8th International IFAC Symposium on
Dynamics and Control of Process Systems
Preprints Vol.1, June 6-8, 2007, Cancún, Mexico

PROFILE CONTROL IN DISTRIBUTED PARAMETER SYSTEMS USING LEXICOGRAPHIC OPTIMIZATION BASED MPC

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Abstract: We have studied control of spatial property in distributed parameter systems using a lexicographic optimization based MPC formulation to prioritize the different sections of the profile. We demonstrate using a hypothetical plug flow reactor that the proposed method has significant benefits when the target profile as a whole is unachievable but parts of which can be satisfied. We have also applied the proposed control strategy for property profile control in a continuous pulp digester of industrial size, which represents a large scale distributed parameter system.

Keywords: distributed parameter system; process control; Kalman filters; model based control; pulp digester

1. INTRODUCTION

Key equipment in large continuous plants typically represent Distributed Parameter Systems (DPS), where the system properties exhibit a significant spatial variation. While most product quality variables are determined by the endpoint properties of the DPS, others may depend on the reaction path assumed during processing. The path dependence is also critical in situations where irreversible phenomena may occur such as catalyst poisoning or gelation. Furthermore, the endpoint itself is a manifestation of the reaction path and a particular path adopted may offer advantages over others. Thus, from an operations perspective, it may be desirable to control not only the endpoint but also the spatial property profile. Examples of DPS include fixed bed reactors, distillation towers and continuous pulp digesters. Despite these advantages of profile control, only the endpoint property is commonly controlled in large-scale distributed parameter systems. For example, control of bottom and top product compositions in distillation column [1, 2] as well as endpoint Kappa number control in a continuous pulp digester [3, 4] have been reported in literature. An obvious advantage of the endpoint property control is that we typically have an online or laboratory measurement of the property of interest and this facilitates its implementation. On the other hand, control of full property profile has received little attention. Despite its advantages, controlling the property profile requires addressing additional challenges. Firstly, the profile must be constructed from available measurements. Secondly, guaranteeing feasibility of the target profile in presence of disturbances is difficult. Thus, while a target endpoint may continue to be achievable for a particular set of disturbances, it is unlikely that the target profile will be achievable for the same set of disturbances. In fact the part of the target profile close to the feed is unlikely to be achieved even for very small perturbations in feed composition.

Reconstruction of the profile may be possible through state estimation. For example, Padhiyar et al. [5] present estimation of the Kappa number profile in the continuous digester using a multi-rate extended Kalman filter. Subsequently, they present control of Kappa number at three different locations along the length of the digester. Doyle and Kayihan [6] show that control of Kappa number at multiple points results in a tightly constrained Kappa number profile. Profile control strategies are also used for control of packed bed reactors to ensure that the reactor hotspot is below the safety limit, which in turn avoids catalyst deactivation [7, 8]. A full profile control of temperature in an FBR has been experimentally demonstrated by [9].

System theoretic properties of DPS and its control have attracted a lot of attention [10]. Broadly, the control approach can be classified into two types. A practical approach includes discretization of the nonlinear PDE followed by synthesis of the controller [11-13]. A drawback of this approach is that system properties such as controllability and observability that depend only on the location of the actuators and sensors, may now depend on the discretization scheme [14]. The other approach is based on maximum principle of Pontryagin principle [15]. In this approach, the controller is synthesized based on the infinite dimensional model of the distributed parameter system. Applicability of this approach is limited to problems of small size and also due to analytical simplifications that may be used. In this work we use the former approach for control of distributed parameter systems. We consider an extended MPC [16] implementation for profile control. When the target profile becomes unachievable, either due to disturbances or input constraints, the controller tuning plays a crucial role in determining the closed loop behaviour. For example, we may provide a higher priority to achieve the endpoint target when the whole profile cannot be achieved. In this work, we propose use of a lexicographic optimisation [17, 18] based MPC to explicitly prioritise the different parts of the profile. Here we split the profiles into sections and solve the MPC control problem as a multi-tiered optimisation, where the different tiers represent priorities of the different parts of the profile. Once an optimal solution is obtained for a particular tier, the next most important objective is optimized by using constraints that ensure that the objective of the preceding tier is maintained at a satisfactory level. This process is continued until the optimal value of the least important objective is achieved. A similar strategy has been implemented in the framework of linear MPC for prioritizing the various goals of the controller such as satisfaction of setpoints and feasibility of constraints [19, 20]. Our work attempts to tailor the lexicographic strategy for use in profile control of distributed parameter systems. For comparison purpose, we also consider the non-lexicographic MPC wherein the full

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profile is controlled using single weighted objective function. We will refer to this strategy as full profile control. The lexicographic optimization based MPC offers significant advantages [19] over profile control when the target profile becomes unachievable by explicitly prioritising the different parts of the profile in a transparent manner. We demonstrate using a simple example that the lexicographic optimisation based MPC spans between endpoint property control and the whole profile control strategies. The added computational expense of solving multiple optimisation problems at each instant is not significant when we use linear MPC or extended MPC where the optimisation problem is a quadratic program.

The paper has been organized as follows. Section 2 introduces a hypothetical plug flow reactor (PFR) example and illustrates the issues pertaining to the available degrees of freedom in the profile control problem. We discuss the three different control law formulations namely, for endpoint control, for profile control using a single objective function, and for profile control with lexicographic method in Section 3. Simulation results for the PFR example are presented in Section 4. To verify the benefits of the lexicographic optimisation based MPC for large-scale distributed parameter systems, we present the profile control in a continuous pulp digester of industrial scale as Section 5 followed by concluding remarks.

2. AN ILLUSTRATIVE EXAMPLE

As a representative DPS, we consider a hypothetical system of a PFR as shown in Fig. 1. The system is approximated by nine CSTRs in series. Reactant A enters the first CSTR and flows down the column to form dimer product P through the following elementary reaction,

$$2A \xrightarrow{k} P \tag{1}$$

Reactant A may also be introduced through trim streams locations in the 4th and 7th CSTRs. We seek to control the profile of the product concentration along the length of the column. The three manipulated variables (MVs), F, F_a, and F_b, can be used to control three independent points along the profile. For example, fixing one concentration from each of the sets $\{C_{p,1}, C_{p,2}, C_{p,3}\}$, $\{C_{p,4}, C_{p,5}, C_{p,6}\}$ and $\{C_{p,7}, C_{p,8}, C_{p,9}\}$ fixes the entire profile. The remaining points of the profile are determined by the interaction of the state variables and the structure of the DPS. If one of the target points of the profile is $C_{p,7}$, then fixing $C_{p,7}$ fixes $C_{p,8}$ and $C_{p,9}$ for the structure depicted in Fig. 1. Hence one cannot control all the three compositions, $C_{p,7}$-$C_{p,9}$. On the other hand, had F_a, and F_b been located at 8th and 9th CSTRs, independent control of $C_{p,7}$, $C_{p,8}$ and $C_{p,9}$ would have been possible. Thus, the idea of the whole profile may be equivalently represented in terms of independent points on the profile and these points must be selected depending on the location and type of the manipulated variables. For the current work, we assume that the target profile can be converted to unique target values of $C_{p,3}$, $C_{p,6}$, and $C_{p,9}$.

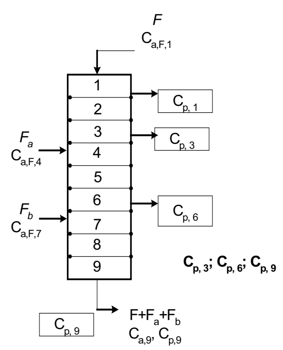

Fig. 1. Schematic diagram of a plug flow reactor. Variables in bold letters are CVs, those in italics are MVs and ones in boxes are the measurements.

3. CONTROL METHODOLOGY

The model of the distributed parameter system may be described in the state space form after spatial discretization as follows:

$$\dot{x} = f(x, u, d) \tag{2}$$

$$y = g(x, d) \tag{3}$$

where x, u, and d are vectors of state, manipulated, and unmeasured disturbance variables, respectively. Available measurements are denoted by y. Nonlinear model predictive control of a distributed parameter system generally represents a formidable task due to requirement of integrating a set of partial differential equations during the online solution of a nonlinear program. To retain the simplicity of quadratic program, Garcia [16] suggested that the future predictions needed in MPC may be obtained by adding the nonlinear unforced response of the system with a forced response based on a linear model. Thus, the approximated model response may be written as,

$$x(t) = \int_{t_0}^{t} f(x(\tau), u(t_0), d(\tau))d\tau + \int_{t_0}^{t} \left.\frac{\partial f}{\partial u}\right|_{u(t_0)} (u(\tau) - u(t_0))d\tau \tag{4}$$

Use of such an approximate form of the model makes the resulting MPC formulation a quadratic program (QP). As we may not have measurements of all the state variables, we use an Extended Kalman Filter (EKF) for state reconstruction. For a recent reference of EKF based MPC formulation and its implementation, the reader is referred to Padhiyar et al. [5]. It was observed in the abovementioned reference that the EKF with an input disturbance model provided reasonable state estimates for a wide range of process and parametric disturbances in a continuous pulp digester. We continue using input disturbance modelling, where it is assumed that the effect of an unmeasured disturbance can be estimated by assuming the disturbance as a load variable. The MPC strategy calculates the control moves by an online solution of the following optimisation problem,

$$\min_{\Delta U_k} \left\| (Y_{k+1/k} - R_{k+1/k}) \right\|_{We}^2 + \left\| \Delta U_k \right\|_{Wu}^2 \tag{5}$$

such that

$$\Delta u_{k+c} = ... = \Delta u_{k+p-1} = 0$$

$$u_{k+l}^{low} \le u_{k+l} \le u_{k+l}^{high}, 0 \le l \le q-1$$

$$\Delta u_{k+l}^{\min} \le \Delta u_{k+l} \le \Delta u_{k+l}^{\max}, 0 \le l \le q-1$$

$$y_{k+l/k}^{low} \le y_{k+l/k} \le y_{k+l/k}^{high}, 1 \le l \le p$$

$$Y_{k+1/k} = [y_{k+1/k}^T \; y_{k+2/k}^T \dots y_{k+p/k}^T]^T$$

where $Y_{k+1/k}$ is a vector of output predictions over the prediction horizon p and R_{k+1} is the corresponding target. If the controlled variable is chosen as only the endpoint, the above problem refers to endpoint property control. On the other hand if $Y_{k+1/k}$ consists of prediction of the whole profile (or the points that uniquely characterize the profile) the control problem becomes one of full profile control. ΔU_k represents an appropriately defined vector of input moves over q future samples, which are optimised at every sampling instant. W_u and W_e are weighting matrices for the manipulated inputs, u_k, $u_{k+1...}$ u_{k+q-1} and the controlled variables, y_k, y_{k+1}, $...y_{k+p}$, respectively.

The lexicographic method of optimisation assumes that the objective function consists of trade-offs, which can be prioritised. Thus, in the full profile control problem, if the whole profile becomes unachievable, we may have alternate solutions, each of which may achieve only parts of the target profile. We may then want to prioritise so that certain parts of the profile are achieved at the cost of other parts. While this is heuristically attempted in conventional MPC applications by differentially weighting the multiple control objectives, a systematic procedure results from the lexicographic method of optimisation [17, 18]. Let us assume that the profile has been split into N sections with $Y^1_{k/k}$, $Y^2_{k/k}$,…, $Y^N_{k/k}$, representing the relevant profile estimates at time instant k. Note that these sections could be overlapping or non-overlapping. The control objective for each section may then be written as,

$$J^i = \sum_{j=1}^{p} W_e^i (Y^i_{k+j/k} - R^i_{k+j})^2 + \sum_{m=1}^{c} W_u^i (\Delta U_{k+m-1})^2 \qquad (6)$$

where i = 1,2, …,N. We assume that the N sections are numbered such that section 1 represents the most important part of the profile, section 2 the next most important part of the profile and so on. The MPC controller based on lexicographic method solves the following problem online,

$$\begin{aligned}
&\min_{\Delta U_k, \Delta U_{k+1}, \cdots, \Delta U_k + c-1} J^1 \\
&s.t.\ \ C^1 \text{ are feasible} \\
&\quad \min_{\Delta U_k, \Delta U_{k+1}, \cdots, \Delta U_k + c-1} J^2 \\
&\quad s.t.\ \ C^2 \text{ are feasible} \\
&\quad y^1_{k+p/k} = y^1_{k+p/k}{}^* \\
&\quad \min_{\Delta U_k, \Delta U_{k+1}, \cdots, \Delta U_k + c-1} J^N \\
&\quad s.t.\ \ C^N \text{ are feasible} \\
&\quad y^1_{k+p/k} = y^1_{k+p/k}{}^* \\
&\quad y^2_{k+p/k} = y^2_{k+p/k}{}^* \\
&\quad\quad \vdots \\
&\quad y^{N-1}_{k+p/k} = y^{N-1}_{k+p/k}{}^*
\end{aligned} \qquad (7)$$

where C^i represents the constraints relevant to the i^{th} objective function corresponding to the i^{th} section of the profile. Typically C^i consist of constraints on the input and output variables. The equality constraints $y^i_{k+p/k} = y^i_{k+p/k}{}^*$ used in the $(i+1)^{st}$ tier of the optimization problem above enforce the lexicographic constraint that the optimal profile at the end of the prediction horizon obtained in the i^{th} tier is maintained. The right hand sides of these lexicographic constraints denote the optimal values obtained in the previous tier. Thus, the profile in the previous section should achieve its optimal value at the end of the prediction horizon. The optimised input corresponding to the current time instant, ΔU_k, obtained at the end of the N^{th} optimisation problem is injected into the plant. Our simulations revealed that the lexicographic constraints are usually active at the optimal solution of the i^{th} optimisation problem. Thus, the lexicographic constraints constrain the evolution of the optimal profile in the i-1 section at steady state to its optimal value while solving the i^{th} optimisation problem.

To summarize, the lexicographic optimisation based MPC finds an optimal solution that explicitly prioritizes the controller objectives. Since in the current work, we use an extended MPC implementation, each of the optimisation problems are QPs for which efficient solvers exist. In the next section, we compare the pros and cons of the exit and full profile control strategies and demonstrate that the lexicographic method based control provides a trade-off between the two.

4. BENEFITS OF LEXICOGRAPHIC OPTIMIZATION BASED MPC: A CASE STUDY

We use a simple example consisting of a PFR presented in Section 2 to test the benefits of the lexicographic optimisation based MPC. We assume the reaction in equation (1) as irreversible and isothermal following elementary kinetics. A mass balance over an infinitesimally small radial slice along the flow direction z yields the following mathematical model for the DPS,

$$\frac{\partial C_p}{\partial t} = \frac{F(z)}{A}\frac{\partial C_p}{\partial z} + kC_a{}^2 \qquad (8)$$

where C_a is the mass concentration of reactant A and C_p the concentration of product P.

As shown in Fig. 1, we simulate the DPS assuming 9 CSTRs in series. We also assume that we have access to four measurements namely product concentrations at 1st, 4th, 7th, and 9th CSTRs. As discussed previously, we could estimate the full profile (product concentrations in the 9 CSTRs) and attempt to control it. Alternatively, we could achieve the same by fixing one concentration from each sets $\{C_{p,1}, C_{p,2}, C_{p,3}\}$, $\{C_{p,4}, C_{p,5}, C_{p,6}\}$ and $\{C_{p,7}, C_{p,8}, C_{p,9}\}$, since this uniquely fixes the entire profile. We have chosen $C_{p,3}$, $C_{p,6}$, and $C_{p,9}$ as the controlled variables for the two profile control approaches and $C_{p,9}$ for exit control approach. The three manipulated inputs include the main feed flowrate at the top of the column and two trim flowrates F_a at the 4th CSTR, and F_b at the 7th CSTR. The three feed flow streams carry pure component A with the nominal feed concentration $C_{a,F,,1}$, $C_{a,F,,4}$, and $C_{a,F,,7}$ at 0.5 kg.m^{-3}.

To showcase the merits of the three control approaches discussed in Section 3 we consider both servo as well as regulatory problems. The initial (Setpoint 1) and final (Setpoint 2) targets used in the servo problem are documented in Table 1. The regulatory problem results from a step disturbance in the feed concentration. In Case Study 1, the injected disturbance is small enough that the target profile (Setpoint 2) continues to be feasible. In Case Study 2, the target profile (Setpoint 2) cannot be achieved due to the large magnitude of the disturbance. Both of these case studies assume no mismatch between plant and controller models.

4. 1. Case Study 1: Achievable target profile

Here the system is switched from Setpoint 1 to the Setpoint 2 at 120 min. A +10 % step disturbance in the concentration of the feed stream at top of the column, $C_{a,F,1}$, is injected at 520 minutes. The closed loop response for such a servo and a regulatory problem using the end point and full profile strategy is shown in Fig. 2. The error penalty matrix for the endpoint property control strategy is $We = 2\text{x}10^5$ and for the full profile control strategy $We = \text{diag}(5\text{x}10^5\ \ 3\text{x}10^5\ \ 2\text{x}10^5)$ corresponding to $C_{p,3}$, $C_{p,6}$, and $C_{p,9}$, respectively. These values of the weight matrix were selected to normalize the deviations from the setpoints for each of the three controlled variables thus ensuring equal contribution in the error penalty term of the objective function. Weighting matrix for inputs is $Wu = \text{diag}(2.5\text{x}10^3\ \ 10^4\ \ 10^4)$. The corresponding manipulated input trajectories are shown in Fig. 3.

It is noted that the controlled variables using endpoint control ($C_{p,9}$) and profile control ($C_{p,3}$, $C_{p,6}$, $C_{p,9}$) strategies successfully switched from their old targets to new targets even in presence of the disturbance. Indeed, the endpoint control strategy does not attempt to control $C_{p,3}$ and $C_{p,6}$

Table 1. Definition of two setpoints and physical limits on manipulated inputs for all the four cases

	$C_{p,3}$, $Kg.m^{-3}x10^2$	$C_{p,6}$, $kg.m^{-3}$ x 10^2	$C_{p,9}$, $kg.m^{-3}$ x 10^2
Setpoint1	7.00	9.17	11.73
Setpoint2	5.68	6.98	8.77
	F, $m^3.min^{-1}$	F_a, $m^3.min^{-1}$	F_b, $m^3.min^{-1}$
Min	0	0	0
Max	2	1	1

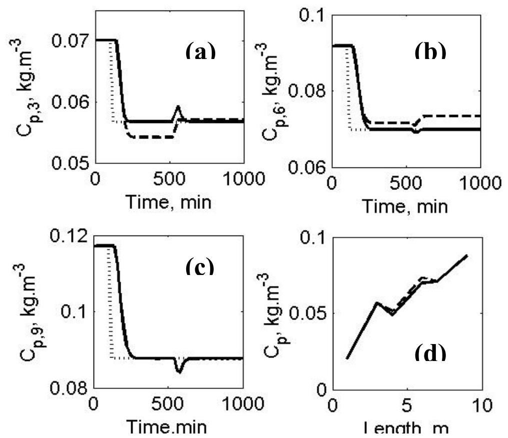

Fig. 2. Case 1: Closed loop response for endpoint control (dashed) and profile control (solid): reference value (dotted)

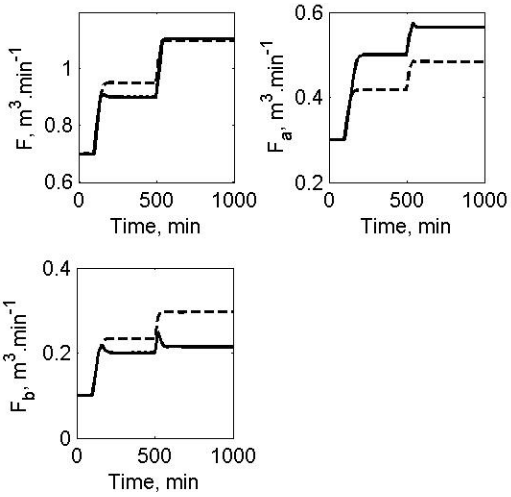

Fig. 3. Case 1: Manipulated variable profile for endpoint control (dashed) and profile control (solid) strategies

resulting in the apparent offset. This clearly demonstrates that the full profile control strategy may successfully control the reaction path and thus offer advantages for overall property control. Fig. 3(d) shows the steady state profile of the product composition along the length of the column at t = 1000 min. Simulation times for exit control and profile control are 0.02 sec, and 0.021 sec respectively. These values are reported for a PIV; 1 Gb RAM and 1.8 GHz processor.

Thus, if the targets are feasible, both profile and endpoint control strategies are able to achieve their objectives satisfactorily. As available degrees of freedom are efficiently utilized in profile control it may be preferable over endpoint property controller. Next we present a case we present a case where a severe disturbance results in infeasibility of Setpoint 2.

4. 2. Case Study 2: Unachievable target profile

As discussed previously, the target profile may become unachievable in presence of large disturbances or limited scope of manipulation. To simulate such a situation, we increase $C_{a,F,1}$ from 0.5 $Kg.m^{-3}$ to 1 $Kg.m^{-3}$ with constraints in manipulated inputs as shown in Table 1. Using identical controller tuning as in Case Study 1, the simulation results are depicted in Fig. 4 and Fig. 5. The full profile control strategy (solid line) attempts to reject the disturbance by reducing the residence time of the reactant and hence increasing the three manipulated variables as seen from Fig. 5. However, the MVs saturate and an offset is observed in all three sections of the PFR. More importantly, the endpoint target properties are also not achieved. On the other hand, since the endpoint controller attempts control of $C_{p,9}$ alone, it successfully uses the degrees of freedom to meet its objective (dashed lines in Fig. 4 and Fig. 5). Although we observe an apparent offset in $C_{p,6}$ and $C_{p,9}$, the controller at least provides the endpoint targets. Since $C_{p,9}$ uniquely characterizes the profile in CSTRs 7-9, it implies that at least in this section, the target has been met. Fig. 4(a) and 5(b) show that the target profiles are not met in CSTRs 1-3 and 4-6 respectively. However, from an operations perspective, the exit control strategy may be superior to the whole profile strategy. The lexicographic method, on the other hand, offers a systematic method to assign the degrees of freedom to the conflicting control objectives. Here, we first divide the PFR into multiple sections. We choose overlapping sections with N=3. Section 1 consists of CSTRs 7-9, section 2 with CSTRs 4-9 and section 3 with CSTRs 1-9. The profile in section 1 has the highest priority and thus the corresponding control problem with objective function J^1 (see Equation (7)) is solved first. This is followed by solution corresponding to sections 2 and 3 along with the appropriate lexicographic constraint. The performance of the lexicographic method is shown as the dashed-dotted line in Fig. 4 and Fig. 5. It is observed from Fig. 4(c) that the lexicographic optimization based controller achieves its target in section 1 even in presence of the disturbance. It then proceeds to achieve the target profile in the middle part of the PFR with the remaining degrees of freedom while maintaining the target profile in the lower part of the PFR (see Fig. 4(b)). Finally, it attempts to obtain the target profile in the top part of the PFR. However, this is unachievable as observed from Fig. 4(a). While the infeasibility of the profile is due to fundamental process limitations, the lexicographic optimisation based MPC systematically achieved the control objectives by

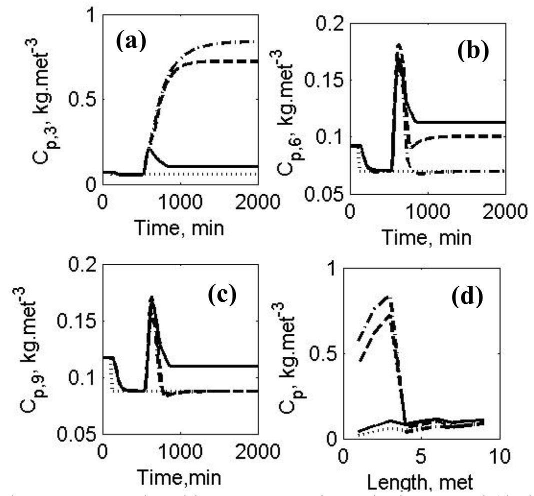

Fig. 4. Case 2: Closed loop response for endpoint control (dashed), profile control with a single objective function (solid), and profile control with lexicographic optimization (dashed dotted) strategies; reference value (dotted)

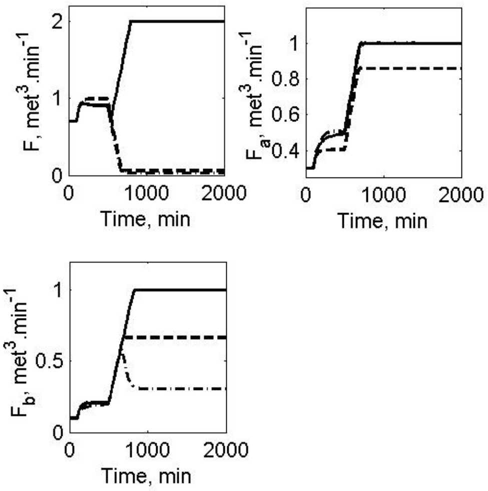

Fig. 5. Case 2: Manipulated variable profiles for endpoint control (dashed), for profile control with a single objective function (solid), and for profile control with lexicographic optimization (dashed dotted) strategies

satisfying section 1 and 2 while failing to meet target in section 3.

Although the lexicographic method solved three QPs, the integration of the PDEs necessary to provide future predictions in Equation (4) need to be performed only once. The simulation times for solving a single iteration in the profile control using a single objective function and using lexicographic optimization are 0.5s and 0.54s respectively. Thus, in our problem the lexicographic method was only 8% more computationally expensive than the full profile control problem.

5. PROFILE CONTROL IN A PULP DIGESTER

Kraft pulping process produces pulp by delignifying wood chips with an aqueous solution of sodium hydroxide and sodium sulfide, called white liquor. The Kappa number measures the extent of delignification and represents one of the key property variables that need to be controlled. A schematic of a dual vessel digester is shown in Fig. 6. Wood chips and white liquor are introduced in the impregnation vessel. Here white liquor diffuses into the pores of wood chips. The chip-liquor mixture flows into the digester vessel wherein majority of the delignification occurs due to higher temperature. The digester vessel is divided into three functional zones namely cook, modified continuous cook (mcc), and extended modified continuous cook (emcc). The wood chips and white liquor flow cocurrently within the cook zone, at the end of which the spent liquor is extracted and sent for recovery. The chips then encounter a countercurrent flow of dilute liquor. The detailed description of the process and mathematical model are provided elsewhere[21, 22]. Material properties of the pulp such as fibre length depend on the history of processing in the digester. Certain safety related issues such as plugging of the digester vessel are also related to the reaction path [23]. In this work, we control Kappa number at the end of cook, mcc, and emcc zone as shown by bold letters in Fig. 6. The cook trim, emcc trim, and mcc temperature represent the manipulated inputs.

In our simulations, we have introduced parametric mismatches in heat transfer coefficient and heat of reaction to simulate plant-model mismatch. Heat transfer coefficient of the plant model is increased from 744.3 to 748.2 KJ.min^{-1}.K^{-1}.m^3 and heat of reaction was reduced from 639.1 to 636.5 KJ.kg^{-1}. Setpoint change is introduced at 2010 min. Definitions of both the setpoints and constraints are provided in Table 2. Weighting matrices W_e and W_u, for output errors and inputs in control calculations are also provided in Table 2. Closed loop response by both the profile control approaches, namely full profile control (solid line) and lexicographic optimization based profile control (dashed-dotted line) are shown in Fig. 7 and corresponding manipulated input profiles in Fig. 8. As can be seen from Fig. 7, the conventional MPC for full profile control fails to achieve any of the three outputs to its setpoint. On the other hand, lexicographic optimization based MPC could bring the exit Kappa number to its set point, while selectively transferring the offset to profiles in the cook and mcc sections.

4. CONCLUSIONS

In this work, we demonstrated a novel formulation of MPC using lexicographic optimisation. This formulation has an advantage of explicitly prioritising the conflicting control objectives, which is of particular concern when the targets become infeasible. We demonstrated the benefits for profile control in a DPS using a simple PFR example and a continuous pulp digester that represents a large-scale DPS. We also showed that the added computation burden is insignificant when using extended MPC.

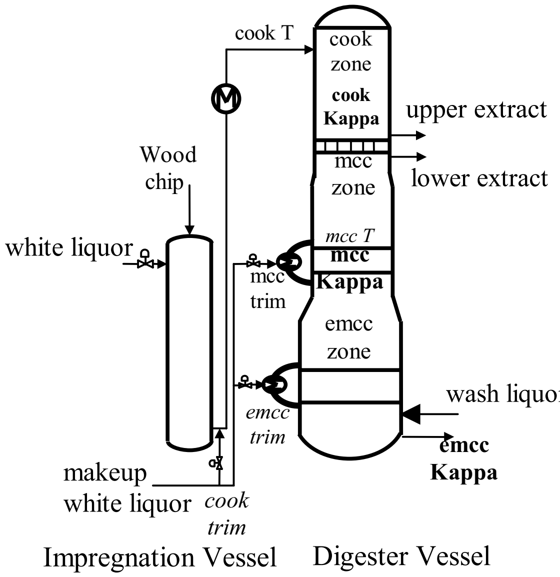

Fig. 6. Schematic of dual vessel pulp digester. Variables in italics represent manipulated variables, in bold font represent controlled outputs.

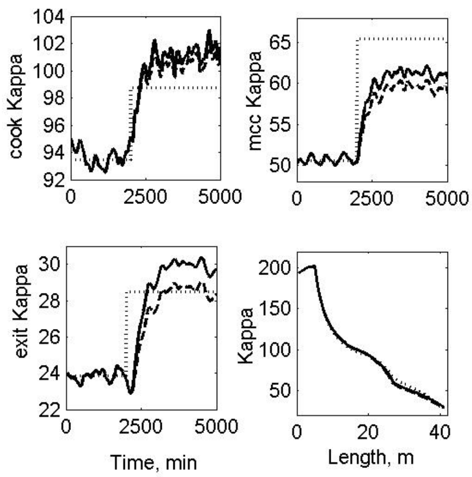

Fig. 7. Case 3: Closed loop response for profile control with a single objective function (solid), and lexicographic optimization (dashed dotted) strategies; reference value (dotted) for the continuous pulp digester

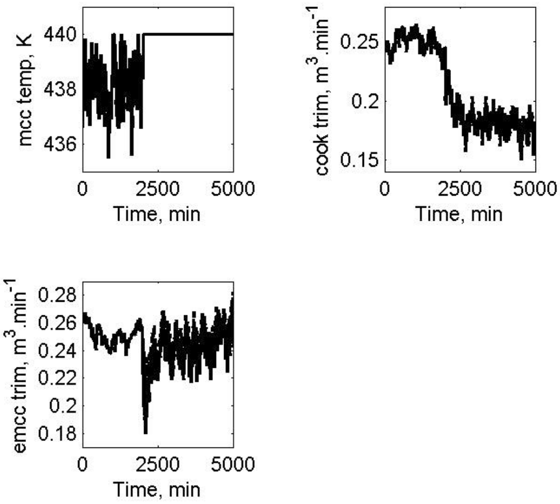

Fig. 8. Case 3: Manipulated inputs profiles for profile control with a single objective function (solid), and lexicographic optimization (dashed dotted) strategies; for the continuous pulp digester.

Table 2. Controller settings for the continuous pulp digester

Setpoints			
	Cook Kappa	*Mcc Kappa*	*Exit Kappa*
Setpoint1	93.5	50.5	23.8
Setpoint2	98.7	65.4	28.5
Input and output weightings			
We	22.9	78.3	351.3
Wu	0.0625	625	1907
Input constraints			
	Emcc T, K	*Cook trim* $m^3.min^{-1}$	*Mcc trim* $m^3.min^{-1}$
Min	400	0	0
Max	440	1	1

ACKNOWLEDGEMENTS

The authors acknowledge funding from the Department of Science and Technology, India, for financial assistance under SERC (Engg. Sciences) (#03DS015).

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