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USING NMPC BASED ON A LOW-ORDER MODEL FOR CONTROLLING PRESSURE DURING OIL WELL DRILLING

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Abstract: During petroleum well drilling operations, the pressure gradient in the well has to be maintained within the pressure restrictions of the formation. The well pressure can be controlled by restricting the drilling fluid flow through a choke valve at the top of the well. This paper proposes a closed-loop control algorithm using a finite horizon nonlinear model predictive control scheme based on a low order well model for this problem. The algorithm is tested on a rigorous high order model. Copyright ©2007 IFAC

Keywords: Drilling; Nonlinear model predictive control; State- and parameter estimation

1. INTRODUCTION

During petroleum well drilling, a drilling fluid is used to transport the cuttings from the drilling process and also to obtain the required downhole pressure due to pressure constraints of the formation. The downhole pressure is controlled by adjusting a choke valve restricting the flow through the well. In a recent study [Nygaard and Nævdal, 2006] a nonlinear model predictive control (NMPC) scheme for controlling the downhole pressure were presented. The NMPC scheme used a detailed, rigorous fluid flow model when performing the predictions. In addition, the scheme also revealed some stability challenges. The current work represents an extension to the results in Nygaard and Nævdal [2006].

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A nonlinear low order model developed for linear controller tuning, is further developed to better describe the liquid fluid flow behavior. This allows the low-order model to be used for prediction in the NMPC scheme instead of the more detailed, rigorous model. This has expected benefits in terms of improved real time capabilities, state observability and robustness.

The present NMPC scheme also uses elements from established NMPC system theory [Mayne et al., 2000, Findeisen et al., 2003, e.g.] for performance and robustness (including stability), and to be able to use shorter control horizons to reduce computational complexity. Note, however, that no proof of closed loop stability is given, however closed-loop simulations are indicating good stability properties.

The following section gives an description of the drilling process. In Section 3, the revised low order model is presented, and in Section 4 the observer is described. In Section 5 the improved NMPC scheme is shown. Results from simulations are given in Section 6, and concluding remarks are found in Section 7.

2. PROCESS DESCRIPTION

When drilling into a formation, the pressure in the well is critical for the success of the drilling process. The pressure in the well p_{well} must be within the operating pressure range of the formation. The upper bound of the pressure range is the formation fracturing pressure p_{frac}, the lower bound is the formation collapse pressure p_{coll}. In managed pressure drilling applications the focus is on controlling the well pressure just above the reservoir pore pressure p_{res} during the whole drilling operation, i.e.

$$p_{coll}(t,x) < p_{res}(t,x) < p_{well}(t,x) < p_{frac}(t,x)$$

where x is the position along the well trajectory and t is the time. The reservoir pore pressure p_{res} is a function of both time and position along the well trajectory. During drilling, a drilling fluid is circulated through the drillstring and drill bit. The drill bit is equipped with a check valve, which prevents the drilling fluid in the annulus to return into the drillstring. The drilling fluid flows through the annulus between the drillstring and the walls of the well. Figure 1 shows a layout of a system setup for drilling into a petroleum reservoir.

The well pressure can be manipulated by the operator by adjusting the drilling fluid flow and density. In addition, the operator can adjust a choke valve, restricting the fluid flow. A further introduction to drilling operations and pressure control can be found in Nygaard [2006]. Drilling operations consists of several different procedures, and the focus in this paper has been on controlling the pressure during a pipe connection procedure, where the fluid flow rate is stopped for about 10-15 minutes.

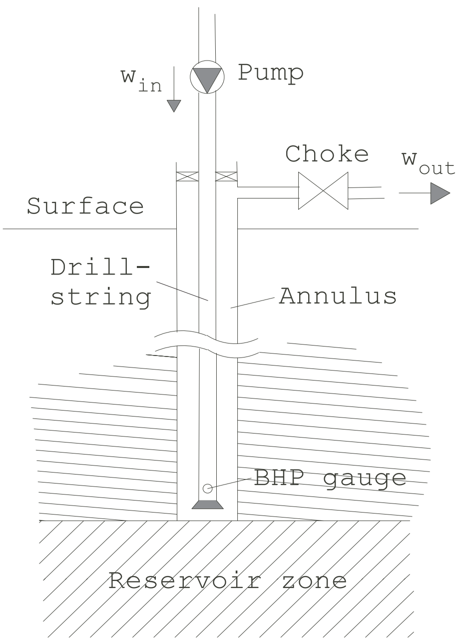

Fig. 1. Drilling of a well into a reservoir.

3. LOW-ORDER WELL MODEL

In Nygaard and Nævdal [2006] a low-order well model for two-phase flow was developed to be used for tuning purposes of a pressure control system using a choke valve as control input. The low-order model was inspired by a low-order two-phase model used for fluid flow stabilization described in Storkaas et al. [2003]. In this paper the model has been revised to describe liquid fluid flow behavior only.

Figure 2 shows how the fluid flow of the well system is divided into three compartments, the drill string and the annulus between the wall of the well and the drill string in addition to the wellhead on the top of the well.

When setting up the low-order model, an explicit calculation scheme is defined by

$$\dot{x} = f(x,u,v) \tag{1}$$

$$y = h(x,u) \tag{2}$$

where x is the state, u contains the manipulated variables, v is the disturbances and y is the measured variables. The state vector is

$$x = (v_d, v_a, v_c)^T \tag{3}$$

where v denotes fluid velocity and L denotes length. Furthermore, subscript d denotes drillstring, a denotes annulus and c denotes choke line. The measured variables

$$y = (p_p, p_{a,bha}, p_{cp})^T \tag{4}$$

where p denoted pressure. Furthermore, subscript p denotes pump, bha denotes bottomhole assem-

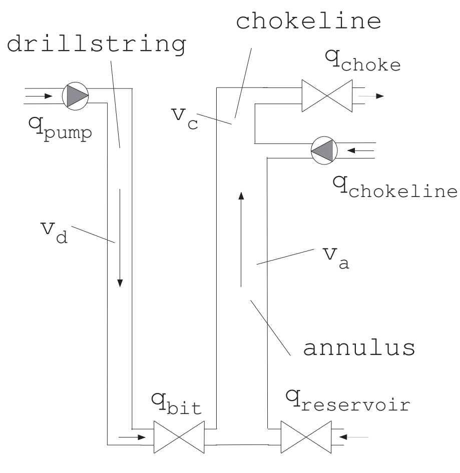

Fig. 2. The fluid flow is divided into three compartments, the drill string, the annulus and the wellhead.

bly and cp denotes choke pump. The manipulated variable is the area of the choke line, given by

$$u = A_c \tag{5}$$

where A denotes area. Pump flow rates and outflux from the well into reservoir is treated as disturbances to the system, giving the disturbance vector

$$v = (q_p, q_r, q_{cp})^T \tag{6}$$

where q denotes volume flow rate, and subscript r denotes reservoir.

When modeling the fluid flow in a pipe segment as a first order system, the system can be modeled as

$$\dot{x} = -\frac{1}{T}x + \frac{K_u}{T}u + \frac{K_v}{T}v \tag{7}$$

where x is the exit velocity of the pipe, u is the controlled input volume rate of the pipe and v is the disturbance such as volume changes or additional fluid inflow, T is the first order system time constant, and K_u is the gain of the control input u and K_v is the gain of the disturbance v.

Using the first order pipe model as a basis for the model design of the well, the model can be presented as:

$$\dot{x}_1 = -\frac{1}{T_d}x_1 + \frac{1}{T_d A_d}v_1 \tag{8a}$$

$$\dot{x}_2 = \frac{A_d}{T_a A_a}x_1 - \frac{1}{T_a}x_2 + \frac{1}{T_a A_a}v_2 \tag{8b}$$

$$\dot{x}_3 = \frac{A_a}{T_c u}x_2 - \frac{1}{T_c}x_3 + \frac{1}{T_c u}v_3 \tag{8c}$$

where subscript od denotes outer drillstring area.

The pressures in the well are influenced by the frictional pressure losses in the drillstring, the annulus and the choke line. The friction pressure in the drillstring p_{fd}, the annulus p_{fa} and the choke line p_{fc} can be expressed using

$$p_{fd} = \frac{\rho f_d L_d |x_1| x_1}{2D_d}$$

$$p_{fa} = \frac{\rho f_a L_a |x_2| x_2}{2D_a}$$

$$p_{fc} = \frac{\rho f_c L_c |x_3| x_3}{4\sqrt{u/\pi}}$$

where ρ is the density, $f_\star$ is the constant friction factor.

The measurement vector for the pump pressure y_1, the pressure at the bottomhole assembly y_2, and the pressure prior to the choke y_3, are given by

$$y_1 = p_{fd} + p_{fa} + p_{fc} + p_0 \tag{9a}$$

$$y_2 = kp_{fc} + \rho g h_d + p_{fa} + p_{fc} + p_0 \tag{9b}$$

$$y_3 = p_{fc} + p_0 \tag{9c}$$

where $h_\star$ is the vertical depth and k is a compressibility factor for the drilling fluid. The correction term kp_{fc} is added to the bottom hole pressure measurement, as the compression of the fluid in the annulus is proportional to the pressure across the choke line.

This model layout is able to capture the most important dynamics that are present during a pipe connection operation. However, an observer system must be designed to allow the model to be used in a model predictive control scheme.

4. OBSERVER BASED ON THE UNSCENTED KALMAN FILTER

The unscented Kalman filter (UKF) is a derivative-free Kalman filter for nonlinear estimation [Julier and Uhlmann, 2004]. The UKF does not use a linearization of the model to calculate the estimation error covariance matrix, but instead it tries to approximate this matrix by introducing sample points.

The nonlinear model is applied to a set of state vectors points, called sigma points. These sigma points are used to calculate the probability distribution of the estimation error. Our implementation of the unscented Kalman filter follows the presentation in Wan and van der Merve [2001], where the unscented Kalman filter is used for parameter estimation. A detailed description of UKF for flow parameter estimation in drilling applications can be found in Gravdal et al. [2005].

Initially, the augmented state vector is assumed to be $\chi_0^a = [\mathbf{x}_0^a, \theta_0^a]$, with an initial estimation error covariance matrix $\mathbf{P}_0^a$. The parameter vector is defined by

$$\theta = (f_d, f_a, f_c, k)^T \tag{10}$$

The remaining unknown parameters, T_d,T_a,T_c, are found by manual comparison with the detailed dynamic well model, and is constant during the whole simulation. The low order model is compared with the detailed dynamic model, and a fairly good match is found. The model parameters, θ, are tuned using the observer system to obtain a good match with the detailed model.

5. CONTROL ALGORITHM

5.1 Nonlinear Model Predictive Control

By now, there seem to be a general consensus that ingredients in a finite horizon NMPC scheme with closed loop stability are a terminal cost bounding the infinite horizon cost ("cost-to-go") and, in many cases, a terminal state constraint [Mayne et al., 2000, Findeisen et al., 2003, e.g.]. The terminal cost and the terminal state constraint are typically found using Lyapunov methods (based on linearization and/or nonlinear system theory) in a given desired equilibrium (corresponding to the *setpoint*(s)).

In the present case, we use these guidelines, but we loosen on some of the requirements: For each given desired equilibrium (given by desired setpoint and measured (and unmeasured) disturbances), we use linearization and LQR methods to calculate a terminal cost that approximate the cost-to-go from the end of the control horizon. To make the optimization problem easier to solve, and to avoid hard and costly computations for each equilibrium, we choose not to use a terminal constraint, but choose instead a longer horizon. Therefore, we are not guaranteed stability, but we have the most important stabilizing ingredients. If we at the end of the horizon are close to the desired equilibrium, the LQR cost based of the linearized system will be a good approximation to the infinite horizon NMPC cost, and this setup will therefore also provide good closed-loop performance.

The NMPC scheme we consider will therefore look like this: Solve, at each sampling instant t_i, the following optimization problem:

$$\min_{\bar{u}(\cdot)} J(\bar{u}(\cdot); x(t_i)) \tag{11a}$$

$$\text{s.t.:} \quad \dot{\bar{x}}(\tau) = f(\bar{x}(\tau), \bar{u}(\tau)), \quad \bar{x}(0) = x(t_i) \tag{11b}$$

$$\bar{u}(\tau) \in \mathbb{U}, \quad \bar{x}(\tau) \in \mathbb{X} \quad \tau \in [0, T_p] \tag{11c}$$

The cost functional J is defined over the control horizon T_p by

$$J(\bar{u}(\cdot); x(t_i)) := \int_0^{T_p} F(\bar{x}(\tau), \bar{u}(\tau)) d\tau + E(\bar{x}(T_p)).$$

The bar denotes internal controller variables, $\bar{x}(\cdot)$ is the solution of the dynamic model (11b) driven by the input $\bar{u}(\cdot) : [0, T_p] \to \mathbb{U}$ with the initial condition $x(t_i)$. The state and control constraints $\mathbb{X}$ and $\mathbb{U}$ are specified in (11c).

The solution to the optimal control problem is written $\bar{u}^\star(\cdot; x(t_i))$. This input is open-loop applied to the system until the next sampling instant t_i,

$$u(t; x(t_i)) = \bar{u}^\star(t - t_i; x(t_i)), \quad t \in [t_i, t_i + \delta) \,. \tag{12}$$

The control $u(t; x(t_i))$ is a feedback, since it is recalculated at each sampling instant using new state measurements.

5.2 Model changes for optimization

We make some changes to the model (8) developed in Section 3 to make it more suitable as optimization model.

First, we note that the input enters as $1/u$ (only) in the model. Therefore, to increase linearity and optimization convexity in the NMPC problem, we use $\tilde{u} = 1/u$ as the optimized input. Note that this does not completely linearize the dynamics, and also the measurements and (as we will see) some constraints are still nonlinear.

Moreover, we write the optimization model on velocity form, that is, we use the derivative of $\tilde{u}$ as optimization input. This has several advantages:

- We avoid a 'direct feedthrough' from input to pressure measurement.
- It allows direct implementation of rate constraints on u. However, since $\frac{d}{dt}\tilde{u} = -1/u^2 \frac{d}{dt} u$, these constraints must be implemented as nonlinear state constraints, which might be a drawback.
- It allows using longer sample intervals between NMPC optimizations, since the input to the process becomes "first-order hold" rather that "zero-order hold".

Disadvantages are that the state dimension is increased (u is a new state, the fourth), and that input constraints become state constraints.

The velocity-form formulation adds an integrator in the control loop, but as we do not control pressure directly (only indirectly through the computation of the desired steady state), we do not obtain integral control this way. Integral control is obtained through parameter estimation in the state estimation.

6. CLOSED LOOP SIMULATIONS

The simulated test case is based on a partly horizontal well that is 2000 m deep and 3600 m long. No drilling is performed during the operations, but the pressure is kept just above the formation pressure, using a downhole pressure reference value of 268 bar. Simulation constants are given in Table 1. The initial observer parameters used in the simulations are $\theta = [0.038, 0.11, 0.08, 1.1]^T$, and the initial estimation error are $\mathbf{P}_0^a = 0$ with a model error standard deviation of $[1 \cdot 10^{-5}, 1 \cdot 10^{-4}, 1 \cdot 10^{-4}, 2 \cdot 10^{-1}]^T$.

The NMPC is implemented using a quadratic stage cost $F(x, u) = x^T Q x + u^T R u$, with $Q = \text{diag}\{0, 0, 10, 0.1\}$ and $R = 10$. We used 5 degrees of freedom over a control horizon of 200 s. Input blocking was used, with more frequent control updates on the first part of the horizon. NMPC sample time was 5 s. The NMPC optimization problem was solved using a sequential method by applying a general-purpose SQP solver, and fixed-step discretizing the dynamic optimization model using a step size of 0.5 s. The time used for optimization are in the same order of magnitude as the NMPC sample time, but there is room for considerable improvements using more tailored optimization algorithms, optimizing choice

Table 1. Well and reservoir data

Parameter	Value
Well vertical depth, h_a	2000 m
Drillstring length, L_d	3600 m
Drillstring inner diameter, D_d	0.0925 m
Drillstring area, A_d	0.0067 m^2
Annulus length, L_a	3600 m
Annulus hydraulic diameter, D_a	0.211 m
Annulus area, A_a	0.0278 m^2
Chokeline length, L_c	3 m
Standard main pump flow rate, q_p	1000 l/min
Standard choke pump flow rate, q_{cp}	200 l/min
Drilling fluid density, ρ	1250 kg/m^3
Downhole reference pressure, p_{ref}	268 bar
Time constant, drillstring, T_d	3.5 s
Time constant, annulus, T_a	10 s
Time constant, choke line, T_c	3 s

of integration method and step size, and compiled implementations.

The scenario is as follows: Initially, no fluid is flowing in the well. After 10 s the main pump is started at 1000 l/min, and the choke pump is started at 200 l/min. At 2 min., the choke valve controller is started. At 6 min., the pipe connection is initiated, and the main pumps are stopped, and the choke line pump flow rate is increased to 400 l/min. When the pipe connection procedure is finished at 16 min., the standard pump flow rates are selected, respectively 1000 l/min and 200 l/min.

Two different simulations are performed. The first simulation is shown in Fig. 3, where the low-order model is used both for generating the measurements and for the predictions. A constraint of ±5 bar (compared to the reference pressure) is included in the NMPC optimization. As can be seen from the simulations, these constraints are slightly broken. The main reason for this is few 'coincidence points' when checking state constraints.

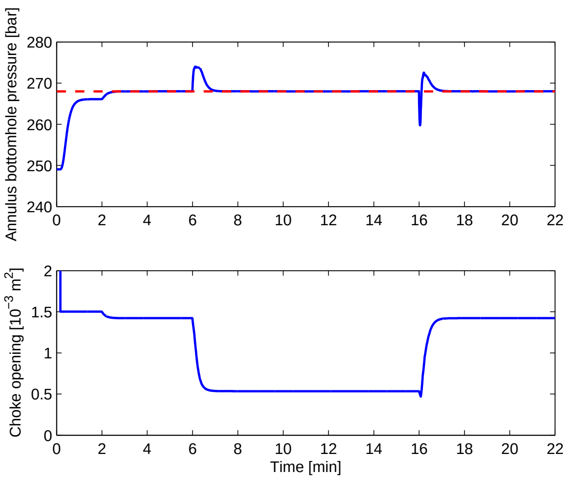

Fig. 3. NMPC simulated with low-order model and perfect state measurements: At the top is the controlled pressure (and the reference), and at the bottom is the choke opening.

The second simulation shown in Fig. 4 is performed using measurements generated by the detailed model. The low order model used in the NMPC is updated using the UKF observer to match the model with the generated measurements. In this case, the pressure constraints were removed from the NMPC optimization problem due to some problems during estimator transients. The estimated parameters are shown in Fig. 5. As can be seen, the model parameters varies some, especially the friction parameter for the choke line, during and after the pipe connection. Due to this, some oscillations in the down hole pressure can be observed during and after the pipe connection. Further examination of the observer system could be a basis for further research.

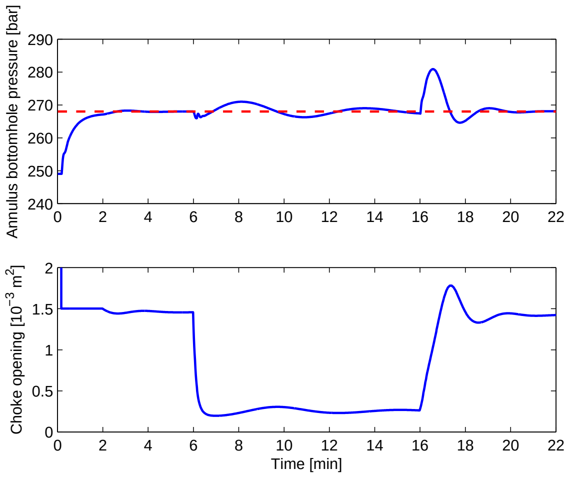

Fig. 4. NMPC simulated with high-order model and state estimation: At the top is the controlled pressure (and the reference), and at the bottom is the choke opening.

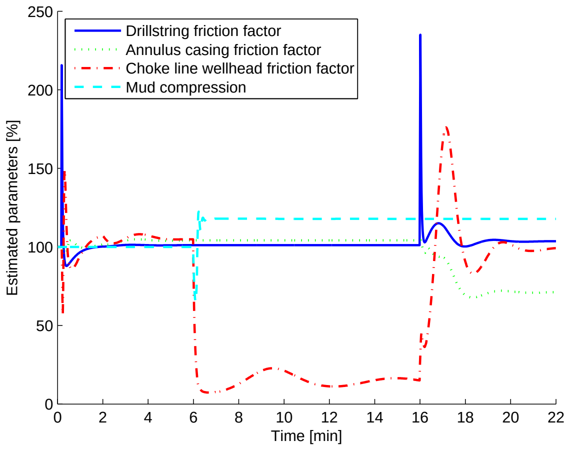

Fig. 5. Model parameters estimated by the UKF.

7. CONCLUDING REMARKS

In this paper, we tested an NMPC solution for controlling pressure during oil well drilling. The

NMPC optimization problem was formulated using a low-order model, and simulated on a rigid, high-order simulator model.

The NMPC scheme is able to successfully control the downhole pressure. The use of an NMPC formulation based on system theory gives us an NMPC optimization problem that is well behaved, and gives a smooth input to the system. When testing on the detailed simulator model, the state- and parameter estimation causes some oscillatory behavior and problems with the constraints. Further work includes better tuning of the UKF used for state- and parameter estimation.

Further research could also focus on including some of the measured disturbances as manipulated variables, such as the main fluid pump rate and the choke line fluid pump rate.

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