modes for solving the robust tracking problem in a nuclear reactor when only the input and the output are available. Due to the appropriate sensor absence, the design is based on a differential neural network observer. The highly nonlinear structure provided by this neural network is linearized using sliding mode. Finally, this linear model is employed for determining a sliding mode control for tracking a reference model. The efficiency of this technique with a guaranteed bound for the averaged tracking error is illustrated by simulation.

Keywords: Nuclear Reactor, Differential Neural Network, Sliding Mode Controller.

1. INTRODUCTION

In the National Institute of Nuclear Research of Mexico (ININ), there is located the most important nuclear research reactor of this country. This reactor is TRIGA Mark III-type and is mainly used for the study of the radiation effects in several substances (activation analysis, aging analysis, etc.). Likewise, it is used to isotope production for medical, industrial and agricultural applications. Basically, the reactor consists of a core immersed in a pool where the water has the double function of moderator and coolant. In the core, it is located the U^{235} fuel combined with zirconium hydride. Inside the core, there exist four rods built with boron. This last material is capable of absorbing neutrons. So, depending on the insertion or extraction of these rods, it is possible to reduce or increase the reactor power.

In general, the control of any nuclear reactor can be classified in two categories: power regulation and trajectory tracking. For the ININ reactor, only the first option is available. Although, some schemes of tracking have been proposed for this reactor in (Pérez, 1994) and (Benítez-Read, 2005), they suppose a complete knowledge of all the parameters and the states of the system. However, in real situations, any design should consider that a nuclear reactor is an uncertain, inherently nonlinear and very complex system with time-varying parameters. Besides, most of the variables associated with a nuclear process are not measurable. Thereby, to solve the tracking problem effectively, it is necessary to resort to some robust techniques. In this paper, the attention will be focused on two of these techniques: neural networks (NN) and sliding modes (SM).

NN are an approach which has generated great enthusiasm as a consequence of their capability of functioning adequately despite a partially (or inclusive totally) incomplete information about plant model. NN could be classified as static (feedforward) or as differential (recurrent) (Haykin, 1994). In the first kind of networks, a system dynamics is approximated by a static mapping; therefore, the network outputs are uniquely determined by the current inputs and the weights. In contrast, differential neural networks (DNN) incorporate feedback in their structure. So, they overcome many problems associated with first ones such as global extrema search. Furthermore, DNN have better properties of approximation. In recent years, it has been proposed to use neural networks in nuclear

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reactors particularly for control (Garis, *et al*., 1998; Khajavi, *et al*., 2001; Boroushaki, *et al.*, 2003) but generally the networks employed have been static or else, in many cases, they have lacked a rigorous proof of stability.

On the other hand, during the last two decades, SM have emerged as other powerful tool for control, identification, and estimation in uncertain environments. Basically, SM consist of the application of a discontinuous control action for reaching and maintaining the dynamics of a system on a, so-called, sliding surface. The major advantages of SM are: low sensitivity to plant parameter variations and disturbance, fast transient behavior, and exponential convergence (Utkin, 1992). However, despite these advantages, only a few applications of this technique have been reported in nuclear literature. So, a SM controller, to control reactor pressure, reactor water level, and turbine power of a nuclear power plant, is proposed in (Huang, 2004). In (Shtessel, 1998), it is discussed a feedback controller based on SM observer for a space nuclear reactor.

In spite of fruitful research in DNN and SM, very few authors have considered the possibility of combining the advantages from these two techniques for obtaining a controller with better performance (Chairez, 2006) and none, to our knowledge, has considered apply this "sliding neuro controller" to the field of nuclear processes. Thereby, in this paper, it is suggested to solve the tracking problem for a nuclear research reactor using the following methodology: to overcome the uncertainty and the lack of appropriate sensors, a differential neural network observer with a sliding mode learning law and a switching correction term is used for estimating the reactor states. Since only the input and the output of the reactor are always available, a simplified, but imprecise, nonlinear third order mathematical model of the reactor is utilized for off-line initial training of DNN. When this process of training has finished, the observer can work without any mathematical model. Next, the structure highly nonlinear provided by this special neural network is linearized using SM. Finally, this simple linear model is utilized to find a SM control for the tracking of a reference model. The workability of suggested approach is illustrated by a simulation example.

2. MATHEMATICAL MODEL

In general, the nuclear reactor dynamics is described by the following, so-called, point kinetics equations with six delayed neutron precursor groups (Hetrick, 1993):

$$\dot{n}_t = \frac{\rho_t - \beta}{\Lambda} n_t + \sum_{i=1}^{6} \lambda_i C_{i,t} \tag{1}$$

$$\dot{C}_{i,t} = \frac{\beta_i}{\Lambda} n_t - \lambda_i C_{i,t}, \qquad i = 1, \dots, 6 \tag{2}$$

where n_t is the neutron power (W), $C_{i,t}$ is the power of the ith group delayed neutron precursor (W), ρ_t is the total reactivity, Λ is the effective prompt neutron lifetime (s), λ_i is the radioactive decay constant of ith group neutron precursor (s^{-1}), β_i is the fraction of ith group delayed neutrons, and β is the total delayed neutron fraction ($\beta=\sum_{i=1}^{6}\beta_i$). It is important to mention that the six group point kinetics equations (1) and (2) are in reality a set of seven ordinary differential equations; accordingly, their manipulation can result difficult. However, it is possible to reduce the system order by combining the six precursor groups into an equivalent single group. First, let us define the effective precursor radioactive decay constant λ as

$$\lambda = \frac{1}{\beta} \sum_{i=1}^{6} \beta_i \lambda_i \tag{3}$$

Next, using (3), the equations (1) and (2) can be simplified into a second order system given by

$$\dot{n}_t = \frac{\rho_t - \beta}{\Lambda} n_t + \lambda C_t \tag{4}$$

$$\dot{C}_t = \frac{\beta}{\Lambda} n_t - \lambda C_t \tag{5}$$

where C_t is the equivalent power of all delayed neutron precursors.

Now then, the total reactivity has two components, the external reactivity $\rho_{ext,t}$ and the internal reactivity $\rho_{int,t}$, that is,

$$\rho_t = \rho_{ext,t} + \rho_{\mathrm{int},t} \tag{6}$$

The external reactivity is related to the position of the control rods. Thus, the external reactivity is considered as the control input of the system. The relationship between the external reactivity and the rod position can be represented through an empirical static function. On the other hand, the internal reactivity is associated with the effects of the temperature feedback. These effects can be described (Hetrick, 1993) by

$$\dot{\rho}_{\mathrm{int},t} = -\alpha K n_t + \alpha K n_0 - \gamma \rho_{\mathrm{int},t} \tag{7}$$

where α is the negative temperature reactivity coefficient (°C^{-1}), K is the reciprocal of the reactor heat capacity (°C/($W{\cdot}s$)), γ is the reciprocal of mean time for heat transfer to the coolant (s^{-1}), and n_0 is the initial power when the external reactivity is equal to zero. For suitability, we consider in this work that $n_0=1W$. The equations (4), (5), and (7) constitute a very simplified third order mathematical model of a nuclear reactor. The nominal parameters corresponding to a TRIGA MARK III research reactor located at National Institute of Nuclear Research of México (Viais, 1994) are as follows: $\alpha=0.01359875\,°C^{-1}$, $\beta=6.433x10^{-3}$, $\lambda=0.4024s^{-1}$, $\Lambda=38\mu s$, $\gamma=0.2s^{-1}$, $K=1/5.21045x10^{4}\,°C/(W{\cdot}s)$ whereas the ranges for the variables of the same reactor operating on standard conditions are: n_t from $1W$ to $1.1MW$, C_t from $420.72W$ to $462.79MW$, $\rho_{int,t}$ from -1.4354 to 0, $\rho_{ext,t}$ from 0 to 1.4354. Defining the state coordinates and the control input as $x_{1,t} := n_t$, $x_{2,t} := C_t$, $x_{3,t} := \rho_{int,t}$, $u_t := \rho_{ext,t}$, the equations (4), (5), and (7) can be represented in the standard state variable form

$$\dot{x}_{1,t} = -\frac{\beta}{\Lambda}x_{1,t} + \lambda x_{2,t} + \frac{1}{\Lambda}x_{1,t}x_{3,t} + \frac{1}{\Lambda}x_{1,t}u_t$$
$$\dot{x}_{2,t} = \frac{\beta}{\Lambda}x_{1,t} - \lambda x_{2,t} \qquad (8)$$
$$\dot{x}_{3,t} = -\alpha K x_{1,t} + \alpha K n_0 - \gamma x_{3,t}$$

3. DIFFENTIAL NEURAL NETWORK

3.1 Uncertain dynamics and basic assumptions

The uncertain nonlinear system which will be controlled, in general, can be represented by

$$\dot{x}_t = f(x_t, u_t, t) + \xi_{1,t}, \quad y_t = Cx_t + \xi_{2,t} \qquad (9)$$

where $x_t \in \Re^n$ is the system state at time $t \geq 0$, $y_t \in \Re^p$ is the system output, $u_t \in \Re^m$ is the control action $(m \leq n)$, $C \in \Re^{p\times n}$ is an a-priory known output matrix $(p \leq n)$ and $f: \Re^n \times \Re^m \times \Re^+ \to \Re^n$. The vectors $\xi_{1,t}$ and $\xi_{2,t}$ characterize mixed uncertainties that may include both unmodelled dynamics and deterministic disturbances. Notice that an alternative representation for (9) always could be

$$\dot{x}_t = A^{(0)}x_t + W^{(0)}\sigma(x_t) + B^{(0)}u_t + B^{(0)\perp}v_1 + \tilde{f}_t \qquad (10)$$

where the parameters $A^{(0)} \in \Re^{n\times n}$, $W^{(0)} \in \Re^{n\times n}$, $B^{(0)} \in \Re^{n\times m}$ are subjected to adjustment, $B^{\perp} := I - (B^{\intercal})^{+}B^{\intercal}$, v_1 is a fixed function of time, the activation vector-function $\sigma(\cdot) := [\sigma_1(\cdot), \ldots, \sigma_n(\cdot)]^{\intercal}$ has sigmoidal components

$$\sigma_j(x) := a_{\sigma j}\left[1 + b_{\sigma j}\exp\left(-\sum_{j=1}^{n} c_{\sigma j}x_{j,t}\right)\right]^{-1} \qquad (11)$$

for $j = 1,\ldots,n$ and

$$\tilde{f}_t := f(x_t,u_t,t) - A^{(0)}x_t - W^{(0)}\sigma(x_t)$$
$$-B^{(0)}u_t - B^{(0)\perp}v_1 + \xi_{1,t}.$$

Hereafter it is supposed that the system (9), aside from being observable and controllable, complies with the following assumptions:

1) System (9) satisfies the (uniform on t) Lipschitz condition, that is,
$$\|f(x,u,t) - f(z,v,t)\| \leq L_1\|x-z\| + L_2\|u-v\|$$
$$x,z \in \Re^n;\ u,v \in \Re^m;\ 0 \leq L_1, L_2 < \infty$$
2) The mixed uncertainties $\xi_{1,t}$ and $\xi_{2,t}$ are bounded, i.e., $\|\xi_{j,t}\|^2_{\Lambda_{\xi_j}} \leq \Upsilon_j$, $\Lambda_{\xi_j} > 0$, j=1,2 (the matrices Λ_{ξ_j} normalize the components to make possible to work with values of different physical nature). Besides, $\xi_{1,t}$ and $\xi_{2,t}$ do not violate the existence of the solution to ODE (9).
3) Admissible controls satisfy $U^{adm} := \left\{u = u(\hat{x}) : \|u\|^2_{\Lambda_u} := u^{\intercal}\Lambda_u u \leq v_0 + v_1'\|\hat{x}\|^2_{\Lambda_{u,x}}\right\}$ where $\hat{x}$ is a state estimation, $v_0, v_1' > 0$, $0 < \Lambda_u \in \Re^{m\times m}, 0 < \Lambda_{u,x} \in \Re^{n\times n}$. Besides, u_t is such that does not violate the existence of the solution to ODE (9).
4) $A^{(0)}$ is Hurwitz, the pair $(A^{(0)}, C)$ is observable, and $(A^{(0)}, B)$ is controllable.
5) $\tilde{f}_t$, the, so-called, "*unmodelled dynamics*", is bounded, specifically, $\|\tilde{f}_t\|^2_{\Lambda_f} \leq \tilde{f}_0 + \tilde{f}_1\|x_t\|^2_{\Lambda_{\tilde{f}}}$, $\Lambda_f > 0$, $\Lambda_{\tilde{f}} > 0$.

It is worth mentioning that the preceding assumptions are generally met for physically meaningful dynamic systems and a nuclear reactor is not an exception.

3.2 Observer structure

DNN observer can be defined as follows:

$$\frac{d}{dt}\hat{x}_t = A^{(0)}\hat{x}_t + W_t\sigma(\hat{x}_t) + B^{(0)}u_t + B^{(0)\perp}v_1 + K_1[y_t - C\hat{x}_t] + K_2 SIGN(y_t - C\hat{x}_t),\ \hat{y}_t = C\hat{x}_t \qquad (12)$$

where $\hat{x}_t \in \Re^n$ is the estimated state, $W_t \in \Re^{n\times n}$ is the weight matrix and

$$sign(z) := \begin{cases} 1 \text{ if } z > 0 \\ -1 \text{ if } z < 0 \\ \in [-1,1] \text{ if } z = 0 \end{cases} \qquad (13)$$

It is possible to see that the structure of the observer (12) consists of three parts:

- the neural network identifier with a single output layer
$$A^{(0)}\hat{x}_t + W_t\sigma(\hat{x}_t) + B^{(0)}u_t + B^{(0)\perp}v_1$$
- the Luenberger tuning term $K_1[y_t - C_t\hat{x}_t]$;
- the sign correction term $K_2\mathrm{sign}(y_t - C\hat{x}_t)$ which is intended to reduce the output external noise effect associated with real data. In general, this term improve the global performance of the observer particularly when the output error $[y_t - C\hat{x}_t]$ is small and consequently the Luenberger term is not effective anymore.

3.3 Off-line training of the network.

Before using DNN observer (12) on-line, it is necessary to select adequate values for $A^{(0)}$, $B^{(0)}$, and $W^{(0)}$. This preliminary process of selection is known as off-line training of DNN. If the off-line knowledge of all states of (9) is not available then at least it is necessary to resort to a simplified and inclusive imprecise model of (9) as the generator of

N training data $(u_{t_k}, x_{t_k})\mid_{k=1,N}$. Basically, the training process consists of two stages: First, using try-to-test method, values for $A^{(0)}$ and $B^{(0)}$ are proposed such that assumption 4 is satisfied. Next, using a least square method, the best nominal value of $W^{(0)}$ is determined. Define this value as $W^{(0)*} =$

$$\arg\ \min \sum_{k=1}^{N} \left\| \tilde{f}_{t_k} \right\|^2 = [Y_{t_1} \cdots Y_{t_N}][\sigma(x_{t_1}) \cdots \sigma(x_{t_N})]^+$$

where $Y_{t_k} := \dot{x}_{t_k} - A^{(0)} x_{t_k} - B^{(0)} u_{t_k} - B^{(0)\perp} v_1$ (here $[\cdot]^+$ means pseudoinverse in Moore-Penrose sense). Starting from t_N we are ready to initiate the learning (or DNN adaptation) process.

3.4 Learning Law

The weight matrix W_t is adjusted by the following learning law (Chairez, 2006):

$$\begin{gathered}\dot{W}_t^{(i,j)} = -k\mu_t S_t^{(i,j)} \mathbf{sign}\left(\tilde{W}_t^{(i,j)}\right),\ i,j = \overline{1,n} \\ \mu_t := \left\| N_\delta P_1 \tilde{W}_t^\intercal \sigma(\hat{x}_t) \right\|_\Pi^2 + 2e_t^\intercal C N_\delta P_1 \tilde{W}_t \sigma(\hat{x}_t) \\ - \| P_2 W_t \sigma(\hat{x}_t) \|_{\Lambda_\sigma}^2 \\ S_t^{(i,j)} = 1/n,\ i,j = \overline{1,n}\ \text{(uniform learning)},\ k > 0 \\ \Pi := C^\intercal \Lambda_{\xi_2} C + \delta \Lambda_1,\ \tilde{W}_t := W_t - W^{(0)*}, \delta > 0 \\ e(t) := y(t) - C\hat{x}(t),\ N_\delta := (C^\intercal C + \delta I)^{-1}\end{gathered} \quad (14)$$

P_j, j=1,2 are the positive solution (if they exists) for the algebraic Riccati equation given by

$$P_j \tilde{A}_j^{(0)*} + \left(\tilde{A}_j^{(0)*}\right)^\intercal P_j + P_j R_j P_j + Q_j = 0 \quad (15)$$

where $\tilde{A}_1^{(0)*} := \left(A^{(0)} + K_1 C\right)$, $\tilde{A}_2^{(0)*} := A^{(0)}$, $R_1 := \bar{W}_{\Lambda_\sigma^{-1}} + \Lambda_{\tilde{f}}^{-1} + \Lambda_{\xi_1}^{-1} + K_1 \Lambda_{\xi_2}^{-1} K_1^\intercal$, $Q_1 := (l_\sigma \|\Lambda_\sigma\| + \delta \|\Lambda_1\|) I_{n\times n} + 2\tilde{f}_1 \Lambda_{\tilde{f}} + \Lambda_D^{-1} + Q_0$, $Q_0 > 0$, $R_2 := K_1 C \Lambda_D C^\intercal K_1^\intercal + B^{(0)} \Lambda_u^{-1} B^{(0)\intercal}$, $Q_2 := \Lambda_\sigma^{-1} + v_1 \Lambda_{u,x} + \left(\tilde{f}_2 + 2\tilde{f}_1\right) \Lambda_{\tilde{f}}$, $\bar{W}_{\Lambda_\sigma^{-1}} := \left(W^{(0)*}\right)^\intercal \Lambda_\sigma^{-1} W^{(0)*}$.

3.5 Main result on the estimation process

One of the principles advantages of (12) and of the corresponding learning law (14) is that we can guarantee that the averaged estimation error is upper bounded.

Theorem 1. If there exist positive definite matrices $\Lambda_{\tilde{f}}, \Lambda_{\xi_1}, \Lambda_{\xi_2}, \Lambda_\sigma, \Lambda_D, \Lambda_u, \Lambda_1, Q_0$ and positive constants δ, k, v_1 such that two matrix Riccati equations (15) have positive definite solutions, then the DNN observer (12) with any matrix K_1 guarantying that the close-loop matrix $\tilde{A}^{(0)*}$ is stable, that is,

$$\tilde{A}^{(0)*} := \left(A^{(0)*} + K_1 C\right) \text{ is Hurwitz} \quad (16)$$

and

$$K_2 = \lambda P_1^{-1} C^\intercal, \lambda > 0 \quad (17)$$

supplied by the learning law (14), provides the following upper bound for the state estimation process:

$$\overline{\lim_{T\to\infty}} \frac{1}{T} \int_{t=0}^{T} \|\Delta_t\|_P^2\, dt \le \rho_Q / \alpha_Q \quad (18)$$

where

$$\begin{aligned}\rho_Q &:= \tilde{f}_0 + v_0 + \Upsilon_1 + 3\Upsilon_2 + 8\lambda\sqrt{n\Upsilon_2} \\ \alpha_Q &:= \lambda_{\min}\left(P_1^{-1/2} Q_0 P_1^{-1/2}\right) > 0\end{aligned} \quad (19)$$

The proof of theorem 1 is achieved by means of Lyapunov-like analysis in (Chairez, 2006).

4. SLIDIG MODE NEUROCONTROLLER

Consider the reference model or desired dynamics given by

$$\dot{x}_t^* = \phi(t, x_t^*),\ x_0^* \text{ is fixed} \quad (20)$$

Basically, the tracking problem consists of selecting a proper control signal such that (9) follows (20) as closely as possible.

4.1 Sliding mode DNN linearization

DNN observer (12) represents one of possible models of the given uncertain system (9). However, the structure of (9) is highly nonlinear what may cause many problems for a control design. Fortunately, it can be demonstrated that this highly nonlinear model may be exactly approached (in finite time) by a simple linear model with a measurable input of a high frequency (generated by a relay type element). To do that, first, represent the DNN observer (9) as

$$\begin{gathered}\frac{d}{dt}\hat{x}_t = A\hat{x}_t + h(\hat{x}_t) + Bu_t + B^\perp v_t^1 \\ h(\hat{x}_t) := W_t \sigma(\hat{x}_t) + K_1(y_t - \hat{y}_t) + K_2 sign(y_t - \hat{y}_t)\end{gathered} \quad (21)$$

And, next, introduce the following *auxiliary* model:

$$\frac{d}{dt}\tilde{x}_t = A\tilde{x}_t + Bu_t + B^\perp v_t^1 + v_t^2 \quad (22)$$

where $\tilde{x}_t$ is the state of this auxiliary model, the matrices A and B are the same as in (21) which are actually known, v^1 and v^2 are auxiliary controls to be selected to provide the closeness of $\tilde{x}_t$ to $\hat{x}_t$.

Theorem 2. If v_t^2 in (22) is selected as

$$\begin{gathered}v_t^2 := \begin{cases} k_t \dfrac{\delta_t}{\|\delta_t\|} & if \quad \delta_t \neq 0 \\ 0 & if \quad \delta_t = 0 \end{cases},\quad \delta_t := \hat{x}_t - \tilde{x}_t \\ k_t := \|A\delta_t + h(\hat{x}_t)\| + 2\rho_c,\quad \rho_c > 0\end{gathered} \quad (23)$$

then for any $t \ge t^* = \dfrac{\|\delta_0\|}{\rho_c}$ we may guarantee the exact matching $\hat{x}_t = \tilde{x}_t$.

Remark 1. To minimize the "chattering effect", it is recommended to use the low-pass filter (LPF) which maintains $\|\delta_t\| \le \varepsilon$ for small enough $\varepsilon > 0$, that is, to use instead of v_t^2 the "averaged" control $v_t^{2(av)}$ generated by the LPF

$$\mu \dot{v}_t^{2(av)} + v_t^{2(av)} = v_t^2,\ v_0^{2(av)} = 0$$

4.2 *The orthogonal compensation control*

Now, the tracking process is analyzed. First, define the tracking error as $d_t := \tilde{x}_t - x_t^*$ where x_t^* is the desired tracking process given by (20). In view of (22), the tracking error dynamics is governed by the following ODE:

$$\dot{d}_t = A d_t + B u_t + B^{\perp} v_t^1 + \zeta_t \qquad (24)$$

where $\zeta_t := A x_t^* - \phi(t, x_t^*) + v_t^2$. Any vector ζ_t always can be represented as its projection to another vector plus its orthogonal projection. Taking into account that $B u_t \perp B^{\perp} v_t^1$ for any u_t and v_t^1, it is possible to define $\zeta_t := B\gamma_{1,t} + B^{\perp}\gamma_{2,t}$ (rank $B^{\perp} = n - m$) where $\gamma_{1,t} = B^{+}\zeta_t, \gamma_{2,t} = \left(B^{\perp}\right)^{+}\zeta_t$ Using this one, the dynamics given by (24) may be presented as

$$\dot{d}_t = A d_t + B\left(u_t + \gamma_{1,t}\right) + B^{\perp}\left(v_t^1 + \gamma_{2,t}\right) \qquad (25)$$

Let us take $u_t := \hat{u}_t - \gamma_{1,t},\ v_t^1 := \hat{v}_t^1 - \gamma_{2,t}$ that transforms (25) into

$$\dot{d}_t = A d_t + B\hat{u}_t + B^{\perp}\hat{v}_t^1 = A d_t + \left[B \vdots B^{\perp}\right]\tilde{u}_t \qquad (26)$$

where $\tilde{u}_t^{\intercal} := \left[\hat{u}_t \vdots \hat{v}_t^1\right] \in R^{n+m}$, $\operatorname{rank}\left[B \vdots B^{\perp}\right] = n$.

Theorem 3. If $\tilde{u}_t$ is selected as

$$\tilde{u}_t := -\left[B \vdots B^{\perp}\right]^{+} A d_t + \tilde{u}_t',\ \tilde{k} > 0$$

$$\tilde{u}_t' := -\tilde{k}\left[B \vdots B^{\perp}\right]^{+} \frac{d_t}{\|d_t\|} \qquad (27)$$

$$\left[B \vdots B^{\perp}\right]^{+} := \left[B \vdots B^{\perp}\right]^{\intercal}\left(\left[B \vdots B^{\perp}\right]\left[B \vdots B^{\perp}\right]^{\intercal}\right)^{-1}$$

then for any $t \ge t_f = \|d_0\| / \left(\sqrt{2}\tilde{k}\right)$ it is possible to guarantee the exact tracking, that is, $d_t = 0$. If the pair (A, B) is stabilizable and $\tilde{u}_t$ is selected as

$$\tilde{u}_t^{\intercal} := \left[\hat{u}_t \vdots \hat{v}_t^1\right] = \left[K d_t \vdots 0\right] \qquad (28)$$

where the matrix K guarantees that $A+BK$ would be stable, then $d_t = e^{(A+BK)t} d_0 \to 0$ when $t \to \infty$.

Remark 2: Here again instead of $\tilde{u}_t'$ it is recommended to use its filtered version obtained by passing this signal through the LPF.

4.3 *Main result on quality of a sliding mode neurocontrol*

Theorem 4. For the class of uncertain nonlinear systems (9) under the assumptions 1-5 the sliding mode neurocontrol (27), which uses the auxiliary signals $\hat{x}_t$ and $\tilde{x}_t$ generated by DNN observer (12) and the ODE (22) respectively, provides the following quality (on "average") of the tracking process with the desired dynamics x_t^* given by (20):

$$\overline{\lim_{T \to \infty}} \frac{1}{T} \int_{t=0}^{T} \|x_t - x_t^*\|_{Q_0}^2\, dt \le 3\left(\frac{\rho_Q}{\alpha_Q} + \varepsilon_1 + \varepsilon_2\right) \qquad (29)$$

where the constants ρ_Q and α_Q are defined in (19) and $\varepsilon_i\ (i = 1,2)$ describes the quality of the low pass filtering (LPF) applied to the sliding-mode controls v_t^2 (23) and $\tilde{u}_t$ (27), that is, if instead of v_t^2 and $\tilde{u}_t$ there are used $v_t^{2(av)}$ and $\tilde{u}_t^{(av)}$ satisfying $\left\|v_t^2 - v_t^{2(av)}\right\| \le \varepsilon_1,\ \left\|\tilde{u}_t - \tilde{u}_t^{(av)}\right\| \le \varepsilon_2$.

5. NUMERICAL EXAMPLE

In this section, the robust tracking process for a nuclear reactor via the sliding mode controller (27) is illustrated. Since it is considered that only the input and the power $x_{1,t}$ are available for measurement, the output of system (8) can be defined as $y_t := x_{1,t}$. Or else, in terms of the state vector, $y_t = \mathbf{C}\mathbf{x}_t$ where $\mathbf{C}$:= [1 0 0] and $\mathbf{x}_t \in \Re^3$. Besides, to overcome the absence of adequate sensors that prevent the off-line knowledge of all states, *the model* (8) *with nominal parameters given in the section 2 and with nominal initial condition* $x_0 = [n_0, \frac{\beta}{\lambda\Lambda} n_0, 0]^T$ *where* n_0 = $1W$ *is used as data generator*. The preliminary training produces the following results:

$$A^{(0)} = \begin{bmatrix} -1.7 & -1.2 & -1 \\ 1 & 0 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}, B^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$W^{(0)*} = \begin{bmatrix} -0.0237 & -0.0258 & 0.2176 \\ 1.078 & 1.502 & 0.1815 \\ -0.0613 & -0.2024 & -0.0154 \end{bmatrix}$$

Now, the observer can work without any mathematical model. In general, to show the robustness properties of the technique studied, the sliding mode neurocontroller (27) is proved when the plant model (8) is simulated on the following new conditions: First, the initial condition is changed such that now $x_0' = [n_0, 10\frac{\beta}{\lambda\Lambda} n_0, 1\times10^{-6}]^T$ where $n_0 = 1W$. Second, the parameter values of (8) are changed to $\alpha = 0.0097°\text{C}^{-1}$, $\beta = 0.0072$, $\lambda = 0.3942 s^{-1}$, and $\Lambda = 30\mu s$ (both γ and K stay equal). Of course, these changes are supposed to be unknown for the controller. Besides, the control structure provided in (Benítez-Read, 2005) is selected as the reference model. By "try-to-test" method, the parameters of DNN observer (12) are selected as follows: K_1 = $[-1.3,\ 17,\ -0.2]^T$, $K_2 = [0.87,\ \text{-}0.24,\ 0.71]^T$,

$k = 2.3$, $P_1 = 7\begin{bmatrix} 5 & 2 & -3 \\ 2 & 10 & 1 \\ -3 & 1 & 4 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 10 & 1 & -4 \\ 1 & 11 & 2 \\ -4 & 2 & 3 \end{bmatrix}$

It is important to mention that due to wide range of state values associated with (8) it is necessary to resort to the normalization of variables in both the preliminary training and on-line estimation process. Such normalization does not affect the results. Instead, it permits to the controller to work satisfactorily. To quantify the global performance of the sliding mode controller, it is defined the following performance index for the tracking process:

$$J_t = \frac{1}{t+\varepsilon}\int_{s=0}^{t} \left\| x_s - x_s^* \right\|_{Q_0}^2 ds, \;\; Q_0 > 0, \;\; \varepsilon = 0.01$$

The simulation results are exhibited in Fig. 1 and Fig. 2 for the tracking of a reference power and the performance index, respectively. The reference power changes from 1W up to 1 MW.

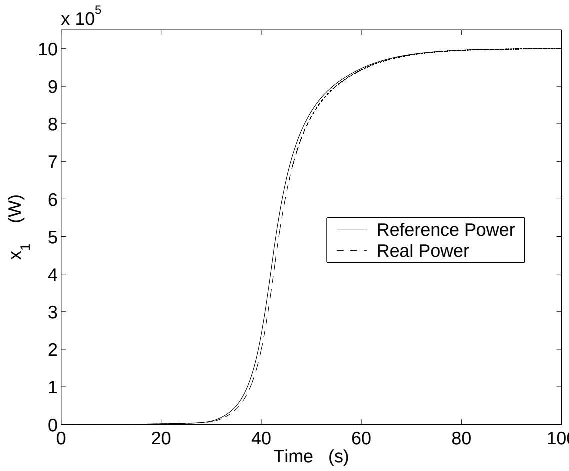

Fig. 1. Control process of the power $x_{1,t}$ for the TRIGA MARK III-Type nuclear reactor.

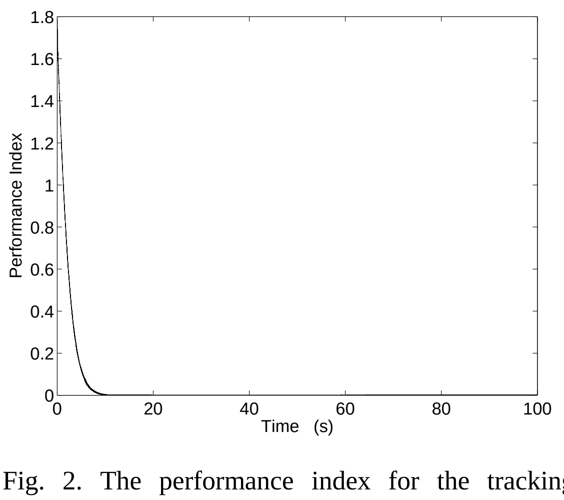

Fig. 2. The performance index for the tracking process.

6. CONCLUSIONS

In this paper, it has been shown the effective use of a sliding mode neurocontroller for the robust tracking in a nuclear research reactor. The controller was applied to the third order nonlinear model of a TRIGA MARK III-Type reactor. The simulation results show a satisfactory performance of the system. So, it is possible conclude that this special technique represents a very promising methodology in nuclear process control.

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