ROBUST FAULT DETECTION AND HANDLING IN CONTROL OF UNCERTAIN TRANSPORT-REACTION PROCESSES

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Abstract: This paper presents an integrated robust fault detection and fault-tolerant control architecture for transport-reaction processes modeled by quasi-linear parabolic partial differential equations with uncertain variables, control constraints and control actuator faults. Using an appropriate reduced-order model that captures the dominant process dynamics, the proposed architecture comprises a family of robustly stabilizing bounded feedback controllers with explicitly characterized stability and uncertainty attenuation properties, a performance-based fault detection scheme and a high-level supervisor that reconfigures the control actuators upon fault detection in a way that maintains robust closed-loop stability. The key idea is to shape the closed-loop performance via robust control in a way that facilitates the design of robust fault detection rules that are less sensitive to the adverse effects of uncertainty. The results are demonstrated using a non-isothermal tubular reactor example with recycle.
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Keywords: Transport-reaction processes, Partial differential equations, Model reduction, Robust fault detection, Nonlinear control, Input constraints, Control reconfiguration.

1. INTRODUCTION

Transport-reaction processes with significant diffusive and dispersive phenomena are typically characterized by strong nonlinearities and spatial variations, and are naturally modeled by quasi-linear parabolic Partial Differential Equations (PDEs). Examples include tubular and packed-bed reactors, as well as chemical vapor deposition and crystal growth processes. Unlike spatially homogeneous processes, the control problem arising in the context of transport-reaction processes often involves the regulation of spatially distributed variables (such as temperature and concentration spatial profiles) using spatially-distributed control actuators and measurement sensors. The need to design control and monitoring systems for these processes has motivated significant research work on the analysis and control of distributed parameter systems (e.g., (Christofides and Daoutidis, 1996; Palazoglu and Karakas, 2000; Christofides, 2001; Hoo and Zheng, 2001; Ruszkowski *et al.*, 2005)).

Compared with these efforts and many others in this area, the problems of fault diagnosis and fault-tolerant control of distributed processes has received limited attention (El-Farra and Christofides, 2004; Demetriou and Kazantzis, 2004). This is an important problem given the vulnerability of process control systems to faults (e.g., in the actuators, sensors or process equipment) and the detrimental effects that such faults can have on the process operating efficiency and, ultimately, on the final product quality. While an extensive body of literature exists on fault diagnosis of chemical processes, most methods have been developed for lumped parameter processes (e.g., (Himmelblau, 1978; Frank, 1990; Kresta *et al.*, 1991; Dunia and Qin, 1998; Tatara and Cinar, 2002; Aradhye *et al.*, 2002; Simani *et al.*, 2003; Cheng *et al.*, 2003; Mehranbod *et al.*, 2005)). At this stage, a unified framework for the integration of fault diagnosis and fault-tolerant control for nonlinear distributed processes remains lacking, thus limiting the achievable control quality and reliability in transport-reaction process operation. To address this problem, we recently developed in (El-Farra, 2006) a model-

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based fault-tolerant control architecture for distributed processes modeled by nonlinear parabolic PDEs with control constraints. The architecture integrates fault detection, feedback and supervisory control on the basis of appropriate reduced-order models that capture the dominant process dynamics. Using singular perturbations techniques, appropriate fault detection thresholds and actuator reconfiguration criteria were establish to guard against false alarms when the control architecture is implemented on the process.

In addition to nonlinearities and spatial variations, another important issue that must be accounted for in the design of model-based fault-tolerant control systems is the presence of plant-model mismatch. Parabolic PDE systems that model transport-reaction processes are uncertain due to the presence of unknown or partially known process parameters and time-varying exogenous disturbances. In the context of fault-tolerant control, the impact of model uncertainty permeates all layers of the control architecture. Within the feedback control layer, for example, the uncertainty alters the stability regions of the nominal controllers and may even render the closed-loop system unstable if not accounted for explicitly in the controller design. Uncertainty can also induce false detection alarms that in turn trigger unnecessary control system reconfiguration leading to closed-loop instability or significant performance degradation.

To address these problems, we develop in this work an integrated robust fault detection and fault-tolerant control (FD-FTC) architecture for transport-reaction processes modeled by systems of nonlinear parabolic PDEs with control constraints, uncertain variables and actuator faults. The central idea is to shape the healthy closed-loop performance, via robust feedback control, in a way that facilitates the design of robust fault detection rules that are less sensitive to the uncertainty. The rest of the paper is organized as follows. Following some preliminaries in Section 2, the robust FD-FTC problem is formulated in Section 3. In Section 4, Galerkins method is used to obtain an approximate, finite-dimensional system that captures the dominant dynamic characteristics of the PDE system. The approximate model is then used in Section 5 to design the FD-FTC structure, including a family of robust feedback controllers with well-characterized stability regions, a performance-based fault detection scheme, and a stability-based supervisory control system. The results are applied to a non-isothermal tubular reactor in Section 6. Due to space limitations, the proofs of the main results will be omitted (see the full version in (Ghantasala and El-Farra, 2007) for the proofs and for a more in-depth discussion of the results).

2. PRELIMINARIES

We focus on transport-reaction processes described by systems of quasi-linear parabolic PDEs of the form:

$$\frac{\partial \bar{x}}{\partial t} = A\frac{\partial^2 \bar{x}}{\partial z^2} + B\frac{\partial \bar{x}}{\partial z} + f(\bar{x}) + W(\bar{x}, d(z)\theta(t)) + \omega b^k(z)[u^k(t) + f_a^k(t)] \quad (1)$$

$$|u^k(t)| \leq u^k_{\max}, k(t) \in \mathcal{K}, |\theta(t)| \leq \theta_b, \quad (2)$$

where $\mathcal{K} := \{1, 2, \cdots, N\}, N < \infty$, subject to the boundary and initial conditions:

$$C_i\bar{x}(\eta_i, t) + D_i\frac{\partial \bar{x}}{\partial z}(\eta_i, t) = 0, i = 1, 2 \quad (3)$$
$$\bar{x}(z, 0) = \bar{x}_0(z)$$

where $\bar{x}(z, t) \in \mathbb{R}^n$ denotes the vector of state variables, $z \in [\eta_1, \eta_2] \subset \mathbb{R}$ is the spatial coordinate, $t \in [0, \infty)$ is the time, $f(\cdot)$ is a nonlinear function, $W(\bar{x}, d(z)\theta(t))$ is a nonlinear vector function, $\theta(t) \in \mathbb{R}^q$ denotes the vector of uncertain variables, which may include uncertain process parameters or exogenous disturbances, $d(z)$ is a known smooth vector function that specifies the positions of action of the uncertain variables, u^k denotes the vector of constrained manipulated inputs (control actuators) associated with the k-th control configuration, $b^k(z)$ is a known smooth vector function that describes how the control action is distributed in $[\eta_1, \eta_2]$, $f_a^k \in \mathbb{R}^m$ denotes the faults in the actuators of the k-th control configuration, $k(t)$ is a discrete variable that takes values in a finite set $\mathcal{K}$ and denotes which control configuration is active at any given time, $|\cdot|$ is the standard Euclidean norm, $u^k_{\max}$ is a positive real number that captures the size of the constraints, θ_b is a positive real number that captures the size of the uncertainty, A, B, C_i, D_i are constant matrices with A positive-definite, and $\bar{x}_0(z)$ is a smooth function of z.

For a precise characterization of the class of PDEs considered, we use standard operator theory to rewrite the PDE of Eqs.1-3 as an infinite-dimensional system of the general form:

$$\dot{x} = \mathcal{A}x + \mathcal{B}^k(u^k + f_a^k) + f(x) + \mathcal{W}(x)\theta \quad (4)$$

where $x(t)$ is the state function defined on an appropriate Hilbert space, $\mathcal{A}$ is the differential operator, $\mathcal{B}$ is the input operator, $f(x)$ is locally Lipschitz and satisfies $f(0) = 0$, and $x(0) = x_0 = \bar{x}_0(z)$. For $\mathcal{A}$, the eigenvalue problem is defined as: $\mathcal{A}\phi_j = \lambda_j\phi_j,\ j = 1, \ldots, \infty$, where λ_j denotes an eigenvalue and ϕ_j denotes an eigenfunction. The eigenspectrum of $\mathcal{A}$, $\sigma(\mathcal{A})$, is defined as the set of all eigenvalues of $\mathcal{A}$. For the majority of diffusion-convection-reaction processes (Ray, 1981; Christofides, 2001), the eigenspectrum is discrete and ordered. Also, for parabolic PDEs (Friedman, 1976), the eigenspectrum of $\mathcal{A}$ can be partitioned as $\sigma(\mathcal{A}) = \sigma_1(\mathcal{A}) \bigcup \sigma_2(\mathcal{A})$, where $\sigma_1(\mathcal{A})$ consists of the first m slow (possibly unstable) eigenvalues, i.e., $\sigma_1(\mathcal{A}) = \{\lambda_1, \ldots, \lambda_m\}$ (with m finite), $|Re\{\lambda_1\}|/|Re\{\lambda_m\}| = O(1)$, and $\sigma_2(\mathcal{A})$ is a stable infinite complement containing the remaining fast eigenvalues, i.e., $Re\{\lambda_{m+1}\} < 0$. Furthermore, the separation between the slow and fast eigenvalues of $\mathcal{A}$ is large, i.e., $|Re\{\lambda_m\}|/|Re\{\lambda_{m+1}\}| = O(\epsilon)$ where $\epsilon < 1$ is a small positive number. These properties imply that the dominant dynamics of the system of Eq.4 can be approximated by a finite-dimensional system, and motivate the use of Galerkins method in Section 4 to derive a reduced-order model that captures the dominant (slow) dynamics of the PDE system.

3. PROBLEM FORMULATION AND OVERVIEW SOLUTION METHODOLOGY

Consider the system of Eq.4 where the uncertain variables are vanishing, in the sense that they do not alter the nominal equilibrium solution. Of the N distinct control actuator configurations available, only one is to be active for control at any given time, while the rest are kept dormant as backup. Each configuration is characterized by a distinct actuator spatial placement. The problems under consideration include how to suppress the effect of uncertainty, how to detect faults in the operating actuator configuration under uncertainty, and, upon detection, how to decide which fall-back actuator configuration should be activated to maintain robust closed-loop stability and achieve fault-tolerance. To address these problems, we proceed as follows. Initially, model reduction techniques are used to obtain an approximate finite-dimensional system that captures the dominant dynamics of the infinite-dimensional system of Eq.4. The reduced-order model is then used to: (1) synthesize, for each actuator configuration, a bounded robust nonlinear feedback controller with well-characterized stability and performance properties, and (2) design a robust fault detection scheme that exploits the uncertainty decoupling capabilities of the controller to detect destabilizing and/or performance-deteriorating faults by comparing the actual evolution of the system with the expected fault-free behavior. Finally, a switching law is devised to orchestrate actuator reconfiguration in a way that respects control constraints and maintains robust closed-loop stability. To simplify the presentation of our results, we focus only on the state feedback problem.

4. MODEL REDUCTION

Let $\mathcal{H}_s$, $\mathcal{H}_f$ be modal subspaces of $\mathcal{A}$, where $\mathcal{H}_s$ is spanned by the first m eigenfunctions and $\mathcal{H}_f$ is spanned by the remaining ones. Defining the orthogonal projection operators, P_s and P_f, such that $x_s = P_s x$, $x_f = P_f x$, the state of the system of Eq.4 can be decomposed as $x = x_s + x_f$. Applying P_s and P_f to the system of Eq.4 and using the decomposition of x, the system of Eq.4 can be decomposed as:

$$\begin{aligned} \dot{x}_s &= F_s(x_s, x_f) + \mathcal{B}_s^k(u^k + f_a^k) + \mathcal{W}_s(x_s, x_f)\theta \quad (5)\\ \dot{x}_f &= F_f(x_s, x_f) + \mathcal{B}_f^k(u^k + f_a^k) + \mathcal{W}_f(x_s, x_f)\theta \end{aligned}$$

where $x_s(0) = P_s x_0$, $x_f(0) = P_f x_0$, $F_s(x_s, x_f) = \mathcal{A}_s x_s + f_s(x_s, x_f)$, $\mathcal{A}_s = P_s\mathcal{A}$ is an $m \times m$ diagonal matrix of the form $\mathcal{A}_s = \text{diag}\{\lambda_j\}$, $\mathcal{B}_s = P_s\mathcal{B}$, $f_s = P_s f$, $\mathcal{W}_s = P_s\mathcal{W}$, $F_f(x_s, x_f) = \mathcal{A}_f x_f + f_f(x_s, x_f)$, $\mathcal{A}_f = P_f\mathcal{A}$ is an unbounded differential operator which is exponentially stable, $\mathcal{B}_f = P_f\mathcal{B}$, $f_f = P_f f$ and $\mathcal{W}_f = P_f\mathcal{W}$. In the remainder of the paper, we will refer to the x_s- and x_f-subsystems as the slow and fast subsystems, respectively. Neglecting the fast and stable x_f-subsystem in Eq.5, the following approximate, m-dimensional slow system is obtained:

$$\dot{\bar{x}}_s = F_s(\bar{x}_s, 0) + \mathcal{B}_s^k(u^k + f_a^k) + \mathcal{W}_s(\bar{x}_s, 0)\theta \quad (6)$$

where the bar symbol in $\bar{x}_s$ denotes that this variable is associated with a finite-dimensional system.

5. REDUCED-ORDER MODEL-BASED DESIGN OF ROBUST FD-FTC STRUCTURE

Having obtained a finite-dimensional system that approximates the dominant dynamics of the infinite-dimensional system, we proceed in this section to describe the design of the various components of the robust FD-FTC architecture.

5.1 *Robust feedback controller synthesis*

The objectives of this step are to: (a) synthesize, for each actuator configuration, a feedback controller that enforces constraint satisfaction and robust stability with an arbitrary degree of attenuation of the effect of uncertainty on the closed-loop system, and (b) explicitly characterize the robust stability region associated with each controller in terms of the constraints, the size of uncertainty and the actuator locations. While several designs can be used to achieve these objectives, we consider, for the sake of a concrete illustration, the following bounded control law introduced in (El-Farra and Christofides, 2003):

$$u^k = -k_r(\bar{x}_s, u_{\max}^k, \theta_b, \xi^k, \chi, \phi)(L_{\mathcal{B}_s^k}V)^T \quad (7)$$

where

$$k_r(\cdot) = \frac{\alpha(\bar{x}_s) + \sqrt{\alpha^2(\bar{x}_s) + (u_{\max}^k\beta(\bar{x}_s, \xi^k))^4}}{\beta^2(\bar{x}_s, \xi^k)\left[1 + \sqrt{1 + (u_{\max}^k\beta(\bar{x}_s, \xi^k))^2}\right]} \quad (8)$$

$$\alpha(\cdot) = L_{F_s}V + (\rho\|\bar{x}_s\| + \chi\theta_b\|L_{\mathcal{W}_s}V\|)\left(\frac{\|\bar{x}_s\|}{\|\bar{x}_s\| + \phi}\right),$$

$\beta(\cdot) = \|(L_{\mathcal{B}_s^k}V)^T\|$, V is a robust control Lyapunov function for the system of Eq.6, $L_{\mathcal{B}_s^k}V$ and $L_{\mathcal{W}_s}V$ are row vectors whose components are the Lie derivatives of V along the columns of $\mathcal{B}_s^k$ and $\mathcal{W}_s^k$, respectively, θ_b is the bound on the uncertainty, and ρ, χ and ϕ are adjustable parameters that satisfy $\rho > 0$, $\chi > 1$ and $\phi > 0$. Let $\Pi(\theta_b, u_{\max}^k, \xi^k)$ be the set defined by:

$$\Pi := \{\bar{x}_s : \alpha(\bar{x}_s, \rho, \chi, \phi, \theta_b) \leq u_{\max}^k\beta(\bar{x}_s, \xi^k)\} \quad (9)$$

and consider the subset:

$$\bar{\Omega}_s(\theta_b, u_{\max}^k, \xi^k) := \{\bar{x}_s : V(\bar{x}_s) \leq c_{\max}^k\} \subseteq \Pi \quad (10)$$

for some $c_{\max}^k > 0$. Proposition 1 that follows characterizes the closed-loop stability properties of the controller of Eqs.7-8.

Proposition 1: *Consider the closed-loop system of Eqs.6-8, for a fixed $k \in \mathcal{K}$, with $f_a^k(t) \equiv 0$ and $\bar{x}_s(0) \in \bar{\Omega}_s(\theta_b, u_{\max}^k, \xi^k)$. Then there exists a positive real number, ϕ^*, such that if $\phi \leq \phi^*$, the origin of the closed-loop system is exponentially stable, i.e., there exists $k_1 > 1$, $k_2 > 0$ such that $\|\bar{x}_s(t)\| \leq k_1\|\bar{x}_s(0)\|e^{-k_2 t} := \beta(\|\bar{x}_s(0)\|, t)$, for all $t \geq 0$.*

Remark 1: The family of control laws in Eqs.7-8 share the same structure but differ in where the control action is applied in the spatial domain. Owing to the dependence of the control action on the actuator locations and the size of the uncertainty, the region of robust stability (i.e., the set of feasible initial conditions that can be steered to the origin under uncertainty and constraints) is parameterized not only by the size of the control constraints, but also by the size of the uncertainty and the locations of control actuators. This parametrization implies that $\bar{\Omega}_s^k$ can be interpreted in several ways. For example, for a given initial condition and magnitude of the uncertain variables, $\bar{\Omega}_s^k$ can be used to determine the set of admissible actuator locations. Alternatively, for fixed actuator locations and size of the uncertain variables, $\bar{\Omega}_s^k$ describes the feasible initial conditions. Furthermore, $u_{\max}^k$, θ_b, ξ^k, can be viewed as "tuning parameters" that control the size of the stability region. For example, inspection of Eqs.9-10 reveals that the tighter the constraints and/or the larger the uncertainty, the smaller the robust stability region. Finally, Eqs.9-10 point to a tradeoff between the size of the stability region (which can be enlarged using small values for the controller tuning parameters χ and ρ), and the controller's robust performance (which requires large values of χ and ρ to achieve the desired degree of uncertainty attenuation). This tradeoff can be managed by proper selection of the actuator locations.

Remark 2: Beyond suppressing the effect of uncertainty and enforcing robust closed-loop stability, an important feature of the robust controllers of Eqs.7-8 is that they provide an explicit characterization of the expected behavior of the closed-loop system in the absence of faults. This characterization, which can be obtained directly from Lyapunov analysis, is expressed in terms of a time-varying bound that captures the evolution of closed-loop state under healthy actuation. As explained in the next section, this feature will facilitate the design of a robust fault detection scheme.

5.2 *Robust performance-based fault detection*

The basic idea in any fault detection scheme is to compare the actual behavior of the monitored system with the behavior expected in the absence of faults and to use the discrepancy between the two, if any, as an indicator of faults. In the absence of uncertainty, the expected behavior can be obtained using a filter that simulates the healthy behavior of the closed-loop system; and the residual in this case is sensitive only to the faults. In the presence of uncertainty, however, replicating the behavior of the uncertain system of Eq.6 is not feasible. Furthermore, unless the filter is re-designed to achieve uncertainty decoupling (which is a difficult task for nonlinear systems), the residual will be sensitive to both the uncertainty and the faults, thus leading to possible false detection alarms.

To achieve robust fault detection and prevent false alarms, we follow a performance-based approach instead. The key idea is to exploit the stability and performance properties of the robust controllers designed in Section 4.1 to characterize the expected healthy behavior and derive a suitable criterion for fault detection. Specifically, we recall from Section 4.1 that, in the absence of faults, the robust controllers of Eqs.7-8 force the decay of the Lyapunov function, V, along the trajectories of the closed-loop uncertain system, according to a well-defined rate. Therefore, deviation from this behavior is an indication that a fault has occurred. This idea is formalized in Proposition 2.

Proposition 2: *Consider the approximate, finite dimensional closed-loop system of Eqs.6-8, for a fixed $k \in \mathcal{K}$, with $\bar{x}_s(0) \in \bar{\Omega}_s^k$, and $\phi \leq \phi^*$, where ϕ^* was defined in Proposition 1. Then if either $\dot{V}(T_d) \geq 0$ or $\|\bar{x}_s(T_d)\| > \beta(\|\bar{x}_s(0)\|, T_d)$, for some $T_d > 0$, where $\beta(\cdot,\cdot)$ was defined in Proposition 1, then $f_a(T_d) \neq 0$.*

Remark 3: The above rule-based fault detection scheme ensures that any fault that negatively impacts either stability or performance is detected. The condition $\dot{V}(T_d) > 0$ detects destabilizing faults that cause an increase in the overall system's energy, while the criterion $\|\bar{x}_s(T_d)\| > \beta(\|\bar{x}_s(0)\|, T_d)$ detects faults that cause a deterioration in the response speed in excess of the minimum allowable speed enforced by the controller. Note that both conditions are needed. For example, if only the first condition is used, faults that slow down the system response (but do not increase V) will go undetected. On the other hand, if only the second condition is used, then faults that cause an increase in V may go undetected for some time. The reason is that, because of the presence of uncertainty and constraints, only a lower (worst-case) bound on the closed-loop response speed can be obtained (this corresponds to an upper bound on the response itself; note, however, that these bounds can be tightened through proper controller tuning). Therefore, if the actual closed-loop uncertain system evolves such that $\|\bar{x}_s\|$ decays at a rate faster than the minimum prescribed, e.g., $\|\bar{x}_s(t)\| \leq \beta_f(\|\bar{x}_s(0)\|, t) < \beta(\|\bar{x}_s(0)\|, t)$, then an increase in V will cause $\|\bar{x}_s\|$ to cross the threshold $\beta(\|\bar{x}_s(0)\|, t)$ after some time; hence the detection delay. The timely detection of faults enhances the ability of the control system to recover from failures through actuator reconfiguration.

Remark 4: It should be noted that only faults that do not cause either an increase in V or a deterioration in the minimum response speed will go undetected. Such faults, however, do not harm stability or performance and therefore require no corrective action. Note also that this detection scheme can be used to detect both partial and complete failures, as well as faults that do not necessarily appear in the control actuators, as long as they influence the evolution of the states.

5.3 *Robust stability-based actuator reconfiguration*

Following fault detection, the supervisor needs to determine which of the available backup configurations can be activated to maintain robust closed-loop stability. Theorem 1 below describes how the fault detection and control reconfiguration rules are integrated to ensure fault-tolerance in the closed-loop reduced system.

Theorem 1: *Consider the approximate, finite dimensional closed-loop system of Eqs.6-8 with $k(0) = j \in \mathcal{K}$, $\phi \leq \phi^*$ and $\bar{x}_s(0) \in \bar{\Omega}_s^j$. Let $T_d^j := \min\{t : \dot{V}(t) \geq 0 \text{ or } \|\bar{x}_s(t)\| > \beta(\|\bar{x}_s(0)\|, t)\}$ be the earliest time that a fault is detected, then the switching rule:*

$$k(t) = \left\{ \begin{array}{ll} j, & 0 \le t < T_d^j \\ \nu \neq j, & t \ge T_d^j, \bar{x}_s(T_d^j) \in \bar{\Omega}_s^\nu \end{array} \right\} \quad (11)$$

exponentially stabilizes the closed-loop system.

Remark 5: The switching law of Eq.11 ensures that the fall-back actuator configuration that is activated and implemented following fault detection is one that guarantees closed-loop stability in the presence of uncertainty and constraints. This is accomplished by choosing a configuration whose robust stability region contains the state at the time of switching. In the event that more than one fall-back configuration satisfies this condition, additional performance criteria (e.g., control effort) could be introduced to further discriminate between the candidate control configurations. Early detection of a fault enhances the chances of taking corrective action. If a fault is not detected in a timely manner, its destabilizing effect could drive the state outside the stability regions of all the backup configurations before the supervisor can take action. In this case, stability cannot be preserved and a process shutdown is unavoidable. Enlarging the stability regions (by adjusting $u_{\max}^k$, ξ_a^k) and/or increasing N, if possible, helps minimize this possibility.

Remark 6: Owing to the characteristic, two time-scale separation between the slow and fast eigenvalues of the spatial differential operator of the PDE system of Eq.1, the FD-FTC architecture designed on the basis of the reduced-order model continues to enforce the desired robustness and fault-tolerance properties when implemented on the actual process (infinite-dimensional system), provided that the separation between the eigenmodes is sufficiently large. This fact can be established rigorously using singular perturbation techniques (see (Ghantasala and El-Farra, 2007)).

6. APPLICATION TO A NON-ISOTHERMAL TUBULAR REACTOR WITH RECYCLE

We consider a non-isothermal tubular reactor where an irreversible first-order reaction takes place. The reaction is exothermic and a cooling jacket is used to remove heat from the reactor. The outlet of the reactor is fed to a separator where the unreacted species is separated from the product and then fed back to the reactor through a recycle loop. The dimensionless process model is given by:

$$\begin{aligned} \frac{\partial \bar{x}_1}{\partial t} &= -\frac{\partial \bar{x}_1}{\partial z} + \frac{1}{Pe_T}\frac{\partial^2 \bar{x}_1}{\partial z^2} + B_T B_C e^{\frac{\gamma \bar{x}_1}{1+\bar{x}_1}}(1+\bar{x}_2) \\ &+ R_1(r, \bar{x}_{1f}, \bar{x}_1(1,t)) + \beta_T(b(z)u(t) - \bar{x}_1) \\ \frac{\partial \bar{x}_2}{\partial t} &= -\frac{\partial \bar{x}_2}{\partial z} + \frac{1}{Pe_C}\frac{\partial^2 \bar{x}_2}{\partial z^2} - B_C e^{\frac{\gamma \bar{x}_1}{1+\bar{x}_1}}(1+\bar{x}_2) \\ &+ R_2(r, \bar{x}_{2f}, \bar{x}_2(1,t)) \end{aligned}$$

subject to the boundary conditions:

$$\begin{aligned} \frac{\partial \bar{x}_1(0,t)}{\partial z} &= Pe_T \bar{x}_1(0,t), \frac{\partial \bar{x}_1(1,t)}{\partial z} = 0 \\ \frac{\partial \bar{x}_2(0,t)}{\partial z} &= Pe_C \bar{x}_2(0,t), \frac{\partial \bar{x}_2(1,t)}{\partial z} = 0 \end{aligned}$$

where $\bar{x}_1$ and $\bar{x}_2$ denote dimensionless temperature and reactant concentration in the reactor, respectively, $\bar{x}_{1f}$ and $\bar{x}_{2f}$ denote dimensionless inlet temperature and inlet reactant concentration, respectively, Pe_T and Pe_C are the heat and mass Peclet numbers, respectively, B_T and B_C denote a dimensionless heat of reaction and a dimensionless pre-exponential factor, respectively, r is the recirculation coefficient ($r = 1$ corresponds to total recycle with zero fresh feed, and $r = 0$ corresponds to no recycle), γ is a dimensionless activation energy, β_T is a dimensionless heat transfer coefficient, $R_i(r, \bar{x}_{if}, \bar{x}_i(1,t)) = \delta(z-0)((1-r)\bar{x}_{if} + r\bar{x}_i(1,t))$, $i = 1, 2$, $\delta(\cdot)$ is the standard Dirac function, u is a dimensionless jacket temperature (chosen to be the manipulated input), and $b(z)$ is the actuator distribution function. For $Pe_T = Pe_C = 7.0$, $B_C = 0.1$, $B_T = 2.5$, $\beta_T = 2.0$, $\gamma = 10.0$, $r = 0.5$, $\bar{x}_{1f} = \bar{x}_{2f} = 0$, it can be verified that the operating, open-loop steady-state is unstable (the linearization around the steady-state possesses one real unstable eigenvalue and infinitely many stable eigenvalues).

The control problem is to stabilize the reactor at a spatially-nonuniform steady-state where the product yield is desirable and the hot-spot temperature is acceptable, by manipulating the jacket temperature, $u(t)$, which is subject to hard constraints and possible actuator failures. The control objective is to be achieved in the presence of time-varying uncertainty in the heat of reaction, i.e. $B_T - B_{T_{\mathrm{nom}}} = \theta(t)$, which, for the purpose of simulations, is taken to be of the form $\theta(t) = \theta_b \sin(t)$ with an upper bound on the uncertainty of $\theta_b = 0.1 B_{T_{\mathrm{nom}}}$ (note that any other time-varying bounded function can be used to simulate the effect of uncertainty). To achieve the control objective, two point control actuators, $(\xi_A = 0, u_{\max}^A = 0.06)$, $(\xi_B = 0.1, u_{\max}^B = 0.06)$, are assumed to be available with A being the primary actuator and B serving as fall-back. Furthermore, we assume that a sufficient number of point measurements of the temperature along the reactor are available so that it is known with sufficient accuracy.

For this system, we consider the first temperature mode as the dominant one, and use Galerkins method to derive an ODE that describes the temporal evolution of the amplitude of the first eigenmode. This ODE is then used for the synthesis of the controllers, the characterization of the stability regions and the design of the fault detection scheme. The synthesis details are omitted due to space limitations. In all simulation runs, the FD-FTC structure is implemented on a 400-th order Galerkins discretization of the PDE model (higher order discretizations led to identical results). It was verified that the controller successfully stabilizes the closed-loop system at the desired steady-state and suppresses the effect of uncertainty when actuator A is used without failures for all times.

To test the efficacy of the proposed FD-FTC structure, failure is introduced into actuator A at $t = 0.25$ (see dashed line in Fig.1(d)). Fig.1(c) shows that shortly after this failure, the Lyapunov function (chosen to

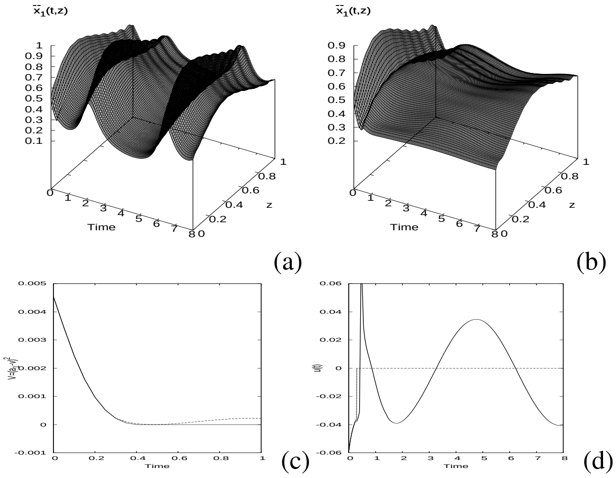

Fig. 1. Top: Reactor temperature profiles when actuator A fails at $t = 0.25$ and (a) no failure compensation takes place, and (b) actuator B is activated. Bottom: (c) Evolution of the Lyapunov function in the presence (dashed) and absence (solid) of faults, and (d) the manipulated input profiles for actuator A (dashed) and actuator B (solid).

be a quadratic function of the first temperature mode) ceases to decrease and begins to increase (dashed line) relative to the expected evolution of V in the absence of faults (solid line). From the result of Proposition 1, this behavior is an indication of a diminished control authority and is used to declare the failure of actuator A at $t = 0.46$ (note that the slight detection delay is a consequence of using only the Lyapunov decay rate condition as the detection criterion – see Remark 3). Robust stability is then preserved by activation of the backup actuator B (see the solid line in Fig.1(d)) whose stability region was verified to contain the first temperature mode at the time of fault detection. The result is depicted in Fig.1(b) which shows that, with timely fault detection and reconfiguration, closed-loop stability of the desired steady-state can be successfully preserved in the presence of uncertainty and faults. For comparison purposes, Fig.1(a) shows the loss of stability when actuator A fails at $t = 0.25$ and no corrective action is taken.

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