# BAYESIAN APPROACH FOR CONSTRAINT ANALYSIS OF MPC AND INDUSTRIAL APPLICATION

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Abstract: Profit margins from plant operations may be improved by changing the constraints so as to increase the degrees of freedom for control. Due to the presence of disturbances the chances of operating the plant outside the set limits cannot be ruled out. Thus, the expected return should be estimated by taking into account the variability. Bayesian Statistics can be used to estimate these probabilities subject to changes in operating constraint limits. The maximum *a posteriori* estimate of the process state due to the change in the operating conditions can be inferred using Bayesian methods and the profits or return thus obtained can be estimated. Also the decisions to obtain target value of the return can be made using the Bayesian methods. 

Keywords: Bayesian Analysis, Constraint tuning, Conditional probability, Control performance monitoring.

## 1. INTRODUCTION

Model Predictive Control (MPC) is an advanced method of process control, which relies on the model of the process to predict the behavior of the Controlled Variables (CVs). By operating the Manipulated Variables (MVs) a cost function is minimized. A multivariable MPC also takes into consideration the interaction between the process variables, thus providing a better control than conventional univariate controls. However, having a multivariable controller is not just enough to serve the purpose. They need to be tuned properly with a proper understanding of the process behavior and the control philosophy adopted for the MPC. There are different commercial controllers available in the market that adopt different control philosophies (Qin and Badgewell, 2003) but they all need to be tuned at the design and engineering level.

MPC controller tuning is primarily being done through the tuning of the penalty matrix on the output errors and/or control moves so as to minimize the squared error of the controller output over the control horizon. The other tuning parameters involved are the prediction and the control horizon. Tuning of MPC controllers with these parameters is done at the engineering level and requires a thorough understanding of the process and the control philosophy of the MPC application used. Though these are the key tuning parameters for an MPC controller, there are other factors, like the CV/MV constraints and variability that also contribute to the performance of an MPC controller. The constraints should be carefully chosen as giving wrong constraint limits in CVs or MVs can lead to poor performance of the controller.

In an MPC controller, CVs generally reflect the desired product qualities, which are to be controlled and optimized within certain limits while the MVs are the handles available for the purpose. Having a value within the limits for the CVs cannot always be called a good control, since within the limits, there also exists an optimum operating point (generally located at the constraints), at which the performance of the MPC controller is maximized. The presence of variability determines how far or close the CVs are to the optimum operating point. Thus, the CVs have probabilities associated with them to be under-spec (values less than low limits), in-spec (values within the limits) or over-spec (values greater than the high limits) depending on which the expected return from the controller can be estimated (Rahim, 2000).

During operations of an MPC controller it is, at times, required to change the constraints of one or more CVs and MVs. This study is to provide a probabilistic method to evaluate the effect of the change in the constraints, on the overall performance of the MPC controller. The study also aims at providing the maximum *a posteriori* (MAP) decision, using Bayesian methods, for the decisions to be made to achieve target value of return. Bayesian Statistics is a branch of statistical inference technique that deals with probabilities of occurrence of certain events given certain set of conditions or observations as discussed next.

## 2. PRELIMINARIES

### *2.1 Defining the Problem*

For illustration purposes, consider two outputs $y_1$ and $y_2$ of a system. Let $y_1$ be the quality variable and $y_2$ be the constraint variable. The current operating data for $y_1$ and $y_2$ are shown in Fig. 1. Fig. 2 shows the approximated probability distribution for the current operating data.

As can be seen from Fig. 1 and Fig. 2 that even though the mean values for $y_1$ and $y_2$ are within the specified limits throughout the period for which the data are collected, there are instances when the process values lie outside the limits. Here since $y_1$ is a quality variable, having its value outside the set limits, if frequently, is undesirable as it can render the product unmarketable.

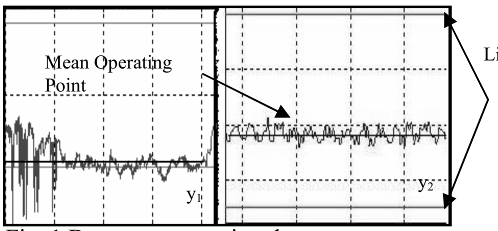

Fig. 1 Base case operating data

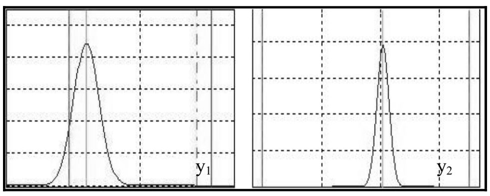

Fig. 2 Process data distribution

It has been observed that, in practice the constraint limits for some CVs/MVs of an MPC controller can be adjusted (tuned), but cannot be changed dramatically and, in addition, not all of them have the equal preference to change the limits. Thus, for the purpose of exploratory study 10% constraint relaxation is being considered for selected CVs and MVs with different preferences (defined by prior). The limit set for the CVs and the MVs and their variability determine the operating point of the MPC controller. Relaxing the limits for one or more process variables provides the controller with increased degrees of freedom and thus may help in improving the expected return even if there is no reduction in the variability of the variables.

### *2.2 Bayesian Analysis*

Named after Thomas Bayes, Bayesian Analysis is a branch of inference that can be applied for decision making and statistical analysis using knowledge of prior events to predict future events. The Bayes theorem forms the backbone of Bayesian analysis. It enables calculating conditional probabilities for a hypothesis (Korb and Nicholson, 2004; Tan 2001). Thus, if $x_j$ is the set of observed variables, $x_i$ is the set of variables whose values we are interested in estimating, $x_k$ is the set of variables in the system, not included in $x_i$ and $x_j$, then inference in a Bayesian analysis means to compute:

$$P\left(x_i = a \mid x_j = b\right) = \frac{P\left(x_i = a, x_j = b\right)}{P\left(x_j = b\right)} = \frac{\sum_{x_k} P\left(x_i = a, x_j = b, x_k\right)}{\sum_{x_i, x_k} P\left(x_i, x_j = b, x_k\right)} \tag{1}$$

where, $P(x_i = a|x_j = b)$ is the probability for node $x_i$ to take value $a$ provided that node $x_j$ takes the value $b$, and $P(x_j = b)$ is the probability for node $x_j$ to take value $b$ and $P(x_i = a,\ x_j = b)$is the probability of conjunction of $x_i$ and $x_j$.

### *2.3 Bayesian Networks*

For a system comprising of random variables, a network connecting all variables can be built, which represents the relationship between the various random variables. A Bayesian network is defined by Korb and Nicholson (2004) as "a graphical structure that allows us to represent and reason about an uncertain domain. The nodes in the network represent a set of random variables." A pair of nodes is connected through directed arcs that represent a relationship between the nodes. The node through which the arc originates is called *the parent node* and the node where it terminates is called *the child node*. The nodes in a Bayesian Network are the variables of interest and the link between them represents the probabilistic dependencies among the nodes. To specify the probability distribution of a Bayesian Network, prior probabilities are to be defined for the root nodes i.e. the nodes with no predecessor, and the

Conditional Probability Distribution Table (CPD or CPT) is defined for all non-root nodes, for all possible combinations of their direct predecessors (Charnaik, 1991). The CPT quantitatively represents the relationship between the parent and the child nodes. A typical Bayes Network is shown in Fig. 3. Node "*C*" has two parent nodes "*A*" and "*B*", and one child node "*D*". Node "*A"* and "*B"* have two states {*1,2*} and node "*C"* has three states {*X,Y,Z*} and node "*D*" has two states {*P,Q*}. The tables beside node "*A*" and "*B*" are their prior probability and that beside node "*C*" and "*D*" represents their CPT.

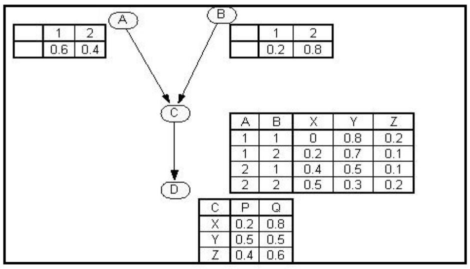

Fig. 3 Typical Bayesian Network

A Bayesian network cannot have directed cycles, i.e. a node cannot be reached again by following the directed arcs. Thus Bayesian Networks are also called Directed Acyclic Graphs (DAGs).

The CPDs quantifies the dependencies between the nodes. They are probability distribution functions $P(x_i \mid Pa_i)$, where $x_i$ is the i[th] node and $Pa_i$ represents all of its parent nodes. There are three types of nodes, two of which are being used in this paper. *1) Chance Nodes*: These nodes represent random variables and are associated with CPT. *2) Utility Nodes*: These nodes represent the value of the utility function (benefit function). The parents for these nodes are the nodes whose outcome directly affects the utility. These nodes are associated with utility table, with the value for each possible instantiation of its parents perhaps including an action taken (Korb and Nicholson, 2004).

## 2. BAYESIAN METHODS FOR MPC CONSTRAINT ANALYSIS AND TUNING

For an APC application with *n* inputs and *m* outputs, let *K* be the steady state gain matrix and $(\bar{y}_{i0}, \bar{u}_{j0})$ be the current mean operating points, which are defined as the base case operating points. Also let the number of CVs and MVs for which it is allowed to change the constraint limits be *a* and *b* respectively. Thus, with *N=a+b* variables available, for which it is possible to change the constraint limits, there are $2^N$ combinations for applying the changes in the limits. Each combination, having a specific optimal return through LQ optimization of the operating point will affect MPC performance.

The LQ optimization is carried out for each possible constraint change, with the CV/MV data collected over $N_l$ time stamps. The objective function is an economic benefit function of MPC, and the constraints are MV and CV constraint limits by taking into account CV/MV variability and process steady state gain relations (Xu, *et al.*, 2006). For the exploratory study, the optimization was carried out for 10% constraint relaxation. The $2^N$ optimization results give the same number of optimum operating points for all the variables.

The Bayesian Network is created with *N* parent nodes, *q* child nodes and 1 utility node, where *q* is the number of quality variables affecting the economic performance of the process. The parent nodes have two states *{yes, no}* where *yes* means "to change the limits" and *no* means "not to change". By changing the constraint limits, the *q* quality CVs are optimized to be operating as close as possible to their optimum. In the same time, other non-quality CVs are also moved due to the interaction. The CVs take continuous values but they are discretized into a finite number of operating zones, in this paper, 6 zones. Therefore, each child node has six states *{Zone 1, Zone 2, Zone 3, Zone 4, Zone 5, Zone 6}* defining the range in which the value of the output variables will lie, illustrated in Fig.4. (The number of zones can be increased depending upon the resolution required.) *Zone 1* and *Zone 6* represent the region below and above the limits, respectively and *Zone 2* to *Zone 5* represent the four zones defined within the limits. Thus, if $L_i$ and $H_i$ are the low and the high limits for a particular CV ($y_i$) then $\Delta_i$ is the span of the range in which its value is to be maintained i.e.

$$\Delta_i = H_i - L_i \tag{2}$$

Then the six zones for the states of the CV can be defined as:

$$\begin{aligned}
&Zone\,1 = -\infty \text{ to } L_i \\
&Zone\,2 = L_i \text{ to } \left(L_i + \frac{\Delta_i}{4}\right) \\
&Zone\,3 = \left(L_i + \frac{\Delta_i}{4}\right) \text{ to } \left(L_i + \frac{\Delta_i}{2}\right) \\
&Zone\,4 = \left(L_i + \frac{\Delta_i}{2}\right) \text{ to } \left(L_i + \frac{3\Delta_i}{4}\right) \\
&Zone\,5 = \left(L_i + \frac{3\Delta_i}{4}\right) \text{ to } H_i \\
&Zone\,6 = H_i \text{ to } \infty
\end{aligned} \tag{3}$$

The prior probability or the priori for the parent nodes can be user defined or obtained from the historical data. It indicates the preference *to change* or *not to change* the limits e.g. if a parent node has a priori of 0.8 for "making a change". This means that the constraint for this variable has 80% tendency to change and 20% not to change.

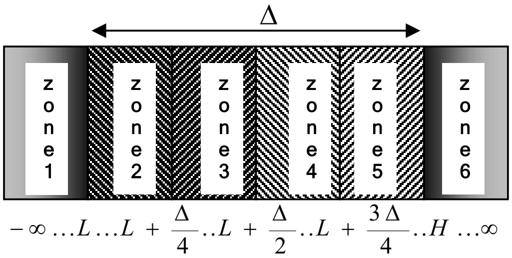

Fig. 4 Division of zones for a child node

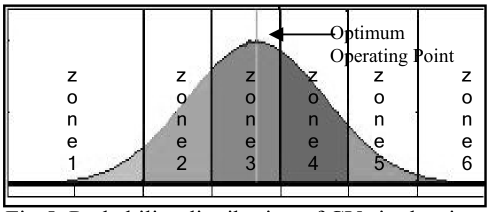

Fig.5: Probability distribution of CVs in the six zones

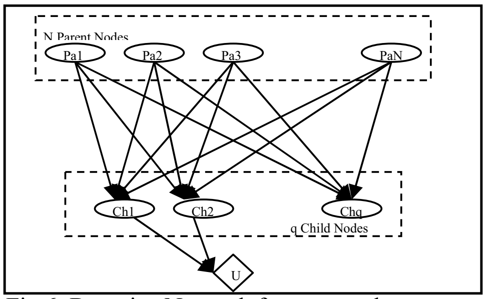

Fig 6: Bayesian Network for $m \times n$ plant

The $2^N$ optimization results obtained for each quality CV, when superimposed with the base case variability and assuming the data to be Gaussian distributed, can be used to obtain its probability distribution to be in each of the six zones. Fig.5 shows the probabilities for any quality variable to be in any of the six zones defined for CVs. The Bayesian Network thus created is shown in Fig. 6 assuming quality variables are independent. *Pa* in the figure represents the parent nodes, *Ch* in the figure is the child node and *U* is the utility node representing benefit functions.

For *q* quality variables and with 6 zone discretization, the utility nodes will have $6^q$ values for each combination of the state of the quality variables. The values taken by the utility node are provided by the mean values taken by the economic objective function in all the 6 zones for each of the *q* quality variable.

If the uncertainties associated with each of the *q* quality variables are mutually independent then, without loss to generality, assuming first *q* CVs as quality variables, the values for the expected return for the process can be obtained as:

$$E(R) = \sum_{i=1}^{q} \Big( C_{i1} \times P_i(y_i \in 1) + \sum_{z=2}^{5} J_{iz} \times P_i(y_i \in z) + C_{i6} \times P_i(y_i \in 6) \Big) \qquad (4)$$

where, $J_{iz}$ is the value of the objective function for $i^{th}$ quality variable to be in the $z^{th}$ zone, $C_{i1}$ and $C_{i6}$ might be the penalty values set for the quality variables to be in *Zone 1* and *Zone 6* respectively and $P_i(y_i \in z)$ is the probability of that quality variable to be in *Zone-z*. $J_{iz}$ can be defined as:

$$J_{iz} = \frac{1}{(h-l)} \int_l^h \left( \alpha_i \times y_i + \beta_i^2 (y_i - \mu_i)^2 \right) dy_i \qquad (5)$$

where, *h, l* are the high and the low limits set for $z^{th}$ zone for the $i^{th}$ quality variable, respectively.

The Bayesian model thus created for the system can now be used for *1) Decision evaluation* i.e. to infer the expected return or the objective function values, if certain decisions regarding to make or not to make the change of limits are made. For the decision evaluation, the decision whether to change or not to change the limits is provided. These decisions are equivalent to the evidences for the Bayesian Network conditional on which, the probabilities of locations of the quality CVs are then estimated. Thus, the expected return can be evaluated using the relation specified in equation (4). *2) Decision making* i.e. to obtain the maximum *a posteriori* explanation for decision making that will help to achieve a target value of expected return. For decision making purposes the target expected return are provided and the corresponding states for the CVs affecting the benefit function are then read from the utility node table for the network. The states thus obtained from the table are the evidences for the Bayesian Network and the maximum *a posteriori* explanation for the limit changes of the parent nodes can be made.

## 4. INDUSTRIAL CASE STUDY

Consider a distillation plant, where the hot feed is flashed into the distillation column to obtain top product, middle distillates and the bottoms. The vapors from the top are cooled and collected into the reflux drum, from where a part is sent back into the column as reflux, and the rest is drawn as the overhead product. The draw from the middle of the column is flashed into a side stripper where it is stripped with steam, to strip off any lighter fractions which have been drawn off along with it from the column. The middle distillates are then drawn from the bottom of the side stripper. The pump-around stream from column removes some excess heat from the column and preheats the feed. Also, the column bottoms may contain some traces of middle distillates, which are stripped off by stripping steam to the main column. The schematic diagram for the system is shown in Fig.7.

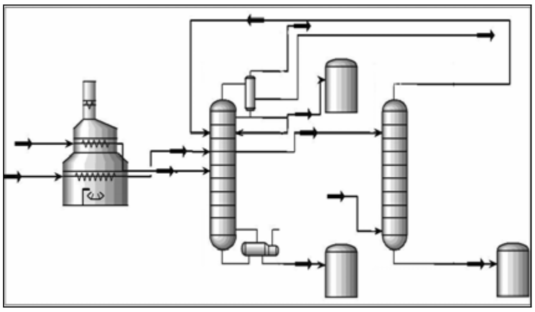

Fig.7 Distillation Column

The MPC application for the process has 8 CVs, 5 MVs. The CV-MV list for the plant and their availability for limit change is given in Table-1. Based on which a total of $2^7$=128 optimizations were carried out offline. CV2 is the only quality variable related to economic performance of the MPC application. The Bayesian Network is created for the system defined with 7 parent nodes, 1 child node and 1 utility node. Table-2 gives the prior probabilities for the changes to be made on the parent nodes. The optimization results are used to create CPT for the child nodes.

The expected return for the system were estimated using equation (4) (with $\alpha_2$=0.30 and $\beta_2$=0), the information provided in the Table 1 and 2, and the conditional probabilities of the CVs in the six zones. For the existing system setup the expected return for the operation are estimated to be 123.20 units.

Table 1: CV-MV List and their limits change allowed

| | Description | Change Lts |
|---|---|---|
| CV1 | Mid Dist Flow | Yes |
| CV2 | Top Prod 90%Pt | No |
| CV3 | Mid Dist 5% Pt | No |
| CV4 | OVHD Vsl Boot Lvl | Yes |
| CV5 | Flood | No |
| CV6 | Col Top Temp | No |
| CV7 | Steam Temp | No |
| CV8 | Col OVHD Temp | Yes |
| MV1 | Col Pr | Yes |
| MV2 | Col Reflux | Yes |
| MV3 | PA Ret temp | Yes |
| MV4 | Str Steam | Yes |
| MV5 | Side Str Stm | No |

Table 2: Prior probability for making changes for parent nodes

| **Node** | **MV1,MV2,MV3,MV4,CV1*, CV4*,CV8*** |
|---|---|
| **Yes** | 0.5 |
| **No** | 0.5 |

*Decision Evaluation:* If the decision is being made to increase the constraint limit of the MV2, the maximum *a posteriori* estimate of CV2 is *Zone 2* and the expected return is 123.80 units.

Thus, it can be inferred that the decision to increase the limits set for MV2 will marginally increase the expected return from the process.

*Decision Making:* If the target is set to increase the return from 123.20 to 123.90 units, then with this as "evidence" the maximum *a posteriori* estimate for the state of the parent nodes that are to have their limits changed are calculated and MV1, MV2, MV3 are expected to have their limits changed. Comparing this result with that of decision evaluation, one can see that to achieve a similar return, the solution needs not to be the same. Bayesian inference is to pick the most probable one in the decision making.

## 5. BINARY DISTILLATION COLUMN CASE STUDY

Consider the simulated binary distillation column (Volk *et.al.*2005, Fig. 8) which is used to separate light petrol and the heavy petrol from the petrol obtained from an upstream desulphurization unit. The hot feed is flashed into the column. The lighter fractions of the feed vaporize and are collected as the top product in the overhead vessel and the heavier components are obtained at the bottoms of the column. The vapors from the top of the column are condensed and a part of the condensed overhead vapors are sent back into the column as the reflux and the rest is drawn as light petrol. The heavier components in the feed do not vaporize and are obtained from the column bottoms. A part of the bottom stream is sent to the reboiler. The vapors from the reboiler are sent back into the column. The balance of the heavier components is drawn from the column as the heavy petrol.

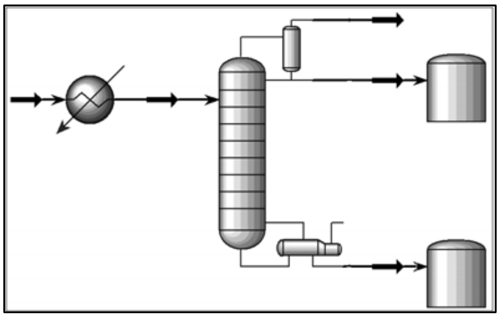

Fig.8 Binary Distillation Column

The MPC controller designed for the system described above has 4 input variables and 10 output variables. Table-3 provides CV-MV list and their availability for limits change. Based on this information, $2^7$=128 optimizations were carried out. As CV2 and CV8 govern the overall economics of the operations of the column, they are chosen as the quality CVs here. The correlation coefficient between these two CVs is 0.0049 and they can be considered as independent. The Bayesian Network is created for the system defined with 7 parent nodes, 2 child nodes and 1 utility node. Table-4 gives the prior probabilities for the changes to be made on the parent nodes. The optimization results are used to create CPT for the child nodes.

Table 3: CV-MV list and their limits change allowed

| | Description | Change Lts |
|---|---|---|
| CV1 | Ref Flow | No |
| CV2 | Lt Petrol FBP | Yes |
| CV3 | Top PCT | No |
| CV4 | Pr. Vlv OP | No |
| CV5 | Bttm PCT | Yes |
| CV6 | Col Pr | Yes |
| CV7 | Feed Temp | Yes |
| CV8 | Reb Furnace Duty | No |
| CV9 | Duty | No |
| CV10 | Bypass Vlv OP | No |
| MV1 | Ref Flow | Yes |
| MV2 | Col Pr | Yes |
| MV3 | Feed Temp Vlv OP | Yes |
| MV4 | Duty Vlv OP | No |

Table 4: Prior probability for making changes for parent nodes

| **Node** | **MV1,MV2,MV3, CV2*, CV5*,CV6*,CV7*** |
|---|---|
| **Yes** | 0.5 |
| **No** | 0.5 |

The expected return for the system were estimated using equation (4) (where ($\alpha_2$,$\beta_2$) and ($\alpha_8$,$\beta_8$) are (0.2364,0) and (0.1714,0) respectively), the information provided in the Table 3 and 4, and the conditional probabilities of CV2 and CV8 in the six zones. For the existing system setup the expected return for the operation are estimated to be 145.36 units.

*Decision Evaluation:* If the decision is being made to increase the constraint limit of the column reflux, MV2, the maximum *a posteriori* estimate of the state of CV2 and CV8 are *Zone 3* and *Zone 4* and the expected return is 173.27 units.

Thus, it can be inferred that the decision to increase the limits set for MV2 will increase the expected return from the process.

*Decision Making:* If the target is set to increase the return from 145.36 units to 165.00 units then the states for the child nodes i.e. CV2 and CV8 is determined as *Zone 1* and *Zone 4*, respectively. With this as evidence the maximum *a posteriori* estimate for the state of the parent nodes that are to have their limits changed are calculated and the parent node 3 i.e. MV3 is expected to have its limits changed.

## 6. CONCLUSION

A Bayesian approach that takes into consideration the process variability to tune the MPC controller for constraint limits change has been developed. The proposed method gives the constraint tuning guidelines by performing the Bayesian analysis of the process variables in the MPC controller.

As the real world is associated with uncertainty, the Bayesian approach of analysis is an appropriate tool that takes into consideration the probabilities for CVs to lie in the different operating ranges defined. The results thus obtained from the analysis are more realistic than simple deterministic profit calculations/optimizations. The Bayesian network built can also be used to assist in decision making when the set target value for objective function is defined.

Two case studies have also been provided that explain the industrial utility of the tool. The results from the studies illustrate its significance and utility for the process engineer for day to day maintenance of the MPC controllers in the plant. The constraint tuning guidelines obtained from the tool can be applied to improve the controller performance.

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