8th International IFAC Symposium on
Dynamics and Control of Process Systems
Preprints Vol.2, June 6-8, 2007, Cancún, Mexico

LOOP-SHAPING PI CONTROLLER REDESIGN USING LEAST-SQUARES MODEL-MATCHING

George Acioli Júnior and Péricles R. Barros

Unidade Acadêmica de Engenharia Elétrica,
Universidade Federal de Campina Grande,
Cx.P. 10105, Campina Grande, PB – BRAZIL.
E-mail: {georgeacioli, prbarros}@dee.ufcg.edu.br

Abstract: In this work it is presented a PI controller loop-shaping redesign technique. The existing closed-loop is excited using a relay experiment which allows one to estimate the current phase-margin, the crossover frequency and a first-order plus time-delay model. Techniques for continuous-time identification are examined. Based on the information obtained from the experiment, a desired loop transfer function is defined. The controller is obtained using least squares model-matching. Constrained optimization is used in the estimation. Simulation examples illustrate the technique.

Keywords: PI controller tuning; Loop-shaping; Continuous-time identification; Closed-loop identification; Time delay process; Relay excitation.

1. INTRODUCTION

PI (and PID) controllers are commonly used to control process which have a simple dynamics and modest performance requirements. Simple experiments like step responses and relay are widely used in classical techniques to tune these controllers as they are very simple to apply. However, these techniques provide good performance only for the process models for which they were tuned. In this work it is presented a PI controller loop-shaping redesign technique. The existing closed-loop is excited using a relay experiment which allows one to estimate the crossover frequency and a first-order plus dead-time model (FOPDT). The current phase-margin can also be estimated, but it will not be used in the design.

Methods of closed-loop identification have been used in industrial applications as they may not cause stops in system operation, unlike open-loop identification. Other reasons which can be listed are demands on safety in process operation, unstable processes and restrictions in production (Ljung, 1999). An additional consideration to accomplish experiments in closed-loop is that the dynamic exhibited by the plant with the old controller is relevant to the new controller design (den Hof and Schrama, 1998) and (Gevers, 1995). Here a continuous-time model is identified. The estimation of continuous-time models from sampled data has received some attention in the last years, motivated by the need of such models to recover physical parameters or to use design techniques developed for continuous-time controllers. An extensive list of references on the subject can be found in (Mensler, 1999), where a detailed survey discusses the advantages of a direct approach in relation to the indirect estimation of a discrete-time model plus a later transformation into a continuous-time model. Although structural constraints such as model order and time delay have been incorporated in continuous time system identification since its origin, the constraints on the estimated parameters were rarely enforced (Wang

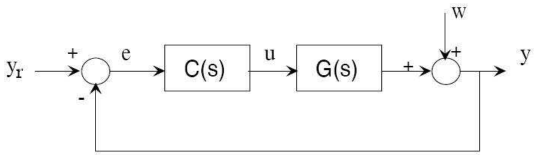

Fig. 1. Closed-loop system.

et al., 2005). In this paper a constrained least-squares minimization which uses frequency data obtained from a relay experiment is used to obtain a model.

When redesigning a controller, specially when using little information from the process transfer function, it is important to evaluate the robustness properties of the existing loop and to make the controller design leaving some safety margins for the case of model errors. One common approach is to evaluate the gain and phase margins and use the information to redesign the controller as the one presented in (de Arruda and Barros, 2003). In the present paper the closed-loop crossover frequency is evaluated and the redesign is based on the shaping of the loop transfer function to match a desired one. The new crossover frequency is derived from the estimated FOPTD model, thus resulting in a closed loop with some stability margin. The loop shaping is performed using a least squares model-matching in the frequency range of interest.

This paper is organized as follows. Initially, the problem is stated, followed by the presentation of the relay experiment. Next, continuous-time identification of FOPDT models is discussed and an algorithm is proposed. The controller redesign is then presented. Simulations examples and conclusions complete the presentation.

2. THE PROBLEM STATEMENT

Considered a closed-loop system shown in Fig. 1, with y_r is the setpoint signal, u is the control signal or process input, y is the process output and w is the disturbance signal. The closed-loop transfer function is $T(s)$, resulting from the combination of a process transfer function $G(s)$ and a controller $C(s)$, while the loop transfer function is $L(s) = C(s)G(s)$.

The controller is assumed to be a PI controller given by

$$C(s) = K_p\left(1 + \frac{1}{T_i s}\right) = \frac{K_p s + K_i}{s} . \quad (1)$$

It is considered that the process dynamics can be captured by a first-order plus dead-time (FOPDT) continuous-time models represented by

$$G(s) = \frac{b}{s+a} e^{-Ls}. \quad (2)$$

It is assumed closed-loop operation and that the excitation is generated from a relay-based experiment to be presented in the sequel. The aim of the paper is to redesign the controller after the assessment of the closed loop.

3. THE LOOP GAIN RELAY EXPERIMENT

The feedback structure applied for frequency point estimation of the loop transfer function is presented in Fig. 2. The conditions of the limit cycle operation are defined by the following proposition.

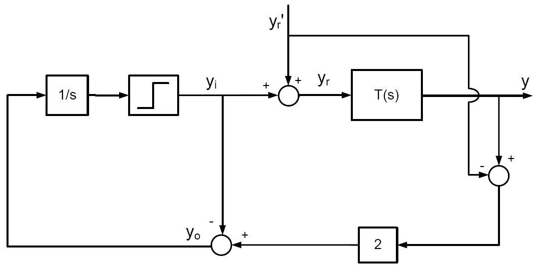

Fig. 2. Relay Closed Loop Experiment.

Assume that for a stable $T(s)$

$$F(s) = 2T(s) - 1 \quad (3)$$

is also stable. Then if a limit cycle is present it oscillates at a frequency ω_g such that

$$|L(j\omega_o)| \approx 1 .$$

Proof: See (Schei, 1994)

The setpoint $y_r(t)$ is the excitation applied to the closed loop $T(s)$. The process transfer function at the crossover frequency is estimated computing the DFT of one period of the process input u and output y when the relay oscillation is present and steady.

4. IDENTIFICATION OF DEAD-TIME SYSTEMS

In this Section continuous-time system identification is presented for FOPDT models. In order to motivate the final algorithm, identification in open-loop is initially reviewed and its limitation evaluated when applied to closed-loop operation.

4.1 Identification of FOPDT Model from a Step Response Experiment

In continuous-time identification, techniques which use such models (with $L = 0$) are named *Integral Methods* (Mensler, 1999). An integral method has been used in (Wang *et al.*, 2000), with the process in open loop and under a step input with amplitude h applied at $t = 0$. The process is assumed to be at steady-state at $t = 0$, so $u(t) = 0$ for $t < 0$

and zero initial conditions are assumed. For this case the model also satisfy

$$y(t) = -a \int_0^t y(\tau)\, d\tau + bht - bhL, \quad (4)$$

In closed-loop, this procedure cannot be applied, so that some additional processing must be made. Under the same conditions (steady-state at $t = 0$, $u(t) = 0$ for $t < 0$ and zero initial conditions) the process model can be written as

$$y(t) = -a\int_0^t y(\tau)\, d\tau + b_1\int_0^t u(\tau)\, d\tau - b_2\int_{t-L}^t u(\tau)\, d\tau. \quad (5)$$

from which a regression model can be obtained on model parameters $\{a, b_1, b_2\}$. To estimate the value of L a iterative algorithm is used. This technique is presented in (Coelho and Barros, 2003).

4.2 A Simple Technique for Identifying a FOPTD Model

In the above techniques the same initial conditions are assumed (steady-state at $t = 0$, $u(t) = 0$ for $t < 0$ and zero initial conditions). The unavoidable presence of noise makes it difficult to exactly determine $y(0)$ and $u(0)$, so that these conditions are violated leading to poor estimates as illustrated in the simulation Section. In order to overcome this limitation, a slightly different technique is proposed. The technique can be used a step response experiment, performed in the open or closed loop, or to a more general one such as the relay experiment used here. The process is assumed to be at steady-state at $t = 0$ and the data is collected until the closed loop system reaches a new steady state. For the purposes of this paper, the following approximation for model 2 is used:

$$G(s) = \frac{b(1 - sL)}{s + a}. \quad (6)$$

This approximation has been used in (Júnior *et al.*, 2006) showing good results and eliminating the iterative algorithm used to estimate the value of L. The process is assumed to satisfy the differential equation

$$\dot{y}(t) + ay(t) = bu(t) - bL\dot{u}(t), \quad (7)$$

where the disturbance term has been discarded.

Integrating Eq.(7) from $\tau = t$ to $\tau = t_f$ yields

$$y(t_f) - y(t) = -a\int_t^{t_f} y(\tau)\, d\tau + b\int_t^{t_f} u(\tau)\, d\tau - bL(u(t_f) - u(t)). \quad (8)$$

Then, define

$$\gamma(t) = y(t_f) - y(t),$$
$$\phi(t) = \left[-\int_t^{t_f} y(\tau)\, d\tau \;\; \int_t^{t_f} u(\tau)\, d\tau \;\; -(u(t_f) - u(t)) \right]^T,$$
$$\theta = \left[a \;\; b \;\; bL \right]^T. \quad (9)$$

and Eq.(8) can be written in regression form

$$\gamma(t) = \phi(t)\,\theta. \quad (10)$$

The estimate can be computed in one step using either least-squares or instrumental variable methods. In order to improve the estimate, the estimated crossover frequency transfer function response is assumed to be true and used as an equality contraint as shown below.

4.3 Identification of FOPDT Model with Frequency Domain Constraints

It is straightforward to introduce equality constraints in least-squares minimization (Nelles, 2001). Here the process frequency response on the first harmonic of the loop gain relay experiment signal is used as a constraint on the estimated model. The frequency response is obtained computing the DFT of process input and output. As expected, the loop transfer function has magnitude close to one around this frequency. The resulting estimate should result in a FOPDT model with good accuracy close to the crossover frequency.

The equality constraint is defined through the following regression vector: $\hat{z} = x^T(\hat{\omega}_g)\,\hat{\theta}$

with

$$\hat{z} = j\hat{\omega}_g \hat{G}(j\hat{\omega}_g); \quad x^T(j\hat{\omega}_g) = \left[-\hat{G}(j\hat{\omega}_g) \;\; 1 \;\; -j\hat{\omega}_g \right]$$
$$\hat{\theta} = \left[a \;\; b \;\; bL \right]^T$$

where $\hat{\omega}_g$ is the crossover frequency estimated using the relay experiment.

Assume the data is grouped in a vector from yielding matrices Y and $\hat{\theta}$. The least-squares optimization problem is given by

$$J = \left(Y - \Phi\hat{\theta}\right)^T \left(Y - \Phi\hat{\theta}\right) \quad (11)$$

subject to $M\theta = \gamma$, which express the equality constraints in a linear form.

In this case, the least-squares optimization problem with constraint is equivalent to minimize in relation to $\hat{\theta}$ and λ the cost function given by

$$J = \left(Y - \Phi\hat{\theta}\right)^T \left(Y - \Phi\hat{\theta}\right) + \lambda(\gamma - M\theta) \quad (12)$$

By defining $E = 2\Phi^T\Phi$ and $F = 2\Phi^T Y$.

The optimization problem solution is given by

$$\lambda^T = \left[ME^{-1}M^T\right]^{-1}\left[\gamma - ME^{-1}F\right]$$
$$\hat{\theta} = [E]^{-1}\left(F + M^T\lambda^T\right).$$

5. THE CONTROLLER REDESIGN

The experiment from last section results in an estimate of the crossover frequency $\hat{\omega}_g$ and a FOPDT model given by the parameters a, b and L. It is desired to redesign the PI controller in order to obtain a loop transfer function

$$L_D(s) = \frac{\omega_d^2(2s + \omega_d)}{s^2(s + 2\omega_d)}e^{-Ls}.$$

This loop transfer function has $20db/decade$ decay over the frequency range $\left\{\frac{\omega_d}{2}, 2\omega_d\right\}$ (slope -1) one decade above and one decade below the desired crossover frequency ω_d. The time delay τ_d is used to yield a causal controller. With a proper choice of ω_d, the proposed design yields robustness margins, while maintaining good performance (Skogestad, 2003). It should be noticed that this loop transfer function yields a high overshoot when using a one degree of freedom controller. A two degree of freedom controller could be used if desired.

Choice of ω_d: Consider frequency ω_1 for which

$$\measuredangle G(j2\omega_1) = -135^o.$$

The new crossover frequency ω_d is computed such that $\omega_d = \alpha\omega_1$. The constant α is a factor to make the loop faster or more robust depending on the designer's choice. In this paper $\alpha = 0.5$ is used.

For a PI controller, a limiting factor in the choice of ω_d is the frequency range for which the process model ceasses to behave like a first order system. The motivation comes from the symmetrical-optimal(S.O.) design as follows: Consider the simplified model

$$G(s) = \frac{K}{s(\tau_\Sigma s + 1)}.$$

The S.O. loop-shaping assumes that the crossover-frequency is defined as $\omega_d = 1/(2\tau_\Sigma)$. It should be noted that at frequency $1/\tau_\Sigma$ the transfer function has phase of $\phi = -135^o$. At this ω_d the transfer function has phase approximately -90^o (Voda and Landau, 1995).

6. SIMULATION EXAMPLES

In this section the controller redesign technique with closed loop identification algorithms defined early are applied to two processes. The cost function used to compare the estimates is

$$\varepsilon = \frac{1}{N}\sum_{k=0}^{N-1}\left[y(kT_s) - \widehat{y}(kT_s)\right]^2$$

where $y(kT_s)$ is the real process output, while $\widehat{y}(kT_s)$ is the estimated process output from a open loop simulation under the same step. In all experiments $T_s = 0.01s$. White noise is added only to the output of the process. The processes and the results are shown below.

6.1 Identification Example

The objective is to show the effectiveness of the technique proposed and the difficulty to exactly determine $y(0)$ and $u(0)$ necessary to satisfy the initial conditions (steady-state at $t = 0$, $u(t) = 0$ for $t < 0$ and zero initial conditions) assumed for the technique represented by Eq. 5. For the simulation the process is given by $G_1(s) = \frac{1}{(s+1)^8}$ and the controller used is $C_1 = \frac{0.7s+1}{s}$. The noise variance is 0.02.

The excitation is generated from a step response experiment performed in a closed-loop system. Firstly, $y_r = 2$ is applied and when the steady state is reached, $y_r = 3$ is applied. The data is collected until the closed-loop system reaches a new steady state. Assuming the closed-loop stable and that an integrator is present in the controller, the new steady state output is equal to y_r.

Figure 3 is given in terms of deviation variables with respect to the initial conditions. Application of the estimator based on regressor of Eq. 5 gives the estimate $G_{ls1}(s) = \frac{0.158}{s+0.19}e^{-4.32s}$.

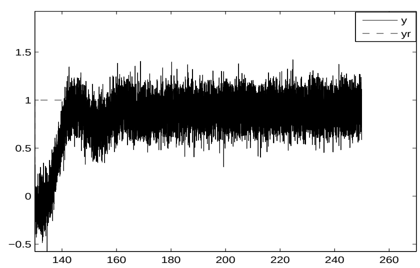

Fig. 3. Data used for estimate Gls1

The use of the proposed estimator applied to raw data as shown in Fig.(4) results in the estimate $G_{ls2}(s) = \frac{0.207}{s+0.207}e^{-4.1s}$.

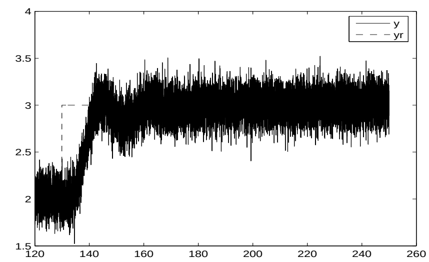

Fig. 4. Data used for estimate Gls2

The step response for estimated models is show in Figure (5) and the mean squared errors are

$\varepsilon_{LS1} = 0.033$ and $\varepsilon_{LS2} = 0.002$, from which it is clear the superiority of the proposed technique.

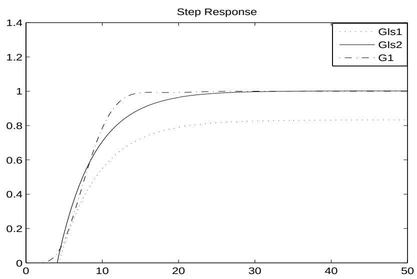

Fig. 5. Step Response for Identification Example

6.2 Controller Redesign Examples

Now the joint identification and controller redesign examples are presented. The relay excitation is applied to the closed-loop.

6.2.1. Example 1 The process is now given by

$$G_2(s) = \frac{(6s+1)(3s+1)}{(10s+1)(8s+1)(s+1)} e^{-0.3s}.$$

and its initial controller is $C_2(s) = \frac{s+0.1}{s}$.

The noise variance is 0.02. The estimates are

$$G_{ls2}(s) = \frac{0.105}{s+0.115} e^{-0.578s}$$
$$G_{ls3}(s) = \frac{0.10}{s+0.108} e^{-0.344s}.$$

The mean squared errors are $\varepsilon_{ls2} = 0.004$ and $\varepsilon_{ls3} = 0.003$.

In this case, the estimated crossover frequency is $\widehat{w}_g = 0.08$ and the process magnitudes are

$$|G_2(j\widehat{w}_g)| = 0.7482$$
$$|G_{ls2}(j\widehat{w}_g)| = 0.7532$$
$$|G_{ls3}(j\widehat{w}_g)| = 0.7472.$$

The identification using constrained least-square minimization provides a better fitting in the crossover frequency and the decreasing of the quadratic error.

For the estimated model $G_{LS3}(s)$, $w_1 = 1.208$ and $w_d = 0.604$ are calculated.

The new controller is $C_n(s) = 1.89(1 + \frac{1}{0.49s})$

The loop gain transfer functions bode diagrams are shown in Figure 6. In this example it can be noted good approximation between the desired (L') and designed ($L1$) loop transfer function around the crossover frequency.

The closed-loop step response is shown in Figure 7. The controller designed have a faster closed-loop response. Notice the comparison with the

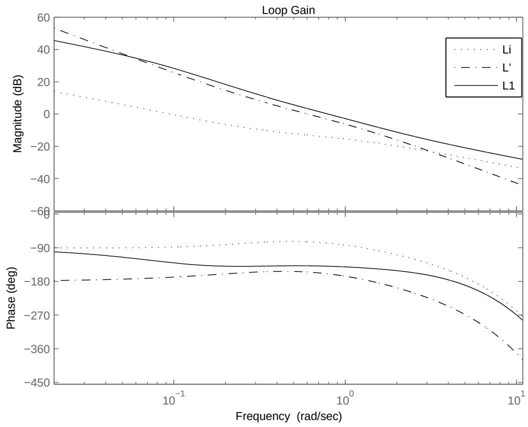

Fig. 6. Loop Gain Transfer Example 2

response that would be obtained with the desired loop transfer function(Cdes) which, as mentioned, has a large overshoot.

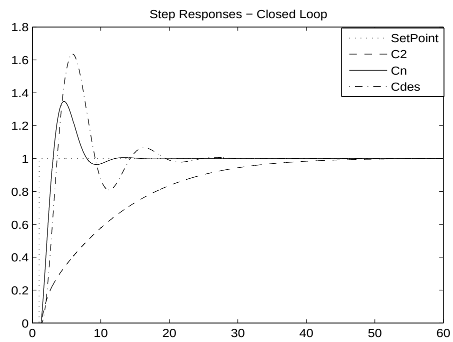

Fig. 7. Step Response for Example 2

6.2.2. Example 2 The process is again

$$G_1(s) = \frac{1}{(s+1)^8}.$$

and its initial controller is the same $C_1(s)$.

The noise variance is 0.01. The estimates without constraints($G_{ls2}(s)$) and with constraints ($G_{ls3}(s)$) are (relay excitation)

$$G_{ls2}(s) = \frac{0.201}{s+0.205} e^{-3.26s}$$
$$G_{ls3}(s) = \frac{0.21}{s+0.208} e^{-3.51s}.$$

The mean squared errors are $\varepsilon_{ls2} = 0.0015$ and $\varepsilon_{ls3} = 8.44e-04$.

In this case, the estimated crossover frequency is $\widehat{w}_g = 0.0847$ and the process magnitudes are

$$|G_1(j\widehat{w}_g)| = 0.9718$$
$$|G_{ls2}(j\widehat{w}_g)| = 0.9054$$
$$|G_{ls3}(j\widehat{w}_g)| = 0.9342.$$

For the estimated model $G_{ls3}(s)$, $w_1 = 0.185$ and $w_d = 0.0925$ are calculated.

The new controller is $C_n(s) = 0.23(1 + \frac{1}{2.09s})$.

The loop gain transfer functions bode diagrams are shown in Figure 8. In this example the results are similar to the first example.

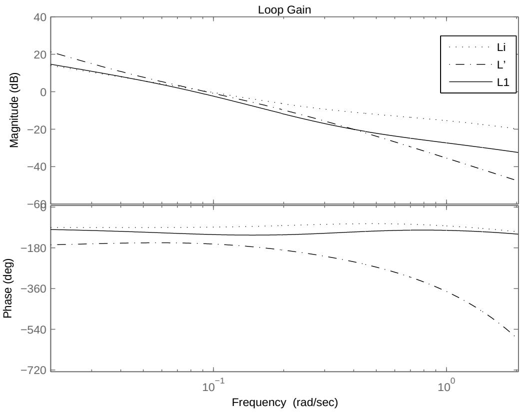

Fig. 8. Loop Gain Transfer Example 2

The closed-loop step response is shown in Figure 9. The controller designed have less oscillatory response.

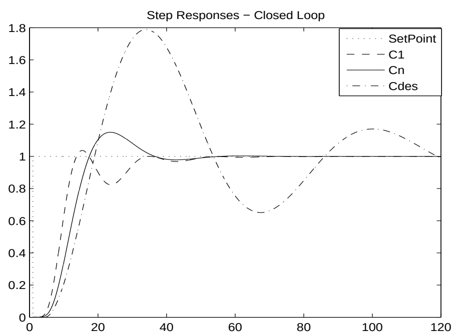

Fig. 9. Step Response for Example 2

Notice the difference between the desired and obtained step responses. This indicates that the FOPDT model is not a good approximation in this case.

7. CONCLUSIONS

In this paper a controller redesign technique was presented. The closed loop is evaluated estimating a simple model with good accuracy close to the crossover frequency. A simple estimation procedure is proposed and uses time data and frequency equality constraints. The obtained model is used to define the desired loop transfer function. The use of least-squares model-matching results in loop shapes closer to the desired specification in the relevant frequency range. Simulation examples illustrate the use of the technique.

ACKNOWLEDGEMENTS

The authors would like to acknowledge financial support from the CNPQ and CENPES/Petrobras.

REFERENCES

Coelho, F. S. and P. R. Barros (2003). Continuous-time identification of first-order plus dead-time models from step response in closed loop. 13th IFAC Symposium on System Identification. Rotterdam (The Netherlands).

de Arruda, Gustavo Henrique Machado and Péricles Rezende Barros (2003). Relay based gain and phase margins PI controller design. *IEEE Transctions on Inst. and Meas. Tech.* **52**(5), 1548–1553.

den Hof, P. M. J. Van and R. J. P. Schrama (1998). Closed loop issues in system identification. *Annual Reviews in Control* **22**, 173–186.

Gevers, Michel (1995). Identification for control. In: *Plenary Talk, 5th IFAC Symposium on Adaptive Systems in Control and Signal Processing*. Budapest, Hungary.

Júnior, G. Acioli, M. A. R. Berger and P. R. Barros (2006). Closed loop continuous-time foptd identification using time-frequency data from relay experiments. International Symposium on Advanced Control of Chemical Processes(ADCHEM). Gramado (Brazil).

Ljung, Lennart (1999). *System Identification: Theory for the User*. 2nd ed.. Prentice Hall. Upper Saddle River, NJ.

Mensler, M. Mensler (1999). Analyse et étude comparative de méthodes d'identification des systèmes à represéntation continue. Développement d'une boîte à outilis logicielle. Thèse du doctorad. Université Henry Poincaré Nancy 1. Nancy,France.

Nelles, Oliver (2001). *Nonlinear System Identification*. 1st ed.. Springer-Verlag. Berlin, Germany.

Schei, Tor Steinar (1994). Automatic tuning of pid controllers based on transfer function estimation. *Automatica* **30**(12), 1983–1989.

Skogestad, Sigurd (2003). *Multivariable Feedback Control*. 2nd ed.. John Wiley & Sons,Ltd.

Voda, A. A. and I. D. Landau (1995). A method for the auto-calibration of PID controllers. *Automatica* **31**(1), 41–53.

Wang, L., P. J. Gawthrop and P. C. Young (2005). Continuous time system identification of nonparametric models with constraints. 16th IFAC World Congress. Prague (Czech Republic).

Wang, Qing-Guo, Bin Hwang and Xin Guo (2000). Auto-tuning of tito decoupling controllers from step tests. *ISA Transactions* **39**, 407–418.