LQG CONTROL WITH RECONFIGURABLE STATE ESTIMATOR UNDER SENSOR AND ACTUATOR FAILURES

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Abstract: Occurrences of sensor / actuator failures can lead to significant degradation in the closed loop performance when conventional feedback controllers are used. In this work, we propose an active failure tolerant LQG (FTLQG) control scheme, which employs model based fault diagnosis for on-line reconfiguration of state estimator on diagnosis of failures. Generalized likelihood ratio (GLR) method proposed in the literature is extended for diagnosis of sensor failures. Recurrence relationships are derived for diagnosing sensor failures, which are amenable for on-line computations. The efficacy of the proposed FTLQG scheme is demonstrated using simulation and experimental studies on a laboratory scale heater-mixer setup.

Keywords: Sensor and Actuator Failures, Generalized Likelihood Ratio, Kalman Filter, LQG

INTRODUCTION

In processing plants, there are various reasons for degraded performance or complete loss of system functions. These include different faults, unknown disturbances, modeling uncertainties or complete failure of system components. The effect of unknown disturbances and modeling uncertainties can be suppressed considerably by appropriate measures like filtering or robust design of controllers. However, sensor and/or actuator failures, which have considerable deteriorating effect on the closed loop performance, are difficult to handle through such a passive approach. Such failures have to be diagnosed on-line as quickly as possible and actively accommodated in order to arrest propagation of their effects.

Active failure tolerance can be achieved by employing fault diagnosis techniques on-line and redesigning/restructuring controller on diagnosis of failures. Variety of active reconfiguration control techniques have been proposed in the literature. Konstantopoulous and Antsaklis(1996) have proposed an active reconfiguration strategy based on eigenstructure assignment. Their approach aims at placing the eigenvalues of the closed loop system at desired locations under variety of failure conditions. Kanev and Verhaegen (2000) have proposed to enumerate all expected failure scenarios and construct models, which describe the dynamics of each failure situation. When a failure occurs this scheme switches to a pre-computed control law corresponding to the current failure situation. This technique works well with systems with relatively few and well understood failures. Yang et al.(2000) have proposed design of reliable LQG controller with sensor failures in which closed loop stability is ensured in the event of sensor failure. Recently, Deshpande et al. (2005) have proposed a fault tolerant nonlinear model predictive control formulation in which modifications are made in the controller objective function and constraint set to account for the loss of a degree of freedom when actuator failures are diagnosed.

In this work, we propose a failure tolerant LQG (FTLQG) controller, which employs model based fault diagnosis for on-line reconfiguration of state estimator. Narasimhan (1987) has shown that generalized likelihood ratio (GLR) method proposed by Wilsky and Jones (1974) can be used for diagnosing actuator failures. In this work, we extend the GLR method for diagnosis of sensor failures. Recurrence relationships are derived for diagnosing sensor failures, which are amenable for on-line computations. The efficacy of the proposed FTLQG scheme is demonstrated using simulation and experimental studies on a laboratory scale heater-mixer setup. The rest of this article is organized as follows. The next section provides a brief review of GLR based FDI scheme and reconfiguration of the state estimators used in FDI and LQG formulation, under sensor failures. Section 2 presents the details of experimental work. The major conclusions reached from experimental work are given in section 3.

1. FAULT DIAGNOSIS AND ESTIMATOR RECONFIGURATION

This section provides a brief review of GLR based FDI scheme and integration of FDI scheme with state estimator.

1.1 Model for Diagnosis and Control

The main component of Fault tolerant control system is a model describing process dynamics, which is used to develop Kalman filter(KF). Let

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}_u\mathbf{u}(k) + \mathbf{L}_p\boldsymbol{\varepsilon}(k) \quad (1)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \boldsymbol{\varepsilon}(k) \quad (2)$$

represent the innovation form of state space model identified from the input output perturbation data obtained under fault free conditions. Here $\mathbf{x}(k) \in R^n$ represents state variables, $\mathbf{u}(k) \in R^m$ represents manipulated inputs to process, $\mathbf{y}(k) \in R^r$ represents measured output and $\boldsymbol{\varepsilon}(k)$ is a white noise sequence with covariance matrix $\mathbf{V}$. This model is equivalent to a process with following state space representation

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}_u\mathbf{u}(k) + \mathbf{w}(k) \quad (3)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k) \quad (4)$$

where $\mathbf{v}(k)$ and $\mathbf{w}(k)$ are zero mean Gaussian white noise sequences with known covariance matrices given as follows

$$\mathbf{R}_1 = E[\mathbf{w}(k)\mathbf{w}(k)^T] = \mathbf{L}_p\mathbf{V}\mathbf{L}_p^T \quad (5)$$

$$\mathbf{R}_{12} = E[\mathbf{w}(k)\mathbf{v}(k)^T] = \mathbf{L}_p\mathbf{V} \quad (6)$$

$$\mathbf{R}_2 = E[\mathbf{v}(k)\mathbf{v}(k)^T] = \mathbf{V} \quad (7)$$

While formulating GLR based FDI scheme, it is assumed that equations (3)-(4) represent plant dynamics under normal operating conditions. These equations are also used to formulate and solve steady state Riccati equation and compute controller gain matrix $\mathbf{K}_\infty$. To handle plant model mismatch arising out of actuator biases or input disturbances in LQG formulation, artificial states are introduced as follows

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}_u\left[\mathbf{u}(k) + \boldsymbol{\beta}(k)\right] + \mathbf{w}(k) \quad (8)$$

$$\boldsymbol{\beta}(k+1) = \boldsymbol{\beta}(k) + \mathbf{w}_\beta(k) \quad (9)$$

$$\mathbf{y}(k) = \mathbf{C}x(k) + \mathbf{v}(k) \quad (10)$$

where $\boldsymbol{\beta} \in R^m$ are artificially introduced input disturbance vectors while $\mathbf{w}_\beta \in R^m$ is a zero mean white noise sequences with covariance $\mathbf{Q}_\beta$. The elements of noise covariance matrix $\mathbf{Q}_\beta$ are tuning parameters, which can be chosen to achieve desired closed loop disturbance rejection characteristics. This augmented model is used to design a state estimator (augmented KF) necessary for implementing the LQG controller. The resulting control law is given as follows

$$\mathbf{u}(k) = \mathbf{u}_s(k) - \mathbf{K}_\infty\left[\widetilde{\mathbf{x}}(k) - \mathbf{x}_s(k)\right] \quad (11)$$

$$\mathbf{u}_s(k) = \mathbf{K}_u^{-1}\mathbf{r}(k) - \boldsymbol{\beta}(k) \quad (12)$$

$$\mathbf{x}_s(k) = (\mathbf{I} - \mathbf{\Phi})^{-1}\Gamma_u\mathbf{K}_u^{-1}\mathbf{r}(k) \quad (13)$$

$$\mathbf{K}_u = \mathbf{C}\left(\mathbf{I} - \mathbf{\Phi}\right)^{-1}\Gamma_u \quad (14)$$

where $\widetilde{\mathbf{x}}(k)$ represents estimated state vector using augmented KF and $\mathbf{r}(k)$ represents setpoint vector. It may be noted that the artificially added states $\boldsymbol{\beta}(k)$ can handle the plant model mismatch that arises due to disturbances but cannot handle the plant model mismatch arising from failure of sensor or actuator.

1.2 Fault Diagnosis

Under normal operating conditions, the state estimates used in FDI scheme are generated using Kalman filter of the form

$$\widehat{\mathbf{x}}(k+1|k) = \mathbf{\Phi}\widehat{\mathbf{x}}(k|k) + \mathbf{\Gamma}_u\mathbf{u}(k) \quad (15)$$

$$\widehat{\mathbf{x}}(k|k) = \mathbf{\Phi}\widehat{\mathbf{x}}(k|k-1) + \mathbf{L}\boldsymbol{\gamma}(k) \quad (16)$$

$$\boldsymbol{\gamma}(k) = \mathbf{y}(k) - \mathbf{C}\widehat{\mathbf{x}}(k|k-1) \quad (17)$$

where $\mathbf{L}$ represents the steady state Kalman gain. When process starts behaving abnormally, the first task is to detect the deviations from the normal operating conditions. To simplify the task of fault detection, it is further assumed that, under normal operating conditions, the innovation sequence from KF, $\{\boldsymbol{\gamma}(k)\}$ is a zero mean Gaussian white noise sequence with covariance $\mathbf{V}(k)$. Under this assumption, a simple statistical test namely fault detection test (FDT) as given in Prakash et. al.(2002) based on the innovations obtained from the normal KF is applied at each time instant to estimate the time of occurrence of a fault. The test statistic for this purpose is given as follows-

$$\epsilon(k) = \boldsymbol{\gamma}(k)^T \mathbf{V}(k)^{-1} \boldsymbol{\gamma}(k) \tag{18}$$

Since it is assumed that innovation sequence is a zero mean Gaussian white noise process, the above test statistic follows a *central chi-square distribution* with $\mathbf{r}$ degrees of freedom, which can be used to fix the threshold. If FDT is rejected, the occurrence of a fault is further confirmed by examining innovation sequence in the time interval $[t, t+N]$. The test statistic given by equation 19 is used for this purpose, which follows a central chi-square distribution with $r(N+1)$ degrees of freedom.

$$\epsilon(t, N) = \sum_{k=t}^{t+N} \boldsymbol{\gamma}(k)^T \mathbf{V}(k)^{-1} \boldsymbol{\gamma}(k) \tag{19}$$

If this test statistic exceeds the threshold, the occurrence of the fault or failure is confirmed. This is referred as fault confirmation test (FCT).

1.3 ***Fault Models***

Once the occurrence of a fault is confirmed, the next step is to isolate the fault and estimate its magnitude. To identify the failures that might have occurred, it is necessary to develop a model for each hypothesized failure that describes its effect on the evolution of the process variables. When j^{th} actuator fails abruptly at instant t, then Narasimhan (1987) have proposed following model for the failure mode

$$\mathbf{u}_{m_j}(k) = \mathbf{m}(k) \tag{20}$$
$$+ \left[b_{m_j} \mathbf{e}_{m_j} - \mathbf{e}_{m_j}^T \mathbf{m}(k) \mathbf{e}_{m_j} \right] \sigma(k-t) \tag{21}$$

where $j \in 1$ to m, b_{m_j} represents constant value at which the j^{th}actuator is stuck and $\mathbf{e}_{m_j}$ represents fault vector with j^{th} element equal to unity and all other elements equal to zero. Note that this model distinguishes between the controller output $\mathbf{m}(k)$ and manipulated input $\mathbf{u}(k)$ entering the process. Here $\sigma(k-t)$ represents unit step function. When j^{th} sensor fails abruptly at instant t, we propose to model the behavior of the measurement vector subsequent to the failure as follows

$$\mathbf{y}_{s_j}(k) = \mathbf{C}\mathbf{x}(k) + v(k)$$
$$+ \left[b_{s_j} \mathbf{e}_{s_j} - \mathbf{e}_{s_j}^T \mathbf{C}\mathbf{x}(k) \mathbf{e}_{s_j} \right] \sigma(k-t) \tag{22}$$

where $j \in 1$ to r, b_{s_j} represents constant value at which the j^{th} sensor is stuck and $\mathbf{e}_{s_j}$ represents fault vector with j^{th} element equal to unity and all other elements equal to zero.

1.4 ***Failure Isolation and Estimation***

Each failure influences the innovations term in a different manner and this fact can be used for fault isolation. In the absence of any failure, innovation sequence is a zero mean Gaussian white noise process. However, if an actuator or sensor gets stuck at a constant value b_{f_j} at time t, the expected values of innovations at any subsequent time can be represented as (Narasimhan, 1987)

$$\boldsymbol{\gamma}_{f_j}(k) = [\boldsymbol{\gamma}(k) + b_{f_j} \mathbf{G}_{f_j}(k;t) \mathbf{e}_{f_j} + \mathbf{g}_{f_j}(k;t)] \tag{23}$$

where $k \geq t$ and subscript f denotes the fault type. $\mathbf{G}_{f_j}(k;t)$ is referred to as *signature matrix* and depends on time t at which a fault occurs and time k at which innovations are computed and also on the fault location. The vector $\mathbf{g}_{f_j}(k;t)$ which we refer to as the fault *signature vector*, also depends on the fault type and location. Similarly, the expected values of state error after occurrence of a fault can be expressed as

$$E(\delta \mathbf{x}_{f_j}(k)) = b_{f_j} \mathbf{J}_{f_j}(k;t) \mathbf{e}_{f_j} + \mathbf{j}_{f_j}(k;t) \tag{24}$$

where $\delta \mathbf{x}_{f_j}(k) = \widehat{\mathbf{x}}(k|k-1) - \mathbf{x}(k)$ and $\mathbf{x}(k)$ represents the true value of the state vector. The *signature matrices* and *signature vectors* for each hypothesized fault can be precomputed based on the appropriate fault model and KF equations. *Signature matrices* and signature vector for state correction and for contributions to innovations in the event of j^{th} sensor failure are as follows

$$\mathbf{J}_{s_j}(k;t) = \boldsymbol{\Phi} \mathbf{J}_{s_j}(k-1;t) + \mathbf{L}\mathbf{G}_{s_j}(k-1;t) \tag{25}$$
$$\mathbf{j}_{s_j}(k;t) = \boldsymbol{\Phi} \mathbf{j}_{s_j}(k-1;t) + \mathbf{L}\mathbf{g}_{s_j}(k-1;t) \tag{26}$$
$$\mathbf{G}_{s_j}(k;t) = \mathbf{I} - \mathbf{C}\boldsymbol{\Phi} \mathbf{J}_{s_j}(k-1;t) \tag{27}$$
$$\mathbf{g}_{s_j}(k;t) = -\mathbf{e}_{s_j}^T \mathbf{C}\mathbf{x}(k) \mathbf{e}_{s_j} - \mathbf{C}\boldsymbol{\Phi} \mathbf{j}_{s_j}(k-1;t) \tag{28}$$

where $\mathbf{I}$ is the identity matrix. The difficulty in using equation (28) is that it requires knowledge of true state vector $\mathbf{x}(k)$. To alleviate this difficulty, we propose to use $\widehat{\mathbf{x}}(k|k)$ given by KF in place of $\mathbf{x}(k)$, under the assumption that observability is not lost with the failed sensor. The detailed derivations for equations (25)-(28) are given in Appendix. Signature matrices and signature vectors in case of actuator failure are given in Narasimhan (1987). Once the occurrence of a fault is confirmed by FCT, GLR method is used for isolating the cause of fault and estimating its magnitude using the innovation sequence in time interval $[t, t+N]$. In this method, for each hypothesized fault the log likelihood ratio,

$$T_{f_j} = \frac{d_{f_j}^2}{c_{f_j}} + \sum_{k=t}^{t+N} 2(\mathbf{g}_{f_j}(k;t)'\mathbf{V}(k)^{-1}\boldsymbol{\gamma}(k)) \\ -\mathbf{g}_{f_j}(k;t)'\mathbf{V}(k)^{-1}\mathbf{g}_{f_j}(k;t)$$

is computed using

$$d_{f_j} = \mathbf{e}_{f_j} \sum_{k=t}^{t+N} \mathbf{G}_{f_j}^{'}(k;t)\mathbf{V}(k)^{-1}\boldsymbol{\gamma}(k)$$

$$c_{f_j} = \mathbf{e}_{f_j}^T \sum_{k=t}^{t+L} \mathbf{G}_{f_j}^{'}(k;t)\mathbf{V}(k)^{-1}\mathbf{G}_{f_j}(k;t)\mathbf{e}_{f_j}$$

The fault with maximum value of this ratio is the fault that is isolated and the corresponding estimate of magnitude is given as $\hat{b}_{f_j} = d_{f_j}/c_{f_j}$ where f denotes the fault type either sensor or actuator failure.

1.5 ***State Estimator Reconfiguration***

Consider a situation where FDT has been rejected at time instant t and subsequently a fault is confirmed to have occurred at time $t+N$ for the first time. Further let us assume that at instant $t+N$, a sensor failure has been diagnosed using GLR method using the data collected in the interval $[t, t+N]$. During the interval $[t, t+N]$, the LQG controller is unaware of the failure and continues to use the faulty sensor measurements for state estimation. Once the occurrence of failure is diagnosed, the measurement from the *failed sensor* is removed from Kalman filters used in FDI scheme and LQG controller. Measurement error covariance matrix **R** and output matrix **C** are also modified accordingly. In effect, we switch over to *inferential control* where the output corresponding to the failed measurement is estimated using other available measurements. It may be noted that the proposed modification is possible only when the system observability is preserved under the sensor failure. In order to avoid repeated detection of the same failure in future, the failed sensor measurement is also excluded from our failure hypothesis in FDI.

In the event of actuator failure, we propose to reconfigure the controller by exploiting additional degrees of freedom that are available in the system. Once actuator failure is detected and confirmed by FDI, the augmented model used by controller and the unaugmented model used by FDI unit are modified to include estimated fault magnitude. Also, subsequent to isolation of an actuator failure, the input corresponding to the failed actuator is held constant at value estimated by the FDI unit.. In case additional degrees of freedom are available, the controller is reconfigured to employ the additional input(s) available for manipulation, subject to controllability condition being satisfied.

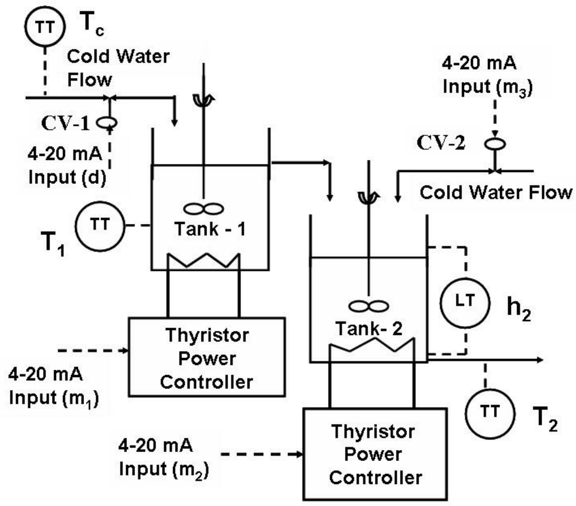

Fig. 1. Schematic of Experimental TwoTank Heater Mixer Setup

2. EXPERIMENTAL VERIFICATION

The experimental **heater-mixer set** up considered for the study consists of two stirred tanks in series as shown in fig 1. A cold water stream is introduced in the first tank. The content of the first tank is heated using 4kWH heating coil. The hot water that overflows the first tank is mixed with cold water stream entering in to second tank. The content of the second tank is heated using another 3kWH heating coil. The heat inputs to both the tanks can be manipulated continuously using thyristor power control units. The cold water inlet flow to both the tanks can be manipulated using pneumatic control valves. The temperatures in the two tanks (T_1 and T_2) and level in second tank (h_2) are measured variables while the heat inputs to first and second tanks (u_1 and u_2) and cold water inlet to Tank 2 (u_3) are treated as manipulated inputs. The cold water flow to first tank is treated as constant input. The detailed model and nominal parameters are given in Srinivasrao et. al., (2005).
For model identification, the steady state operating point of the process is chosen as [$T_1 = 56^0$, $T_2 = 52^0$ and $h_2 = 0.36\ m$] and each of the nominal steady state input to the plant has been set to 12 mA. The three inputs were perturbed simultaneously with random binary signals (RBS) of amplitude 2.5 mA, 2.5 mA and 2 mA respectively in the frequency band [0, 0.005]. A linear state space model having four states was identified using System Identification Toolbox in Matlab.

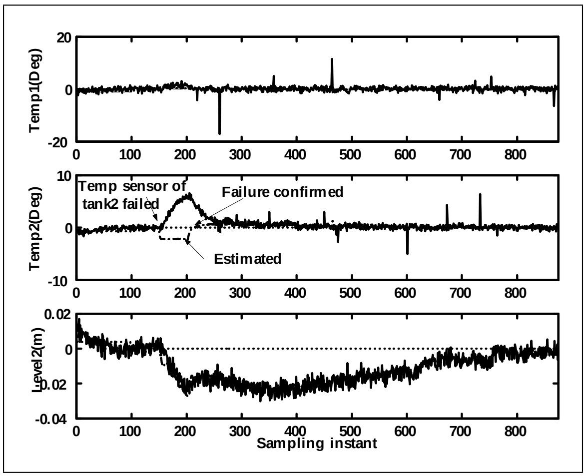

Fig. 2. Experimental Validation of FTLQG : Outputs

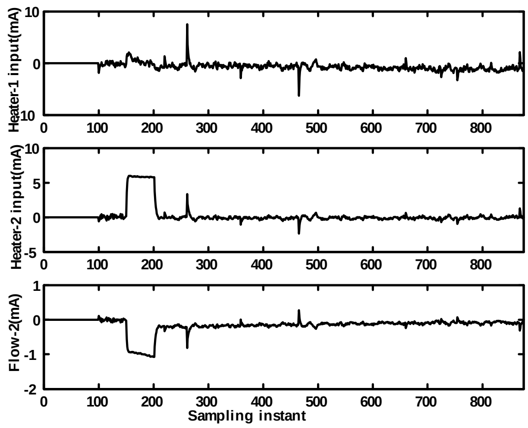

Fig. 3. Experimental Validation of FTLQG : Inputs

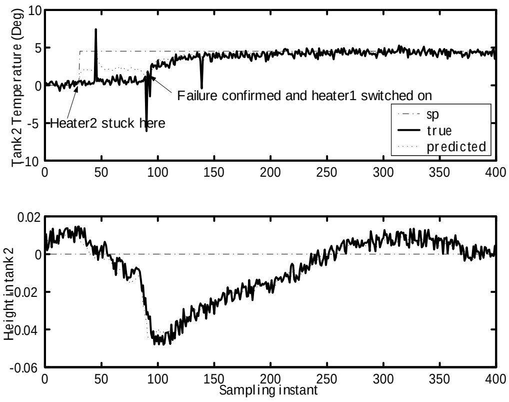

Fig. 4. **Process outputs under failure of actuator for heat input1**

2.1 ***Sensor Failure:***

To evaluate the performance of FTLQG based on this model, a sensor failure was simulated by artificially holding the temperature measurements T_2 constant in the control computer at 6^0 below its steady state value subsequent to 145^{th} sampling instant. Failures have been hypothesized in sensors (T_1, T_2, h_2) and actuators (u_1, u_2, u_3). GLR based failure detection test has been used whose parameters are listed in Table 1.

Table 1. GLR Parameters

Variable description	Value
Simulation time	1250 sampling instants
Window length N	50 sampling instants
Level of significance for FDT	0.5
Level of significance for FCT	0.005

The objective of FTLQG is to track the desired set point trajectories for temperature of liquid in Tank 1 and 2 (T_1, T_2) and level of liquid in Tank2 (h_2) in the face of **failure of temperature sensor**. As evident from Figures (2) and (3), the proposed FTLQG is able to track T_2 setpoint using its inferred value after diagnosis and accommodation of the failure.

2.2 ***Actuator Failure:***

In this case, the objective of FTLQG is to track set point trajectories of $T_2$and h_2 by manipulating heat input to Tank 2 (u_2) and flow to Tank2 (u_3). In the event of **failure of actuator** for heat input to Tank 2 (u_2), the objective of FTLQG was to detect and confirm the failed heater for Tank 2 and reconfigure the controller online by switching to another LQG control law that brings T_2 and h_2 to their set points by manipulating heater input in Tank1 (u_1) and flow input to Tank 2 (u_3). Failures were hypothesized in sensors (T_2, h_2) and actuators (u_2, u_3). As shown in Figures 4 and 5, input to heater in Tank 2 was stuck at 25th sampling instant where the true input was -0.53 mA. The set point change for T_2 was given at 30^{th} instant. The failure was isolated by the FDI unit at 85^{th} sampling instant with an estimated magnitude of $-0.601\ mA$. After failure isolation, new LQG controller was implemented, which manipulates heater input in Tank1 (u_1) and flow input to Tank 2 (u_3). It is evident from figure 4 that the reconfigured control law is able to track the desired set point change.

3. CONCLUSIONS

Analysis of experimental results reveals that the proposed GLR based FDI method is able to detect and isolate the failed sensor and actuator correctly. The proposed FTLQG is able to recover the performance degradation caused by failed sensor. This is achieved by removing the faulty sensor measurements from measurement set used for state estimation and continuing control using the inferred value of failed measurement. The proposed FTLQG is also able to reconfigure itself online under actuator failure and meet the desired performance specifications.

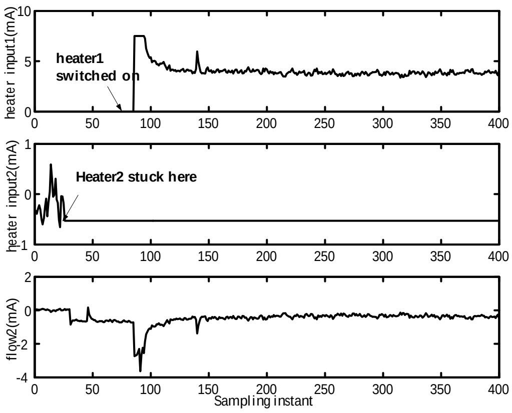

Fig. 5. **Manipulated inputs under failure of actuator for heat input1**

4. APPENDIX: COMPUTATION OF SIGNATURE MATRICES

Let us assume that j^{th} sensor fails at instant t. Then, for all $k \geq t$, the process states are given by equation 3 and the output be given as

$$\mathbf{y}_{s_j}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k) + \left[b_{s_j}\mathbf{e}_{s_j} - \mathbf{e}_{s_j}^T\mathbf{C}\mathbf{x}(k)\mathbf{e}_{s_j}\right]\sigma(k-t) \quad (29)$$

where $\mathbf{e}_{s_j}$ is a vector of faults having a nonzero entry only at j^{th} location and b_{s_j} is a constant value at which the sensor is stuck. Let us also assume that the FDT has been rejected at time t and FCT has been rejected at instant $t+N$. During the interval $[t, t+N]$, the state estimates are still generated using the *normal* state estimator (15)-(17). Under these conditions, it is desired to develop recurrence relationships for describing the effect of failure on the evolution of system and estimator variables. Let the difference between true state and estimated state under sensor failure be given as follows

$$\delta\widehat{\mathbf{x}}_{s_j}(k) = \widehat{\mathbf{x}}_{s_j}(k|k) - \mathbf{x}(k) \quad (30)$$

where $\widehat{\mathbf{x}}_{s_j}(k|k)$ is the estimated state under sensor failure and $\mathbf{x}(k)$ is the true state. Then, using standard Kalman filter equations, we can obtain the following relations:

$$\widehat{\mathbf{x}}_{s_j}(k|k) = \widehat{\mathbf{x}}_{s_j}(k|k-1) + \mathbf{L}\boldsymbol{\gamma}_{s_j}(k) \quad (31)$$

$$\widehat{\mathbf{x}}_{s_j}(k|k-1) = \boldsymbol{\Phi}\widehat{\mathbf{x}}_{s_j}(k-1|k-1) + \boldsymbol{\Gamma}_u\mathbf{u}(k-1) \quad (32)$$

$$\boldsymbol{\gamma}_{s_j}(k) = \mathbf{y}_{s_j}(k) - \mathbf{C}\widehat{\mathbf{x}}_{s_j}(k|k-1)) \quad (33)$$

where $\mathbf{L}$ is the Kalman gain and $\boldsymbol{\gamma}_{s_j}(k)$ is the innovation vector at k under j^{th} sensor failure. From equations 29 to 33 and equation 3, we can write

$$\boldsymbol{\gamma}_{s_j}(k) = \mathbf{v}(k) - \mathbf{C}\boldsymbol{\Phi}\delta\widehat{x}_{s_j}(k-1) + b_{s_j}\mathbf{e}_{s_j} - \mathbf{e}_{s_j}^T\mathbf{C}\mathbf{x}(k)\mathbf{e}_{s_j} \quad (34)$$

and

$$\delta\widehat{\mathbf{x}}_{s_j}(k) = (\mathbf{I} - \mathbf{L}\mathbf{C}\boldsymbol{\Phi})\delta\widehat{\mathbf{x}}_{s_j}(k-1) + \mathbf{L}b_{s_j}\mathbf{e}_{s_j} - \mathbf{L}\mathbf{e}_{s_j}^T\mathbf{C}\mathbf{x}(k)\mathbf{e}_{s_j} \quad (35)$$

Now, let us define the linear dependence of expected values $E[(\delta\widehat{\mathbf{x}}_{s_j}(k))]$ and $E[(\boldsymbol{\gamma}_{s_j}(k))]$ on the failure by following relations:

$$E[(\delta\widehat{\mathbf{x}}_{s_j}(k))] = b_{s_j}\mathbf{J}_{s_j}(k;t)\mathbf{e}_{s_j} + \mathbf{j}_{s_j}(k,t) \quad (36)$$

$$E[(\boldsymbol{\gamma}_{s_j}(k))] = b_{s_j}\mathbf{G}_{s_j}(k;t)\mathbf{e}_{s_j} + \mathbf{g}_{s_j}(k,t) \quad (37)$$

where $\mathbf{J}_{s_j}(k;t)$ and $\mathbf{G}_{s_j}(k;t)$ are the signature matrices and $\mathbf{j}_{s_j}(k,t)$ and $\mathbf{g}_{s_j}(k,t)$ are the signature vectors for state correction and contributions to innovations in the event of j^{th} sensor failure, respectively. From equations 34 to 37 we can obtain the relations given in equations 25 to 28

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