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CONTROL SCHEME BASED ON INTERNAL PREDICTION FOR UNSTABLE LINEAR TIME-DELAY SYSTEMS

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Abstract: This work considers the regulator problem for unstable linear input-delay systems. A discrete time control strategy intended to compensate the effects of the involved time delay and to stabilize the overall closed loop system, is designed. The proposed control strategy provides a prediction of "all" the internal information in the system, which is used to solve the considered regulation problem in a similar way that it is used in a classical Smith predictor compensator

Keywords: Time delay, Prediction, Observers

1. INTRODUCTION

Time delays appear commonly in process control problems, because of the distance velocity lags, recycle loops, and composition analysis loops, or in the approximation of higher-order systems with a lower-order system with a time delay. Also, time delay systems can be due to natural modeling, for example, in the case of population and chemical processes modeling (Kolmanovskii and Myshkis 1992). Many controllers have been developed for stable processes. When a time delay affects the input (or output) signal of the system, a common approach is to eliminate the effect of the delayed signal to deal with a system free of delay. An approximation approach uses Taylor or Pade expansions of the delay operator (Marshall 1979, Hu and Wang 2002). For linear systems, the most common strategy is the so-called Smith predictor compensator (SPC) (Smith 1957, Palmor 1996), which provides an estimate of future outputs to be incorporated within a feedback control function (see Figure 1). The main drawback of the original SPC was related to the class of systems were it could be implemented, since it was restricted to stable plants. In order to overcome this problem, a modification that allows the consideration of processes with an integrator and long time delay was reported (Astrom *et al.* 1994, Matausek and Micic 1996, Normey-Rico and Camacho 2001). A SPC for unstable plants was proposed by (Xian *et al.* 2005). Further results by considering a discrete-time representation of the process was studied in (Torrico and Normey-Rico 2005).

This paper focuses on the regulator problem for unstable time delayed linear systems. Note that the classical SPC cannot be used for this class of systems because the process instability forbids a stable cancellation of the time delay operator. To solve this situation, a discrete predictor schema is proposed in order to estimate "all" the internal information, i.e., starting from a discrete state space model of the plant and the delay, not only the discrete states of the plant are estimated but the "internal" delayed information.

To carry out the control strategy, using the internal estimated information obtained, a discrete static state feedback control is achieved.

The paper is organized as follows. Section 2 presents the class of systems and the classical SPC is briefly recalled. Section 3 develops the prediction discrete-time strategy. To show the performance of the proposed discrete time control strategy, some simulation experiments are presented in Section 4. Finally, Section 5, presents some conclusions.

2. PROBLEM FORMULATION

This section presents the class of systems involving time delays at the input signal (or equivalently, at the output). Consider the following class of (possibly unstable) SISO linear systems with delayed input:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}u(t-\tau) \\ y(t) &= \bar{C}\bar{x}(t) \end{aligned} \tag{1}$$

where $\bar{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output, and $\tau \geq 0$ is the time-delay associated to the input. $\bar{A} \in \mathbb{R}^{n\times n}$, $\bar{B} \in \mathbb{R}^{n\times 1}$ and $\bar{C} \in \mathbb{R}^{1\times n}$ are matrices and vectors of system parameters and are assumed to be known. The input-output representation of system (1) can be obtained as usually by considering the Laplace transform of (1) that leads to the following expression:

$$\begin{aligned} s\bar{X}(s) &= \bar{A}\bar{X}(s) + \bar{B}e^{-\tau s}U(s) \\ Y(s) &= \bar{C}\bar{X}(s). \end{aligned}$$

This expression can be rewritten as

$$\begin{aligned} \frac{Y(s)}{U(s)} &= \bar{C}\left[sI - \bar{A}\right]^{-1}\bar{B}e^{-\tau s} \\ &= \frac{N(s)}{D(s)}e^{-\tau s} = G(s)e^{-\tau s} \end{aligned} \tag{2}$$

where $N(s)$ and $D(s)$ are polynomials in the variable s. Note that a traditional output feedback control strategy as

$$U(s) = [R(s) - Y(s)]Q(s)$$

leads to a closed loop system of the form

$$\frac{Y(s)}{R(s)} = \frac{Q(s)G(s)e^{-\tau s}}{1 + Q(s)G(s)e^{-\tau s}}$$

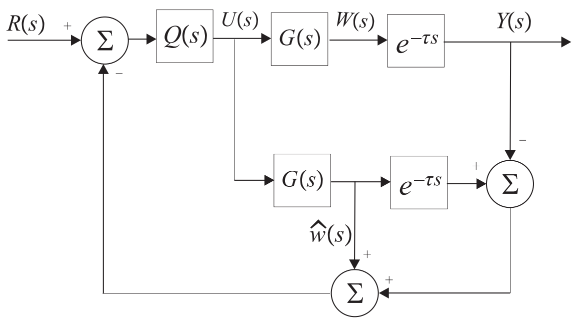

Fig. 1. The Smith Scheme

where the term $e^{-\tau s}$ in the denominator complicates the stability analysis of the feedback system.

2.1 Smith predictor compensator

A classical SPC for a system of the class (2) is shown in Figure 1. The main idea behind a SPC strategy is based on the modeling of the system as

$$W(s) = G(s)U(s) \tag{3a}$$

$$Y(s) = e^{-\tau s}W(s), \tag{3b}$$

and to design an estimator (predictor) for the intermediate signal $W(s)$ (not available for measurement). The smith predictor control scheme use this signal in the controller $Q(s)$ depicted also in Figure 1, in order to compensate the effects of the time delay $e^{-\tau s}$ on the overall closed loop system. It is easy to see that the closed loop system is for the compensation strategy of Figure 1 it is given by,

$$\frac{Y(s)}{R(s)} = \frac{G(s)Q(s)}{1 + G(s)Q(s)}e^{-\tau s}$$

Under ideal conditions (*i.e.*, exact knowledge), the SPC allows to keep out of the closed loop the time delay term. Unfortunately, the classical Smith predictor scheme is able to deal only with stable plants (Palmor 1996, Smith 1957), and several modifications of the same strategy can only manage with some special class of unstable systems (Astrom *et al.* 1994, Majhi and Atherton 1998, Matausek and Micic 1996, Torrico and Normey-Rico 2005, Xian *et al.* 2005).

In that follows, a methodology is presented in order to provide a discrete prediction of the internal plant information before the delay. But also the intermediate delay information is estimated in order to use "all" the internal information to implement an estimated "extended" state feedback controller.

3. PREDICTION STRATEGY

In order to describe the alternative prediction strategy proposed in this work, consider the discretization (Astrom and Wittenmark 1997) of system (1) (equivalently, 2) subject to a sampling period T and sunder the assumption that the input time delay satisfies $\tau = \alpha T$. For doing this, consider now $G(z)$, the z-transform of $G(s)$, under the action of a sampling and hold device (a zero order hold for instance). A discrete state representation (observable and controllable) can easily be obtained for $G(z)$ (or equivalently for system (1) with $\tau = 0$ (system without delay) as,

$$\begin{aligned} v(k+1) &= A_p v(k) + B_p u(k) \\ w(k) &= C_p v(k) + D_p u(k) \end{aligned} \tag{4}$$

Note that in the case of a zero order hold, the matrices A_P, B_P, C_P can be directly obtained from system (1) with,

$$\begin{aligned} A_p &= e^{\bar{A}T} = \mathcal{L}^{-1}(sI - \bar{A})^{-1} \mid_{t=T} \\ B_p &= \int_0^T e^{\bar{A}(T-\tau)} \bar{B} d\tau \\ C_p &= \bar{C},\ D_p = \bar{D} \end{aligned}$$

where $\mathcal{L}^{-1}$ is the inverse of the Laplace operator. In order to simplify the developments of the paper it is assumed that the discretization process produces a system for which $D_p = 0$.

The discrete-time representation for the delay term e^{-hs} can be easily found as z^{-k}. A discrete state space representation for this model produces,

$$\begin{aligned} x_d(k+1) &= A_d x_d(k) + B_d u_d(k) \\ y(k) &= C_d x_d(k) \end{aligned} \tag{5}$$

with

$$x_d(k) = \left[\, x_{d1}(k)\ x_{d2}(k)\ \cdots\ x_{d\alpha}(k) \,\right]^T$$

$$A_d = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{\alpha \times \alpha}$$

$$B_d = \left[\, 0 \ \cdots \ 0 \ 1 \,\right]^T \in \mathbb{R}^{\alpha \times 1}$$
$$C_d = \left[\, 1 \ 0 \ \cdots \ 0 \,\right] \in \mathbb{R}^{1 \times \alpha}.$$

From the basic properties of the z-transform, the cascade systems $G(z)$ and $z^{-\alpha}$ (or (4) and (5)) are equivalent to system $G(z)z^{-\alpha}$. Then, considering $u_d(k) = w(k)$ the complete discrete-time model (observable) for system (1) takes the form,

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{6}$$

where $x(k) = [\, x_d(k)^T\ v(k)^T \,]^T$, and the α-row of A is $[\, 0\ ..\ 0\ C_p \,]$ and $x_\alpha(k) = w(k)$. Then, a discrete state predictor for the original system can be design as a state observer for (6), described as follows. Let us define,

$$\hat{x}(k) = [\, \widehat{x_d}(k)^T\ \widehat{v}(k)^T \,]^T,$$

the prediction $\hat{x}(k)$ of the signal $x(k)$ is obtained by a predictor of the form,

$$\begin{aligned} \widehat{x_d}(k+1) &= A_d \widehat{x_d}(k) \\ &+ B_d \hat{w}(k) - G_1 e_y(k) \end{aligned} \tag{7}$$

$$\begin{aligned} \hat{v}(k+1) &= A_p \hat{v}(k) + B_p u(k) - G_2 e_y(k) \\ \hat{w}(k) &= C_p \hat{v}(k) \end{aligned} \tag{8}$$

where

$$e_y(k) = \hat{y}(k) - y(k),\ \hat{y}(k) = \hat{x}_1(k)$$

and

$$\begin{aligned} G_1 &= [\, g_1\ g_2\ ..\ g_\alpha \,]^T \\ G_2 &= [\, g_{\alpha+1}\ g_{\alpha+2}\ ..\ g_{\alpha+n} \,]^T. \end{aligned}$$

Or simply,

$$\begin{aligned} \widehat{x}(k+1) &= A\widehat{x}(k) + Bu(k) - G[\hat{y}(k) - y(k)] \\ \hat{y}(k) &= C\widehat{x}(k) \end{aligned} \tag{9}$$

with $G = [\, G_1^T \mid G_2^T \,]^T$, $G_1 \in \mathbb{R}^{\alpha \times 1}$ $G_2 \in \mathbb{R}^{n \times 1}$.

The structure of the proposed predictor it is shown in Figure 2.

Considering the preceding developing it is now possible to formally state the following lemma, without need of any additional proof.

Lemma 1. Consider system (1) and the compensator (7)-(8). It is always possible to find a real valued vector $G = [\, G_1^T \mid G_2^T \,]^T$, $G_1 \in \mathbb{R}^{\alpha \times 1}$ $G_2 \in \mathbb{R}^{n \times 1}$ such that the output $\widehat{x}(k)$ of the compensator (7)-(8) provides the estimation of the signal $x(k)$ for the original system (6). This is, $\underset{K \to \infty}{lim} [\widehat{x}(k) - x(k)] = 0$.

Note that $\widehat{x_d}(kT)$ as a estimation of $x_d(kT) = \left[\, x_{d1}(kT)\ x_{d2}(kT)\ \cdots\ x_{d\alpha}(kT) \,\right]^T$, is a prediction of the delayed output $y(kT)$, i.e., $\widehat{x_{d1}}(kT) = \widehat{y}(kT), \widehat{x_{d2}}(kT) = \widehat{y}(kT+T), ... \widehat{x_{d\alpha}}(kT) = \widehat{y}(kT+\alpha T) = \hat{w}(kT)$. Then, the main idea of this work is to use this information added to $\hat{v}(k)$, the state estimated of the plant in order to build the control law, just computing F such that the spectrum of $(A - BF)$ is a stable set (and placing the roots following performance specifications) . Then we can implement the discrete control law as $u(k) = r(k) - F\widehat{x}(k)$.

4. SIMULATION RESULTS

The aim of this section is to present some academic examples, all of them containing a deadtime in the forward path, to illustrate the goodness of the proposed prediction strategy. The first case study consists of an unstable first order system taken from (Xian *et al.* 2005). The performance of the system with the proposed predictor

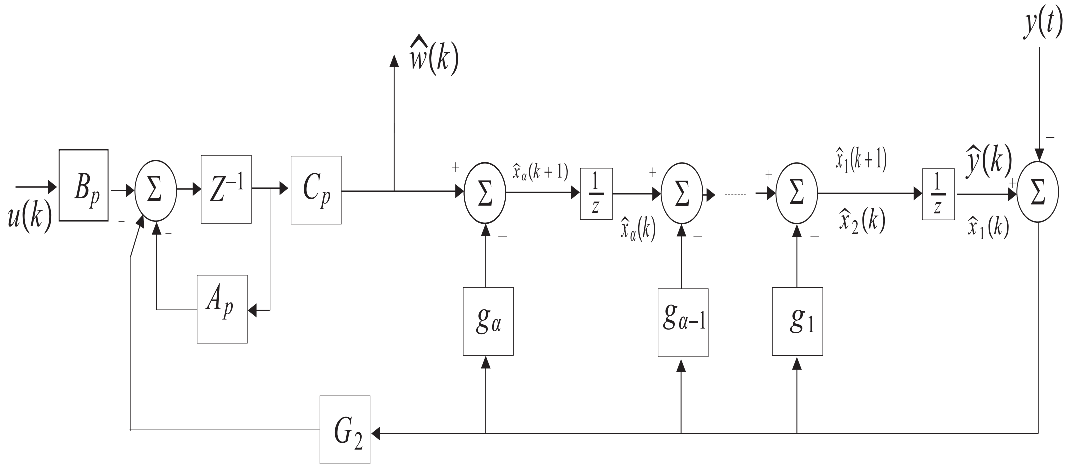

Fig. 2. General predictor structure

are compared with the results in (Xian *et al.* 2005) using the same compensator used in the referred paper.

The second case consists of a more complicate plant: a second order unstable system. It is exposed to parametric perturbations and to different initial conditions.

Example 1. Consider the unstable delayed system(Xian *et al.* 2005),

$$\frac{Y(s)}{U(s)} = \frac{4e^{-5s}}{10s-1} = G_1(s)e^{-5s}. \qquad (10)$$

A discrete-time version of this system, considering a ZOH is given by,

$$G(z) = \frac{0.4207z^{-5}}{(z-1.105)} = G_1(z)z^{-5}.$$

Note that the term $G_1(z)$ corresponds to the subsystem without delay $G_1(s)$ and z^{-5} correspond to the discretization of the time delay term e^{-5s}. Now, an observable representation in state variables can be obtained by considering,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4207 \\ 0 & 0 & 0 & 0 & 0 & 1.105 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T,$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$D = [0].$$

In order to stabilize $(A - GC)$, the vector G is computed as,

$$G = [1.005, 1.1105, 1.2271, 1.356, 1.4984, 3.9355]$$

that locates the poles of the system at

$$[0.1, 0, 0, 0, 0, 0].$$

The controlled here proposed is then

$$u(kT) = (1/.8923)r(kT) - (1.5)F\widehat{x}(kT) \qquad (11)$$

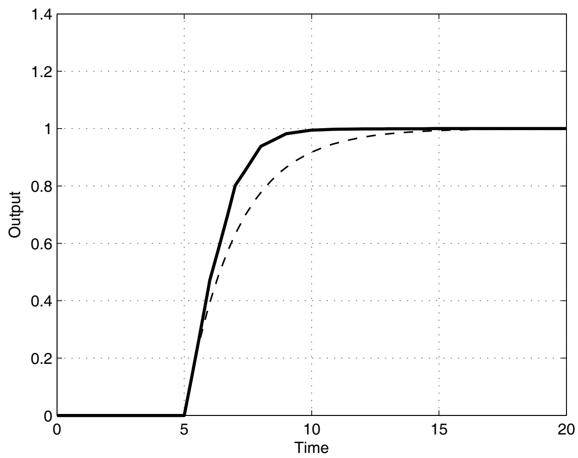

Fig. 3. $y(t)$ (solid line) of system (10)-(11) versus the one (dashed line) obtained in (Xian et. al 2005).

with F computed in order to stabilize $(A - BF)$. With F = [0, −0.0003, 0.00059, −0.0713, 0.4754, 0.4050] the poles are relocated at

$$\{0.2,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1\}.$$

The controller used in (Xian *et al.* 2005) is

$$u(kT) = (1.25)r(kT) - (1.5)\hat{w}(kT) \qquad (12)$$

with $r(k)$ a unitary step input.

In the following tree figures it is compared the output $y(t)$ (solid line) of system (10)-(11) versus the output (dashed line) obtained in (Xian *et al.* 2005). In Figure 3 we can observe that although the performance of both systems is very similar, the controller here proposed provides a faster answer. In Figure 4 is possible to see that when the time delay is set to $\tau = 4.5$ sec. the controller here proposed presents a better performance. A similar conclusion can be obtained in Figure 5 where a time delay of $\tau = 5.5$ sec. is used.

Example 2. Consider now the unstable second order time-delay system,

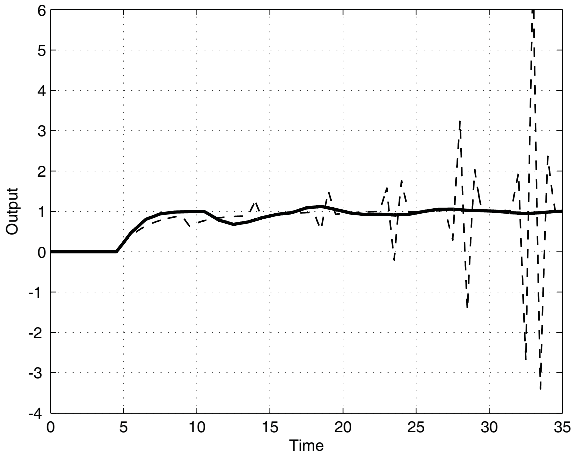

Fig. 4. Time delay $\tau = 4.5$

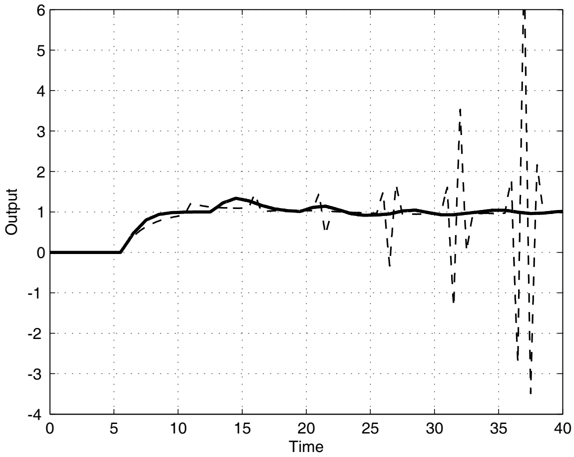

Fig. 5. Time delay $\tau = 5.5$

$$\frac{Y(s)}{U(s)} = \frac{(s+2)e^{-0.4s}}{(s+1)(s-2)}.$$

A discrete-time version of this system, considering a ZOH and $T = 0.1\,\mathrm{sec}$. is,

$$G(z) = \frac{0.11588(z-0.8182)z^{-4}}{(z-0.9048)(z-1.221)}.$$

In this case the observable representation in state space variables can be obtained as,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0948 & 0.1159 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1.1050 & 2.1260 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, D = [0].$$

The closed loop poles of the predictor are located at $[0.1, 0, 0, 0, 0, 0]$ by considering a vector,

$$G = [2.026,\ 3.202,\ 4.569,\ 6.175,\ 52.799,\ 112.909].$$

The controlled here proposed is

$$u(kT) = r(kT) - (1.5)F\widehat{x}(kT) \qquad (13)$$

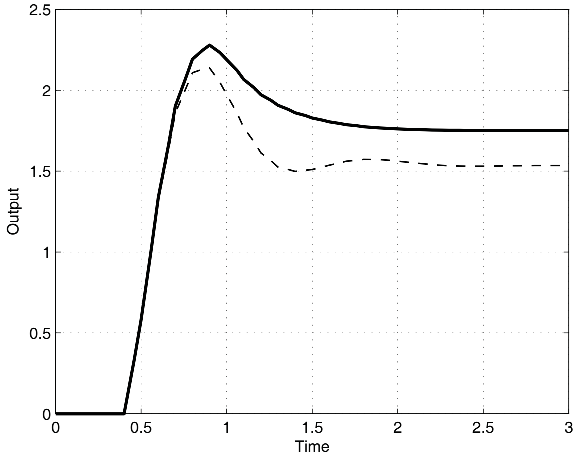

Fig. 6. Ideal case (solid line) and perturbed case (dashed line)

with F computed in order to stabilize $(A - BF)$. With $F = [-0.0076,\ 0.1768,\ -1.4969,\ 5.9230,\ -0.0415,\ 0.0260]$ the poles are relocated at $\{0.6,\ 0.5,\ 0.4,\ 0.3,\ 0.2,\ 0.1\}$.

In Figure 6 it is shown performance of the control strategy: The solid line shows the case of ideal conditions and the dashed is the response of the system when the plant parameters are perturbed in the following way:

$$G_p(s) = \frac{(s+2)e^{-0.4s}}{(s+0.9)(s-1.8)}.$$

It is possible to appreciate the adequate performance of the system in both situations.

5. CONCLUSIONS

In this work is considered the regulation problem for a liner time-invariant time-lag system. Based on the exact discrete time model of the system it is proposed a discrete-time control strategy intended to compensate the effects of the involved time delay and to stabilize the overall closed loop system. The regulation strategy is based on the design of a discrete time observer that provides a prediction of "all" the internal information in the system (the discrete states variables). The propose methodology provides a simple way to extend the ideas of the classical smith predictor to a wider class of time-lag systems that are open loop unstable.

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