EFFECT OF EXCITATION FREQUENCY IN PERTURBATION-BASED EXTREMUM SEEKING METHODS

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Abstract: In perturbation-based extremum-seeking methods, an excitation signal is added to the input, and the gradient, computed from the correlation between the input and output variations, is forced to zero. It is shown here that the distance between the optimum and solution reached by the perturbation method is proportional to the square of the frequency of excitation and does not go to zero with the amplitude of the excitation. However, for Wiener/Hammerstein approximations, the error will indeed go to zero with the excitation amplitude. Simulation results on a simple reaction system are used to illustrate the concepts presented in this work.

Keywords: Real-time Optimization, Extremum seeking, Perturbations Method, Reactor control, Chemical Reactors

1. INTRODUCTION

Real-time optimization has seen a resurgence of interest in the recent years. The traditional approach is the model-based repeated optimization where the model is adapted using the available measurements and numerical optimization is performed on the updated model (Marlin and Hrymak 1996, Zhang *et al.* 2002).

An alternative approach to real-time optimization known as the "extremum seeking" allows treating the optimization problem as a *control problem.* The optimization problem becomes one of regulating the norm of the gradient at zero. Controlling a system at a point with zero steady state gain is closely related to dual adaptive control (Allison *et al.* 1995) and probing control (Velut and Hagander 2004). The crucial point is the computation of this gradient for which either an adapted model of the system is used for analytical evaluation (Guay and Zhang 2003), or the system is perturbed in order to numerically compute the gradient (Krstic and Wang 2000, Aryur and Krstic 2003). These methods have both been successively applied to the on-line optimization of (bio-)chemical processes (Guay *et al.* 2004, Wang *et al.* 1999).

This paper deals with the extremum-seeking methods based on perturbations. The renewed popularity of perturbation-based methods (Blackman 1962) is mainly due to the publication of (Krstic and Wang 2000) where a formal proof of convergence has been established. Therein, it has been shown that the system on the average converges to a neighborhood of the optimum, the size of this neighborhood being determined by the amplitude of the excitation signal. However, not much has been said on the dependence of size of this neighborhood on the frequency of excitation since the system is assumed to be quasi-static compared to the excitation frequency.

In this paper, the frequency dependence of the neighborhood is quantified, and it is shown that for a general nonlinear system its size is proportional to the square of the excitation frequency. The implication of this result is that even when the amplitude of the excitation signal goes to zero, the neighborhood will not shrink to zero. On the other hand, this neighborhood does indeed shrink to zero with the amplitude of the excitation signal when Wiener/Hammerstein models are used.

The paper is organized as follows: The next section introduces the perturbation method for extremum seeking. Section 3 presents the main results of this paper regarding the dependence of the averaged solution on the excitation frequency. A simple example is presented in Section 4 and Section 5 concludes the paper.

2. EXTREMUM SEEKING USING PERTURBATIONS

The problem addressed is the steady-state optimization of a nonlinear dynamic system as stated below:

$$\min_{\theta} J(x,\theta) \qquad (1)$$
$$s.t.\ \dot{x} = F(x,\theta) \equiv 0$$

where $x \in \mathbb{R}^n$ is the state, $\theta \in \mathbb{R}^m$ is the control input, $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a smooth function describing the dynamics and $J : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ the objective function.

To solve this optimization problem online, the following extremum-seeking controller is derived from the necessary conditions of optimality, under the assumption that the function J is convex.

$$\dot{\theta} = k\frac{dJ}{d\theta} = k\left(\frac{\partial J}{\partial \theta} - \frac{\partial J}{\partial x}\left(\frac{\partial F}{\partial x}\right)^{-1}\frac{\partial F}{\partial \theta}\right) \qquad (2)$$

The main challenge of extremum-seeking is the gradient estimation and the perturbation based schemes add an excitation signal to the input in order to extract this information (Fig 1). Note that the objective function is supposed to be directly measured ($y = J(x,\theta)$).

A high pass filter with cutoff frequency ω_h isolates the variations of this optimized variable from its average value. The state that represents the high pass filter is denoted by η. This signal is then modulated by the same excitation signal. A low pass filter with cutoff frequency ω_l and output ξ will filter the resulting signal in order to get the required gradient, $\xi = \frac{dJ}{d\theta}$. Finally, an integral controller with gain k drives this estimated gradient to zero.

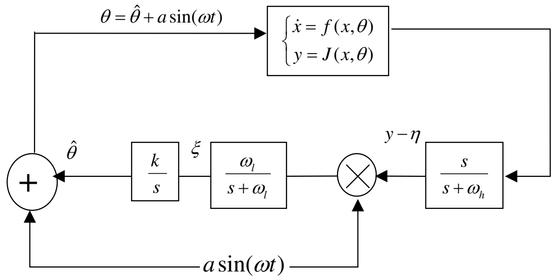

Fig. 1. Extremum seeking control via perturbation method inspired from (Krstic and Wang 2000).

The scheme can be summarized using the following equations:

$$\dot{\hat{\theta}} = k\xi, \qquad \theta = \hat{\theta} + a\sin(\omega t) \qquad (3)$$
$$\dot{\xi} = -\omega_l\xi + \omega_l(y-\eta)a\sin(\omega t) \qquad (4)$$
$$\dot{\eta} = -\omega_h\eta + \omega_h y \qquad (5)$$

The value of the states at steady-state obtained from $\dot{x} = F(x,\theta) \equiv 0$ is given by $x = l(\theta)$. Then, the cost function at steady-state is given by $y = J(l(\theta),\theta)$.

Next the deviation variables are defined: $\tilde{\theta} = \hat{\theta} - \theta^*$, $\tilde{y} = y - y^*$ and $\tilde{\eta} = \eta - y^*$. Then, the relationship between $\tilde{y}$ and $\tilde{\theta}$ is expressed as $\tilde{y} = J(l(\theta^* + \tilde{\theta}), \theta^* + \tilde{\theta}) - J(l(\theta^*),\theta^*) \equiv \nu(\tilde{\theta})$.

Assuming that x is at steady state, the averaged system for the three remaining variables (θ, ξ, and η) is obtained by taking the average of the right hand side over $[0, \frac{2\pi}{\omega}]$. The averaged states are denoted by the superscript $(\cdot)^a$. The averaged system reads (Khalil 2002):

$$\frac{d}{dt}\begin{bmatrix}\tilde{\theta}^a\\ \xi^a\\ \tilde{\eta}^a\end{bmatrix} = \begin{bmatrix} k\xi^a \\ -\omega_l\xi^a + \frac{\omega_l\omega}{2\pi}a\int_0^{\frac{2\pi}{\omega}}\nu(\tilde{\theta})\sin(\omega t)dt \\ -\omega_h\tilde{\eta}^a + \frac{\omega_h\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\nu(\tilde{\theta})dt\end{bmatrix} \qquad (6)$$

where $\tilde{\theta} = \tilde{\theta}^a + a\sin(\omega t)$.

Convergence is established in (Krstic and Wang 2000) through the following steps:

- The exponential stability of the equilibrium point of the above averaged system ($\tilde{\theta}^a$, ξ^a, and $\tilde{\eta}^a$) is first proved.
- From there on, the exponentially stability of (θ, ξ, and η) (non-averaged) is established using the averaging theorem (Khalil 2002).
- This non-averaged system (θ, ξ, and η) acts as the "slow" manifold, while the original system $\dot{x} = F(x,\theta)$ acts as the boundary layer system which is assumed to be exponentially

stable. Then, singular perturbation ideas are applied to show that their interconnection is also exponentially stable (Khalil 2002).

In present work, what is of interest is not the stability properties, but the position of the equilibrium (zero-correlation point). So, the part of the result from (Krstic and Wang 2000) that deals with the position of the equilibrium is repeated in the following proposition.

Proposition 1. (Krstic and Wang 2000) The equilibrium of the averaged model (6) is a function of the amplitude of the excitation signal:

$$\tilde{\theta}^a = -\frac{\nu'''(0)}{8\nu''(0)}a^2 + O(a^3) \tag{7}$$

3. DEPENDENCE OF THE SOLUTION ON THE PERTURBATION FREQUENCY

In the analysis presented in Section 2, it was assumed that x and y are at steady state, i.e. the relationship $\tilde{y} = \nu(\tilde{\theta})$ is considered algebraic. Then, the Taylor series approximation around the origin to the first order is given by:

$$\begin{aligned}\tilde{y} &= \nu(\tilde{\theta}^a + a\sin(\omega t)) \qquad (8)\\ &= \nu(0) + \nu'(0)\tilde{\theta}^a + \nu'(0)\, a\, \sin(\omega t) + O(a^2)\end{aligned}$$

Note that here the constant value $\widetilde{\theta}^a$ and the sinusoidal excitation $a\ \sin(\omega t)$ have the same gain, i.e, the gain is independent of the frequency of excitation. This, however, is true only if the system dynamics are neglected.

In this section, we show dependence of the zero-correlation point on the frequency of the excitation signal, by taking into account the variation of the gain of the system with frequency. For this, instead of a static relationship $\tilde{y} = \nu(\tilde{\theta})$, a dynamic relationship $\tilde{y} = P(\tilde{\theta})$ will be assumed. The averaged system (6) becomes :

$$\frac{d}{dt}\begin{bmatrix}\tilde{\theta}^a\\ \xi^a\\ \tilde{\eta}^a\end{bmatrix} = \begin{bmatrix} k\xi^a \\ -\omega_l\xi^a + \frac{\omega_l\omega}{2\pi}a\int_0^{\frac{2\pi}{\omega}} P(\tilde{\theta})\sin(\omega t)dt \\ -\omega_h\tilde{\eta}^a + \frac{\omega_h\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} P(\tilde{\theta})dt\end{bmatrix} \tag{9}$$

3.1 General nonlinear system

Next, it will be shown that for a general nonlinear system the difference between the optimum and the zero-correlation point is proportional to the square of the frequency.

Theorem 1. The equilibrium of the averaged model given by (9) is a function of the amplitude and the frequency of the excitation signal:

$$\tilde{\theta}^a = \alpha a^2 + \beta\omega^2 + \gamma a^2\omega + \delta\omega^3 + O([\omega\ a]^4) \tag{10}$$

where α, β, γ, and δ are constants that could be computed from the Taylor series development of the dynamic operator P. Also, $O([\omega\ a]^4)$ is used to represent $O([\omega\ a]^4) = O(a^4) + O(\omega^4) + O(\omega^2a^2) + O(\omega a^3) + O(a\omega^3)$.

Proof: The equilibrium of the averaged system (9) corresponds to $\xi^a = 0$, $\int_0^{\frac{2\pi}{\omega}} P(\widetilde{\theta})\sin(\omega t)dt = 0$, and $\tilde{\eta}^a = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} P(\widetilde{\theta})dt$. The value of $\widetilde{\theta}^a$, is obtained from the second condition and is analyzed below:

$$\int_0^{\frac{2\pi}{\omega}} P(\widetilde{\theta}^a + a\sin(\omega t))\sin(\omega t)dt = 0 \tag{11}$$

To compute the integral (11), fourth order Taylor series approximation of $P(\widetilde{\theta})$ around the equilibrium point will be used. The order of truncation is determined in such a manner that the only terms that are of order 4 are neglected.

$$\begin{aligned}P(\tilde{\theta}) &= P(0) + P'(0)\tilde{\theta} + \frac{1}{2}P''(0)\tilde{\theta}^2\\ &+\frac{1}{3!}P^{(3)}(0)\tilde{\theta}^3 + \frac{1}{4!}P^{(4)}(0)\tilde{\theta}^4 + O([\omega\ a]^4)\end{aligned} \tag{12}$$

where $P^{(i)}(0)$ is the i^{th} derivative of P evaluated at the optimum. Note that $P^{(i)}(0)$ are linear operators. Also in the above development, $\sin^j(\omega t)$ should be rewritten in terms of $\sin(j\omega t)$ and $\cos(j\omega t)$. Then, the various terms of (12) can be expressed as follows:

$$P'(0)\widetilde{\theta} = G_{10}\widetilde{\theta}^a + G_{11}\sin(\omega t + \phi_{11})a \tag{13}$$

$$\begin{aligned}P''(0)\widetilde{\theta}^2 &= G_{20}\left(\widetilde{\theta^a}^2 + \frac{a^2}{2}\right)\\ &+G_{21}\sin(\omega t + \phi_{21})\ 2a\widetilde{\theta^a}\\ &-G_{22}\cos(2\omega t + \phi_{22})\frac{a^2}{2}\end{aligned} \tag{14}$$

$$\begin{aligned}P^{(3)}(0)\widetilde{\theta}^3 &= G_{30}(\widetilde{\theta^a}^3 + \frac{3a^2\widetilde{\theta^a}}{2})\\ &+G_{31}\sin(\omega t + \phi_{31})\left(3a\widetilde{\theta^a}^2 + \frac{3a^3}{4}\right)\\ &-G_{32}\cos(2\omega t + \phi_{32})\frac{3a^2\widetilde{\theta^a}}{2}\\ &-G_{33}\sin(3\omega t + \phi_{33})\frac{a^3}{4}\end{aligned} \tag{15}$$

$$\begin{aligned}P^{(4)}(0)\widetilde{\theta}^4 &= G_{40}\left(\widetilde{\theta^a}^4 + 3a^2\widetilde{\theta^a}^2 + \frac{3a^4}{8}\right)\\ &+G_{41}\sin(\omega t + \phi_{41})\left(4a\widetilde{\theta^a}^3 + 3a^3\widetilde{\theta^a}\right)\\ &-G_{42}\cos(2\omega t + \phi_{42})\left(3a^2\widetilde{\theta^a}^2 + \frac{a^4}{2}\right)\\ &-G_{43}\sin(3\omega t + \phi_{43})\ a^3\widetilde{\theta^a}\\ &+G_{44}\cos(4\omega t + \phi_{44})\frac{a^4}{8}\end{aligned} \tag{16}$$

where G_{ij} and ϕ_{ij} are the gain and phase shift caused by $P^{(i)}$ at the frequency $j\omega$, respectively.

Using (12)-(16) in (11) leads to the following equilibrium condition:

$$\sum_{i=1}^{4} G_{i1}\cos(\phi_{i1}) \sum_{j=0}^{floor(\frac{i-1}{2})} \mu_{ij}\widetilde{\theta^a}^{\,i-2j-1} a^{2j} = 0 \tag{17}$$

where μ_{ij} are the coefficients that can be derived from (13)-(16). A third-order expansion of $\widetilde{\theta^a}$ is used:

$$\begin{aligned}\widetilde{\theta^a} = {} & b_0 a + b_1\omega + b_2 a\omega + b_3 a^2 + b_4\omega^2 + \\ & b_5 a^2\omega + b_6 a\omega^2 + b_7 a^3 + b_8\omega^3 + O([\omega\ a]^4)\end{aligned} \tag{18}$$

In addition, G_{ij} and $\cos(\phi_{ij})$ are also developed in terms of ω:

$$\begin{aligned}G_{ij} = {} & G_{ij0} + \omega G_{ij1} + \omega^2 G_{ij2} + \\ & \omega^3 G_{ij3} + O(\omega^4)\end{aligned} \tag{19}$$

$$\begin{aligned}\cos(\phi_{ij}) = {} & C_{ij0} + \omega C_{ij1} + \omega^2 C_{ij2} + \\ & \omega^3 C_{ij3} + O(\omega^4)\end{aligned} \tag{20}$$

Next, $G_{110} = 0$ and $C_{110} = 0$ are used in the development. G_{110} is the static gain of the first derivative which is zero by the definition of the optimum. C_{110} is the cosine of the phase shift caused by G_{11} as $s \to 0$. Let the numerator of the transfer function representing the first derivative be $N(s) = (n_0 + n_1 s + n_2 s^2 + \cdots)$. As $G_{110} = 0$, $n_0 = 0$. Then, s becomes a factor of the numerator and this leads to $\lim_{s\to 0} G_{11}(s) \propto s$. So, G_{11} will have a phase shift of $\frac{\pi}{2}$ as $s \to 0$ and its cosine would be zero.

Solving for the values of b_i gives

$$b_i = 0, \qquad i = \{0, 1, 2, 6, 7\} \tag{21}$$

$$b_3 = -\frac{1}{8}\frac{G_{310}C_{310}}{G_{210}C_{210}} \qquad b_4 = -\frac{G_{111}C_{111}}{G_{210}C_{210}} \tag{22}$$

$$\begin{aligned}b_5 = {} & \frac{C_{310}}{8}\frac{(G_{310}G_{211} - G_{311}G_{210})}{G_{210}^2 C_{210}} \\ & -\frac{G_{310}}{8}\frac{(C_{311}C_{210} + C_{310}C_{211})}{G_{210}C_{210}^2}\end{aligned} \tag{23}$$

$$\begin{aligned}b_8 = {} & G_{111}\frac{(C_{111}C_{211} - C_{112}C_{210})}{G_{210}C_{210}^2} \\ & -C_{111}\frac{(G_{111}G_{211} + G_{112}G_{210})}{G_{210}^2 C_{210}}\end{aligned} \tag{24}$$

Retaining only the non-zero terms, it can be seen that $\alpha = b_3$, $\beta = b_4$, $\gamma = b_5$ and $\delta = b_8$. □

An important consequence of Theorem 1 is that even if $a \to 0$, the zero-correlation point does not go to optimum, the error being a function of ω^2:

$$\lim_{a\to 0}\widetilde{\theta^a} = \beta\omega^2 + \delta\omega^3 + O(\omega^4)$$

On the other hand, since the equilibrium condition (17) has only even powers of a (due to the presence of $2j$ in the exponent), the following result can be deduced.

Lemma 1. In the expansion of $\tilde{\theta}^a$ given by (18), the coefficient of all terms that have an odd exponent of a are zero.

This lemma can be proved by induction, the detailed proof being omitted for the sake of brevity.

3.2 Wiener and Hammerstein models

Wiener or Hammerstein models are widely used to represent nonlinear dynamic systems (Wittenmark and Evans 2001). Such models have linear dynamics and a static nonlinearity. The difference between Wiener and Hammerstein models come from the order in which the linear and nonlinear blocks are placed. The Wiener model consists of a static nonlinearity followed by a linear dynamics, while in the Hammerstein model, the linear dynamics are placed first.

The interesting aspect of these models is that, at the optimum, the gain of the static part, which multiplies the linear dynamics, is zero. So, the overall operator $G_{11} = 0$ for all frequencies. When this fact is used in Theorem 1, it can be seen that the coefficients of ω^2 and ω^3, i.e. β and δ are zeros. This means that $\tilde{\theta}^a = \alpha a^2 + \gamma a^2\omega + O(a^4) + O(\omega^4) + O(\omega^2 a^2) + O(\omega a^3) + O(a\omega^3)$. Also, from Lemma 1, $O(\omega a^3) + O(a\omega^3) = 0$. Next, it will be shown that when $G_{11} = 0$ for all frequencies, $\tilde{\theta}^a$ does not depend on $O(\omega^4)$ either.

Theorem 2. If the nonlinear dynamic system is represented by a Wiener or a Hammerstein model, then, the equilibrium of (9) is given by,

$$\tilde{\theta}^a = \alpha a^2 + \gamma a^2\omega + O(a^2\omega^2) + O(a^4) \tag{25}$$

The above theorem can also be proved by induction, by noting that an addition of a term like ω^i in $\tilde{\theta}^a$ will appear alone in the second derivative and from (17) it should be zero.

The consequence of this result is that for a Wiener or a Hammerstein model the distance between the zero-correlation point and the optimum goes to zero as $a \to 0$,

$$\lim_{a\to 0}\tilde{\theta}^a = 0$$

However, note that even for a Wiener or Hammerstein representation, for $a \neq 0$, the equilibrium point will in fact be affected by the frequency.

Another interesting result can be stated when the static nonlinearity of a Wiener or Hammerstein model is symmetric around the optimum.

Lemma 2. Let the nonlinear dynamic system be represented by a Wiener or a Hammerstein model, wherein the static nonlinearity is an even function of $\tilde{\theta}$. Then, $\tilde{\theta}^a = 0$ is an the equilibrium of (9).

Proof: This result stems from the following facts: (i) The odd derivatives of the static nonlinearity are zero at the origin. (ii) Wiener/Hammerstein representation indicates that the gain for these derivatives is zero at all frequencies. (iii) Removing the odd derivatives, $\tilde{\theta}^a$ becomes a factor of (17). So, $\tilde{\theta}^a = 0$ is a solution. □

This result states that the error comes from the asymmetry of the static nonlinearity. However, note that nothing much can be said if the nonlinearity cannot be represented by a Wiener or Hammerstein model.

The above development shows that, from an optimization perspective, certain nonlinear systems behave differently from their Wiener/ Hammerstein approximations. At the steady-state optimum, the gain is zero for all frequencies with Wiener or Hammerstein approximation. However, at the steady-state optimum of a nonlinear dynamic system, though the static gain is zero, the gain at other frequencies is typically non-zero. This aspect can be captured by representing a nonlinear dynamic system as a sum of two Wiener or Hammerstein models as shown in Fig. 2. The first branch has a zero gain at the static optimum, while the second has nonzero gain.

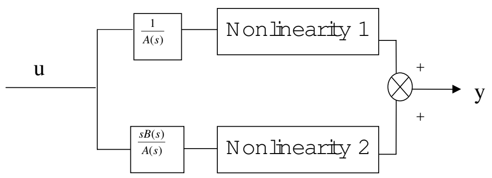

Fig. 2. Approximation of a general nonlinear system using two parallel Hammerstein models.

4. ILLUSTRATIVE EXAMPLE

4.1 Description of the system

A simple isothermal reaction system in a continuous stirred tank reactor (CSTR) will be considered to illustrate the dependence of the zero-correlation point with frequency.

- Reaction system: $A \rightarrow B \rightarrow C$, $2A \rightarrow D$.
- Model equations:

$$\frac{dC_A}{dt} = D(C_{Ain} - C_A) - k_1 C_A - 2k_3 C_A^2$$
$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B - DC_B \qquad (26)$$

- Variables: C_X, concentration of species X; D, Dilution rate; k_i, rate constants; C_{Ain}, inlet concentration.
- Objective: Maximize the concentration of product B, C_B.
- Manipulated variable: D.
- Parameter values: $k_1 = 24$ h^{-1}; $k_2 = 24$ h^{-1}; $k_3 = 0.5$ l mol^{-1} h^{-1}; C_{Ain}= 1 mol l^{-1}.

4.2 Linearization

The system under study is linearized to obtain the following transfer function between C_B and D:

$$G(s) = \frac{C_{B0}s + \alpha}{(s + D_0 + k_1 + 2k_3C_{A0})(s + D_0 + k_2)} \qquad (27)$$

where $\alpha = C_{B0}D_0 + k_1 (C_{B0} + C_{A0} - C_{Ain}) + 2k_3C_{A0}C_{B0}$, and the subscript $(\cdot)_0$ is used to represent the point around which the linearization is performed.

Note that at the optimum α is zero and changes sign. So, this particular system cannot be represented by Wiener or Hammerstein models, and has to be approximated locally by two parallel branches as illustrated in Fig. 2.

4.3 Simulation results

As summarized in Table 1 and Fig. 3 below, several excitation signal frequencies are used and the obtained zero-correlation points are compared to the static optimum dilution rate 24.016 h^{-1}. The interesting point to note is that the error is small until a certain cutoff frequency, and increases drastically when the frequency is larger than this value.

Two scenarios are considered. When the frequency is very high, i.e. 3.16 h^{-1} (Fig. 4), the obtained equilibrium value, 14.566 h^{-1}, is far from the optimum. The frequency of the output signal is the same as the input frequency, and the input and output have a phase lag of $\frac{\pi}{2}$. When the frequency is very low, i.e. 0.05 h^{-1} (Fig. 5) the equilibrium value is relatively close to the optimum. Also, note that the output has a strong "double frequency" component, which is what is expected if the system had been static.

Table 1. Influence excitation frequency on the equilibrium value

Excitation frequency[h^{-1}]	k_{opt}	τ_h [h]	τ_l [h]	Equilibrium [h^{-1}]
0.0500	14.142	80.000	80.000	24.019
0.5000	141.42	8.0000	8.0000	23.875
1.0000	282.84	4.0000	4.0000	23.324
2.0000	565.68	2.0000	2.0000	20.842
3.1600	893.78	1.2658	1.2658	14.585

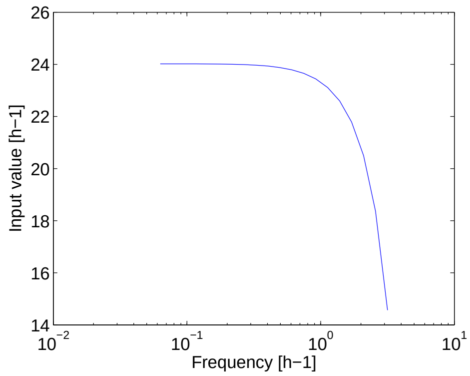

Fig. 3. Evolution of the solution as a function of the excitation signal frequency.

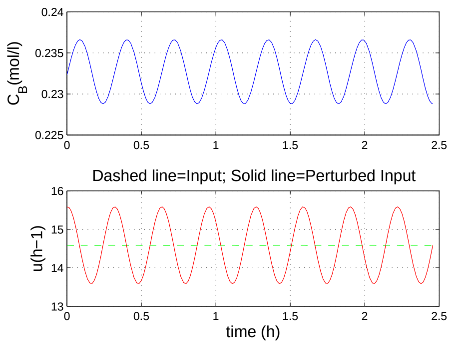

Fig. 4. Input and output signals for a high frequency excitation, $f = 3.16h^{-1}$.

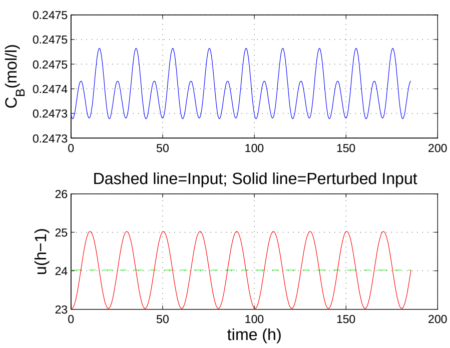

Fig. 5. Input and output signals for a low frequency excitation, $f = 0.05h^{-1}$.

5. CONCLUSIONS

The objective of this work was to show that the perturbation-based extremum-seeking algorithm on the average does not converge to the optimum but only close to it. The error for a general nonlinear dynamic system is proportional not only to the square of the excitation amplitude but also to the square of frequency of excitation. The main point here is that slower optimization frequency is not only required for stability purposes but also for accuracy. With this inference, the frequency of excitation should be low, which in turn makes the optimization slower if accuracy is demanded.

On the other hand, for the particular case of the Wiener/Hammerstein models, the error is always multiplied by the square of amplitude of the excitation signal. So, by choosing a low amplitude for the excitation signal, of course limited by the noise, the effect of frequency could be minimized. Faster convergence can possibly be achieved.

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