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NONPARAMETRIC METHOD FOR IDENTIFICATION OF MIMO HAMMERSTEIN MODELS

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Abstract: A new nonparametric approach to identify multivariable Hammerstein models is presented in this paper. The linear dynamic subsystem is identified and represented by its finite impulse response (FIR) model, and, the static nonlinearity is identified and represented as an MIMO input-output mapping. By specially designed test signals, the estimation of FIRs for multivariable linear subsystems can be conducted under a SISO framework and can be decoupled from the identification of the static nonlinearity. Due to the nonparametric nature, the representation of MIMO Hammerstein model may not be unique. By making uses of this fact, several parameters can be adjusted to shape the model for achieving engineer's requirement. These above-mentioned representations can be used to obtain an exact process model or an apparent model suitable for control design.

Keywords: system identification, Hammerstein model, MIMO, nonparametric.

1. INTRODUCTION

Most chemical processes are better represented by nonlinear models, which can be able to describe the global behaviour of the system over the whole operating range. Thus, many research activities have focused on developing identification methods of nonlinear systems. One of the most frequently used nonlinear model structure is the Hammerstein model, which are composed of a memoryless static nonlinearity followed by a linear dynamical system.

Many techniques have been proposed for the identification of Hammerstein systems. Most of the methods focus on single-input-single-output (SISO) systems, while only some methods can handle multi-input-multi-output (MIMO) systems (e.g. Verhaegen and Westwick, 1996; Al-Duwaish and Karim, 1997). These techniques mainly differ in the way the static nonlinearity is represented and in the type of optimization problem that is finally resulted. In parametric approaches, the static nonlinearity is expressed in terms of a given functional form (e.g. polynomial, neural network, expansion of basis functions) with a set of parameters. However, a priori knowledge of the process is required to select an appropriate form. Regardless of the parameterization scheme chosen, the resulted optimization problem (sometimes non-convex) is usually difficult to solve and the global convergence of the estimated parameters is not always guaranteed. Recently, Lee et al. (2005) proposed an identification method by a special test signal that enables the decoupling of the identification of the linear dynamical part from that of static nonlinearity. Chan et al. (2006) use cardinal cubic spline functions to model the static nonlinearity and then the nonlinear identification problem is converted into a linear one. Although the optimization problem has been simplified, the parameterization of static nonlinearity is still required.

Motivated by the above problems for Hammerstein model identification, a new nonparametric approach is presented in this paper. The parameterization of system is not required and complicate optimization problem is avoided. By specially designed test

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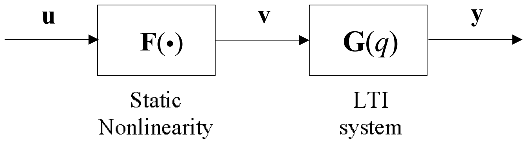

Fig. 1. Hammerstein model.

signals, the identification of multivariable linear subsystem can be conducted under a SISO framework and can be decoupled from the identification of the static nonlinearity. Due to the nonparametric nature, the representation of MIMO Hammerstein model may not be unique. Several parameters within the model can be adjusted to obtain an exact process model or an apparent model for control system design.

2. PROBLEM FORMULATION

A Hammerstein system is schematically represented in Fig. 1. The system consists of a static nonlinearity $\mathbf{F}(\bullet)$ followed by a linear time invariant (LTI) system $\mathbf{G}(q)$ which contains all the dynamics of the process. Let $\mathbf{H}(\ell)$ be the impulse response matrix of a $m \times n$ (m inputs, n outputs) LTI system with elements $\{h_{ij}(\ell)\}_{i=1,2,\cdots n;\, j=1,2,\cdots m}$, and $\mathbf{F}(\bullet) = \{f_j(\bullet)\}_{j=1,2,\cdots,m}$: $\mathbb{R}^m \to \mathbb{R}^m$ be a nonlinear vector function. The input-output relationship is then given by

$$\begin{aligned} \mathbf{y}(k) &= \sum_{\ell=0}^{L-1} \mathbf{H}(\ell)\,\mathbf{v}(k-\ell) + \mathbf{e}(k) \\ \mathbf{v}(k) &= \mathbf{F}\left(\mathbf{u}(k)\right) \end{aligned} \tag{1}$$

where $\mathbf{u}(k)$, $\mathbf{v}(k) \in \mathbb{R}^m$, $\mathbf{y}(k)$ and $\mathbf{e}(k) \in \mathbb{R}^n$ are the system input, intermediate input (unobserved), output and noise, respectively, and L is the length of the longest $h_{ij}(\ell)$. The identification problem is to find $\mathbf{H}(\ell)$ and input-output mapping of $\mathbf{F}(\bullet)$ based on a devised input $\mathbf{u}(k)$ and the observed output $\mathbf{y}(k)$. This is a nonparametric approach because no specific model structure is imposed on either the nonlinearity or the LTI system. The only assumptions are that the LTI system is stable and $\mathbf{F}(\mathbf{0}) = \mathbf{0}$.

In order to simplify the presentation of our method, the following content of this paper is focused on the identification of a 2×2 Hammerstein system. However, the proposed method can be applied to a general $m \times n$ system straightforwardly.

3. IDENTIFICATION OF LTI SYSTEM

The impulse response matrix of a multivariable LTI system can be estimated by least-squares method (Hsia, 1977) provided that its input and output data are given. For Hammerstein system, the intermediate variable $\mathbf{v}(k)$ is unobserved so that the least-squares method cannot be directly applied. Thus, we propose a special test signal to excite the Hammerstein system for the estimation of its impulse response matrix.

3.1 Process Excitation

A sequential multi-stage process excitation using a special test signal is proposed. The number of stage equals the number of input variable m. At the Jth stage, the Jth input $u_J(k)$ is a pseudo-random binary sequence (PRBS) of which one step value must be zero, while all other inputs $u_j(k)$, $j \neq J$, are zero. It follows that each intermediate variable $v_j(k)$, $j = 1, 2, \cdots, m$, is also a PRBS of which one step value is zero and has the same switch time with $u_J(k)$. As we will show in the following, such test signal can make the identification of LTI system be separated from the nonlinearity.

3.2 Estimation of Impulse Response Matrix

Consider the identification of LTI system of a 2×2 Hammerstein process. Denoting a PRBS with its two step values being a and b as $\mathrm{PRBS}(a,b)$, the input at the first stage is $u_1(k) = \mathrm{PRBS}(\bar{u}_1, 0)$, $u_2(k) = 0$ and, hence

$$\begin{aligned} v_1(k) &= \mathrm{PRBS}\big(f_1(\bar{u}_1, 0), 0\big) \\ v_2(k) &= \mathrm{PRBS}\big(f_2(\bar{u}_1, 0), 0\big) \end{aligned} \tag{2}$$

with the same switch time as $u_1(k)$. The input at the second stage is $u_1(k) = 0$, $u_2(k) = \mathrm{PRBS}(\bar{u}_2, 0)$ and, hence

$$\begin{aligned} v_1(k) &= \mathrm{PRBS}\big(f_1(0, \bar{u}_2), 0\big) \\ v_2(k) &= \mathrm{PRBS}\big(f_2(0, \bar{u}_2), 0\big) \end{aligned} \tag{3}$$

with the same switch time as $u_2(k)$. Notice that the nonlinear functions in $\mathbf{F}(\bullet)$ can be scaled by any nonzero constants because the steady-state gains of the LTI system will compensate it accordingly. Here, we assume, without loss of generality, $f_1(\bar{u}_1, 0) = \bar{u}_1$ and $f_2(0, \bar{u}_2) = \bar{u}_2$. Thus, the intermediate variables at the first stage (Eq.(2)) become $v_1(k) = u_1(k)$ and $v_2(k) = p_2 u_1(k)$, where $p_2 = \bar{u}_2 f_2(\bar{u}_1, 0) \big/ f_2(0, \bar{u}_2)\bar{u}_1$. Similarly, the intermediate variables at the second stage (Eq.(3)) become $v_1(k) = p_1 u_2(k)$ and $v_2(k) = u_2(k)$, where $p_1 = \bar{u}_1 f_1(0, \bar{u}_2) \big/ f_1(\bar{u}_1, 0)\bar{u}_2$.

The input-output relationship of a 2×2 LTI system in Eq.(1) can be written as (the noise term is omitted for simplicity)

$$\begin{aligned} y_1(k) &= \sum_{\ell=0}^{L-1} \left[h_{11}(\ell) v_1(k-\ell) + h_{12}(\ell) v_2(k-\ell) \right] \\ y_2(k) &= \sum_{\ell=0}^{L-1} \left[h_{21}(\ell) v_1(k-\ell) + h_{22}(\ell) v_2(k-\ell) \right] \end{aligned} \tag{4}$$

At the first and second stages, Eq.(4) becomes

$$\begin{aligned} y_1(k) &= \sum_{\ell=0}^{L-1}\left[h_{11}(\ell)+p_2 h_{12}(\ell)\right]u_1(k-\ell) \\ y_2(k) &= \sum_{\ell=0}^{L-1}\left[h_{21}(\ell)+p_2 h_{22}(\ell)\right]u_1(k-\ell) \end{aligned} \tag{5}$$

and

$$\begin{aligned} y_1(k) &= \sum_{\ell=0}^{L-1}\left[p_1 h_{11}(\ell)+h_{12}(\ell)\right]u_2(k-\ell) \\ y_2(k) &= \sum_{\ell=0}^{L-1}\left[p_1 h_{21}(\ell)+h_{22}(\ell)\right]u_2(k-\ell) \end{aligned} \tag{6}$$

respectively. It is seen that Eqs.(5) and (6) represent four SISO LTI systems where the input and output data of each system are known. Let

$$\begin{aligned} h_1^{(1)}(\ell) &= h_{11}(\ell)+p_2 h_{12}(\ell) \\ h_2^{(1)}(\ell) &= h_{21}(\ell)+p_2 h_{22}(\ell) \\ h_1^{(2)}(\ell) &= p_1 h_{11}(\ell)+h_{12}(\ell) \\ h_2^{(2)}(\ell) &= p_1 h_{21}(\ell)+h_{22}(\ell) \end{aligned} \tag{7}$$

At the first stage, $h_1^{(1)}(\ell)$ and $h_2^{(1)}(\ell)$ can be identified, for $\ell = 0,1,\cdots,L-1$, based on data (u_1, y_1) and (u_1, y_2), respectively; at the second stage, $h_1^{(2)}(\ell)$ and $h_2^{(2)}(\ell)$ can be identified based on data (u_2, y_1) and (u_2, y_2), respectively. These SISO identification tasks can be done by standard least-squares method (Hsia, 1977) or subspace-based least-squares algorithm (Jeng and Huang, 2006). Rewrite Eq.(7) in matrix form as

$$\mathbf{Q}(\ell) = \mathbf{H}(\ell)\,\mathbf{P} \tag{8}$$

where

$$\begin{aligned} \mathbf{Q}(\ell) &= \begin{bmatrix} h_1^{(1)}(\ell) & h_1^{(2)}(\ell) \\ h_2^{(1)}(\ell) & h_2^{(2)}(\ell) \end{bmatrix} \\ \mathbf{H}(\ell) &= \begin{bmatrix} h_{11}(\ell) & h_{12}(\ell) \\ h_{21}(\ell) & h_{22}(\ell) \end{bmatrix} \\ \mathbf{P} &= \begin{bmatrix} 1 & p_1 \\ p_2 & 1 \end{bmatrix} \end{aligned} \tag{9}$$

or,

$$\mathbf{H}(\ell) = \mathbf{Q}(\ell)\,\mathbf{P}^{-1}, \quad \ell = 0,1,\cdots,L-1 \tag{10}$$

Notice that $\mathbf{P}$ is a square matrix with all the diagonal elements as unity. Since the elements of $\mathbf{Q}(\ell)$ are identified, the impulse response matrix $\mathbf{H}(\ell)$ can be computed by Eq.(10) with given values of p_1 and p_2 provided $p_1 p_2 \neq 1$. The selection of values of p_1 and p_2 will be discussed later.

4. IDENTIFICATION OF NONLINEARITY

With the arbitrarily given values of p_1 and p_2, denoted as $\tilde{p}_1$ and $\tilde{p}_2$ ($\tilde{p}_1\tilde{p}_2 \neq 1$), the LTI system is estimated by Eq.(10) as

$$\tilde{\mathbf{H}}(\ell) = \mathbf{Q}(\ell)\,\tilde{\mathbf{P}}^{-1} \tag{11}$$

Then, based on $\tilde{\mathbf{H}}(\ell)$, it is possible to reconstruct the unobserved intermediate variable $\tilde{\mathbf{v}}(k)$ from output $\mathbf{y}(k)$. As a result, the input-output mapping of the nonlinearity $\tilde{\mathbf{F}}(\bullet): \mathbf{u}(k) \to \tilde{\mathbf{v}}(k)$ can be built by exciting the system with devised $\mathbf{u}(k)$ which covers the region of interest.

4.1 Process Excitation

The accuracy of extrapolation of a nonlinear function is not guaranteed. Thus, the distribution of introduced input signal to excite the nonlinear function must cover the whole space expanded by input variables. In addition, higher density has to be used in the region where high model accuracy is desired. If there is not enough a priori knowledge about the process, uniformly distributed signal is recommended as test input. In this paper, multivariable multi-step signal is used. For example, if the numbers of possible value of u_1 and u_2 are n_1 and n_2, respectively, then the test signal contains a total of $n_1 n_2$ steps which are all combinations of u_1 and u_2, with one sampling interval for each step. The sequence of these steps can be randomly ordered.

4.2 Estimation of Nonlinear Mapping

Assume the introduced multi-step signal contains $N+1$ steps, i.e. $\mathbf{u}(k), k = 0,1,\cdots,N$, and $\mathbf{u}(k) = \mathbf{0}$ for $k > N$. Then, the LTI system of Eq.(4) can be rewritten, based on $\tilde{\mathbf{H}}(\ell)$ and $\tilde{\mathbf{v}}(k)$, as matrix form

$$\boldsymbol{\eta} = \tilde{\boldsymbol{\Phi}}\,\tilde{\boldsymbol{\upsilon}} \tag{12}$$

where

$$\tilde{\boldsymbol{\Phi}} = \begin{bmatrix} \tilde{\boldsymbol{\Phi}}_{11} & \tilde{\boldsymbol{\Phi}}_{12} \\ \tilde{\boldsymbol{\Phi}}_{21} & \tilde{\boldsymbol{\Phi}}_{22} \end{bmatrix}, \quad \tilde{\boldsymbol{\upsilon}} = \begin{bmatrix} \tilde{\boldsymbol{\upsilon}}_1 \\ \tilde{\boldsymbol{\upsilon}}_2 \end{bmatrix}, \quad \boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{\eta}_1 \\ \boldsymbol{\eta}_2 \end{bmatrix} \tag{13}$$

with

$$\tilde{\boldsymbol{\Phi}}_{ij} = \begin{bmatrix} \tilde{h}_{ij}(0) & 0 & \cdots & 0 \\ \tilde{h}_{ij}(1) & \tilde{h}_{ij}(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{h}_{ij}(N) & \tilde{h}_{ij}(N-1) & \cdots & \tilde{h}_{ij}(0) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{h}_{ij}(N+L-1) & \tilde{h}_{ij}(N+L-2) & \cdots & \tilde{h}_{ij}(L-1) \end{bmatrix}$$

$$\tilde{h}_{ij}(\ell) = 0 \quad \text{for } \ell \geq L$$

$$\tilde{\boldsymbol{\upsilon}}_j = \begin{bmatrix} \tilde{v}_j(0) & \tilde{v}_j(1) & \cdots & \tilde{v}_j(N) \end{bmatrix}^T$$

$$\boldsymbol{\eta}_i = \begin{bmatrix} y_i(0) & y_i(1) & \cdots & y_i(N+L-1) \end{bmatrix}^T$$

Notice that if the LTI system has time delay, the rows, where all elements are zero, of $\tilde{\boldsymbol{\Phi}}$ and $\boldsymbol{\eta}$ have to be removed. Since the row number of $\tilde{\boldsymbol{\Phi}}$ is always larger than the column number and all elements of $\tilde{\boldsymbol{\Phi}}$ are identified, we can now estimate the unobserved intermediate variable $\tilde{\boldsymbol{\upsilon}}$ by the method of least-squares, i.e.

$$\hat{\tilde{\boldsymbol{\upsilon}}} = \left(\tilde{\boldsymbol{\Phi}}^T\tilde{\boldsymbol{\Phi}}\right)^{-1}\tilde{\boldsymbol{\Phi}}^T\boldsymbol{\eta} \tag{14}$$

Thus, the nonlinearity is identified as the mapping of $\tilde{\mathbf{F}}(\bullet):\mathbf{u}(k)\rightarrow\hat{\tilde{\mathbf{v}}}(k)$. Furthermore, a functional form of the nonlinearity, e.g. multivariable polynomial with cross-terms, can be found to fit this data set, if it is desired.

4.3 Measurement Noise

If the noise $\mathbf{e}(k)$ is random white noise with zero mean, then $\hat{\tilde{\boldsymbol{\upsilon}}}$ is an unbiased estimate of $\tilde{\boldsymbol{\upsilon}}$. Moreover, if the row number of $\tilde{\boldsymbol{\Phi}}$ approaches infinity, $\hat{\tilde{\boldsymbol{\upsilon}}}$ is a consistent estimate of $\tilde{\boldsymbol{\upsilon}}$. To increase the row number of $\tilde{\boldsymbol{\Phi}}$, we can proceed the test using the signal of which the steps are identical to those of previously used input, but the sequence is randomly scrambled. In this way, more equations for the unknown $\tilde{\boldsymbol{\upsilon}}$ can be set up in Eq.(12) and hence a more consistent estimate of $\tilde{\boldsymbol{\upsilon}}$ could be resulted. However, this is at the cost of prolonged experiment time. In case of strong noise level, it is suggested that the measurement is first passed through a filter to reduce the effect of noise so that the model accuracy can be maintained without extra test. Another way to deal with the noise is fitting the estimated data set with a nonlinear function because the noise will be filtered out by the fitting procedure.

5. DETERMINATION OF MODEL REPRESENTATION

According to the identification algorithm presented in Sections 3 and 4, the resulted Hammerstein model is represented in Fig. 2. In this model, the LTI system consists of two blocks, $\mathbf{Q}(\ell)$ and $\tilde{\mathbf{P}}^{-1}$, where $\mathbf{Q}(\ell)$ is well-determined by several SISO FIR identification tasks. However, the off-diagonal elements of $\tilde{\mathbf{P}}$ can be arbitrarily given provided that $\tilde{\mathbf{P}}$ is non-singular. Based on the selected $\tilde{\mathbf{P}}$, the associated intermediate variable $\tilde{\mathbf{v}}$ is estimated and then a nonlinearity $\tilde{\mathbf{F}}(\bullet)$ is resulted accordingly.

Since the proposed identification method is a nonparametric one, the static nonlinearity and LTI system are not restricted to any specific structures. Thus, these two components of a MIMO Hammerstein model are not uniquely determined if only input and output data are given. In other words, for a given input set, many combinations of different static nonlinearity $\tilde{\mathbf{F}}(\bullet)$ and LTI system $\tilde{\mathbf{H}}(\ell)$ can produce identical output with that of the actual nonlinear process. Therefore, it gives engineers the flexibility to choose a desired model representation. In the proposed model shown in Fig. 2, this flexibility is achieved by selecting the off-diagonal elements of $\tilde{\mathbf{P}}$, as discussed in the following.

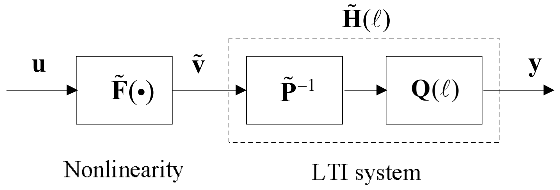

Fig. 2. Representation of identified Hammerstein model.

5.1 Exact Model Representation

The result of identification can be made unique if a prior knowledge about the model structure of the LTI system is incorporated. Generally, the linear dynamics can be assumed as the ARX form with known order (r_{ij}, s_{ij}) and delay d_{ij}, i.e.

$$\begin{aligned} G_{ij}(q) &= \frac{y_i(t)}{u_j(t)} = \frac{B_{ij}(q)q^{-d_{ij}}}{A_{ij}(q)} \\ &= \frac{\left(b_{ij,0}+b_{ij,1}q^{-1}+b_{ij,2}q^{-2}+\cdots+b_{ij,s_{ij}}q^{-s_{ij}}\right)q^{-d_{ij}}}{1-a_{ij,1}q^{-1}-a_{ij,2}q^{-2}-\cdots-a_{ij,r_{ij}}q^{-r_{ij}}} \end{aligned} \tag{15}$$

The corresponding FIR, for $\ell > (d_{ij}+s_{ij})$ satisfies the following relation:

$$\begin{aligned} h_{ij}(\ell) &= a_{ij,1}h_{ij}(\ell-1)+a_{ij,2}h_{ij}(\ell-2) \\ &\quad +\cdots+a_{ij,r_{ij}}h_{ij}(\ell-r_{ij}) \end{aligned} \tag{16}$$

Thus, the off-diagonal elements of $\mathbf{P}$ can be determined such that the condition in Eq.(16) is satisfied. Although the parameters a_{ij} are unknown, they can be computed from $\tilde{h}_{ij}(\ell)$ by the method of least-squares if the off-diagonal elements of $\tilde{\mathbf{P}}$ are given. That is

$$\mathbf{a}_{ij} = \left(\boldsymbol{\Gamma}_{ij}^{T}\boldsymbol{\Gamma}_{ij}\right)^{-1}\boldsymbol{\Gamma}_{ij}^{T}\boldsymbol{\varphi}_{ij} \tag{17}$$

where

$$\mathbf{a}_{ij} = \begin{bmatrix} a_{ij,1} & a_{ij,2} & \cdots & a_{ij,r_{ij}} \end{bmatrix}^T$$

$$\boldsymbol{\varphi}_{ij} = \begin{bmatrix} \tilde{h}_{ij}(d_{ij}+s_{ij}+1) & \tilde{h}_{ij}(d_{ij}+s_{ij}+2) & \cdots & \tilde{h}_{ij}(L) \end{bmatrix}^T$$

$$\boldsymbol{\Gamma}_{ij} = \begin{bmatrix} \tilde{h}_{ij}(d_{ij}+s_{ij}) & \tilde{h}_{ij}(d_{ij}+s_{ij}-1) & \cdots & \tilde{h}_{ij}(d_{ij}+s_{ij}-r_{ij}+1) \\ \tilde{h}_{ij}(d_{ij}+s_{ij}+1) & \tilde{h}_{ij}(d_{ij}+s_{ij}) & \cdots & \tilde{h}_{ij}(d_{ij}+s_{ij}-r_{ij}+2) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{h}_{ij}(L-1) & \tilde{h}_{ij}(L-2) & \cdots & \tilde{h}_{ij}(L-r_{ij}) \end{bmatrix}$$

Then, the off-diagonal elements of $\mathbf{P}$ can be estimated by solving an optimization problem. For a 2×2 system, it is

$$\{p_1, p_2\} = \arg\min_{\tilde{p}_1,\tilde{p}_1}\left(\sum_{i=1}^{2}\sum_{j=1}^{2}\left\|\boldsymbol{\varphi}_{ij}-\boldsymbol{\Gamma}_{ij}\mathbf{a}_{ij}\right\|_2\right) \tag{18}$$

As a result, a unique representation of Hammerstein model, i.e. the exact model representation, is obtained. If the model order and delay are unknown, the above optimization problem can be solved repetitively using different sets of (r_{ij}, s_{ij}, d_{ij}) until the residual is smaller than required.

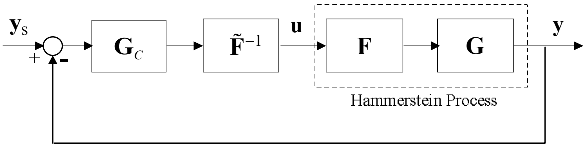

Fig. 3. Control structure of Hammerstein system.

5.2 Model Representation for Control Design

The result of process identification enables us to design a model-based control system, such as model predictive control (MPC). Since many model representations can exhibit identical input-output relationship, identification of the exact model representation described previously may not be our primary goal. Alternatively, identifying a model representation which can help us to achieve some control relevant purposes (e.g. better control performance, simpler control design) is usually more desirable.

The most straightforward nonlinear control strategy for Hammerstein process is applying an inverse function of the static nonlinearity model, $\tilde{\mathbf{F}}^{-1}(\bullet)$, precedent to the Hammerstein process and then designing a linear controller, $\mathbf{G}_C$, based on the LTI model, $\tilde{\mathbf{H}}(\ell)$. This control structure is shown in Fig. 3, where $\mathbf{G}_C$ can be conventional PID controller or MPC controller. Assume the nonlinearity $\tilde{\mathbf{F}}(\bullet)$ is invertible so that the design of $\mathbf{G}_C$ depends only on $\tilde{\mathbf{H}}(\ell)$. Thus, the off-diagonal elements of $\tilde{\mathbf{P}}$ can be determined to meet the desired characteristic imposed on the LTI system $\tilde{\mathbf{H}}(\ell)$. Two cases are discussed in the following, but engineers can specify other conditions based on their demands.

Steady-State Decoupling. By the proposed method, if the system is square, the MIMO LTI system can be modelled as a decoupled system at steady-state without using additional decoupler in the loop, i.e. the LTI model is "self-decoupled" at steady-state. Let $K_i^{(j)} = \sum_{\ell=0}^{L-1} h_i^{(j)}(\ell)$ be the steady-state gain of the dynamics described by $h_i^{(j)}(\ell)$ and $\mathbf{K}$ be the steady-state gain matrix of $\mathbf{Q}(\ell)$ in Eq.(9) with

$$\mathbf{K} = \begin{bmatrix} K_1^{(1)} & K_1^{(2)} \\ K_2^{(1)} & K_2^{(2)} \end{bmatrix} \tag{19}$$

By Eq.(11), if $\tilde{\mathbf{P}} = \tilde{\mathbf{P}}_d$ is chosen such that $\mathbf{K}\tilde{\mathbf{P}}_d^{-1}$ is a diagonal matrix, the LTI system described by $\tilde{\mathbf{H}}(\ell)$ is decoupled at steady-state. In other words, a steady-state decoupler, $\tilde{\mathbf{P}}_d$, has been automatically imbedded into the model. Let $\mathbf{D} = \mathbf{K}\tilde{\mathbf{P}}_d^{-1}$. The off-diagonal elements of $\tilde{\mathbf{P}}_d$ are obtained by solving equations of

$$\left\{D_{ij}\right\}_{i=1,2,\cdots,n;\ j=1,2,\cdots,m;\ i\neq j} = 0 \tag{20}$$

For a 2×2 system, the resulted $\tilde{\mathbf{P}}_d$ is

$$\tilde{\mathbf{P}}_d = \begin{bmatrix} 1 & K_2^{(1)}/K_2^{(2)} \\ K_1^{(2)}/K_1^{(1)} & 1 \end{bmatrix} \tag{21}$$

Diagonally Dominant. The diagonal dominance of a MIMO model is usually desired because it means slight interactions between loops, so that better control performance can be achieved. To this end, the off-diagonal elements of $\tilde{\mathbf{P}}$ can be estimated by solving the following optimization problem to reduce the loop interactions

$$\left\{\tilde{p}_1^*, \tilde{p}_2^*\right\} = \arg\min_{\tilde{p}_1, \tilde{p}_2} \left(w_1 \frac{\left\|\tilde{\mathbf{h}}_{12}\right\|_2}{\left\|\tilde{\mathbf{h}}_{11}\right\|_2} + w_2 \frac{\left\|\tilde{\mathbf{h}}_{21}\right\|_2}{\left\|\tilde{\mathbf{h}}_{22}\right\|_2} \right) \tag{22}$$

where w_1, w_2 are weighting factors of two loops and $\tilde{\mathbf{h}}_{ij} = \left[\tilde{h}_{ij}(0) \quad \tilde{h}_{ij}(1) \quad \cdots \quad \tilde{h}_{ij}(L-1)\right]^T$. Alternatively, similar to steady-state decoupling, $\tilde{\mathbf{P}}$ can also be estimated to maximize the diagonal dominance of the LTI model at a certain frequency ω by

$$\tilde{\mathbf{P}}_{dd} = \begin{bmatrix} 1 & K_2^{(1)}(\omega)/K_2^{(2)}(\omega) \\ K_1^{(2)}(\omega)/K_1^{(1)}(\omega) & 1 \end{bmatrix} \tag{23}$$

where $K_i^{(j)}(\omega) = \left|\sum_{\ell=0}^{L-1} h_i^{(j)}(\ell)\, e^{-j\ell\omega}\right|$.

6. SIMULATION EXAMPLE

Consider a nonlinear process described by Hammerstein system as follows:

$$\mathbf{G}(q) = \begin{bmatrix} \dfrac{0.1q^{-1}+0.2q^{-2}}{1-1.2q^{-1}+0.35q^{-2}} & \dfrac{q^{-1}}{1-0.7q^{-1}} \\ \dfrac{0.3q^{-1}+0.2q^{-2}}{1-0.8q^{-1}} & \dfrac{q^{-1}+0.5q^{-2}}{1+0.4q^{-2}} \end{bmatrix}$$

$$\mathbf{F}(\mathbf{u}) = \begin{bmatrix} u_1^3 - u_1u_2 + 2u_2^2 \\ 0.582\left(e^{(u_1+u_2)} - 1\right) \end{bmatrix}$$

The inputs for estimation of impulse response matrix are $u_1(k) = \mathrm{PRBS}(1,0)$, $u_2(k) = 0$ at the first stage and $u_1(k) = 0$, $u_2(k) = \mathrm{PRBS}(1,0)$ at the second stage. The input for identification of the nonlinearity is a multivariable multi-step signal which covers all the combination of $u_1 = u_2 = \{-2:0.2:2\}$, i.e. a total of $21\times 21 = 441$ steps. The sampling interval is takes as one. To simulate the measurement noise, random white noise is added to the output, with $|e_1(k)| \le 0.2$ and $|e_2(k)| \le 0.2$. First, the matrix $\mathbf{Q}(\ell)$ is identified from the PRBS test. Here, two model representations of LTI system, i.e. exact (Model 1) and steady-state decoupled (Model 2) ones, are used for illustration. For identifying the exact one, the order (r_{ij}, s_{ij}) of ARX dynamics is assumed known. The results obtained by Eq.(18) are $p_1 = 1.973$ and $p_2 = 0.965$. Notice that their exact values are $p_1 = 2$ and $p_2 = 1$. For steady-state

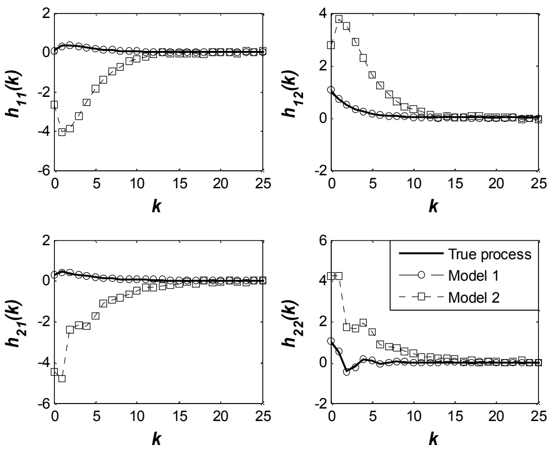

Fig. 4. Impulse response of LTI system.

decoupled model, the results computed from Eq.(21) are $p_1 = 0.592$ and $p_2 = 1.357$. The impulse response sequences of these two models together with that of original process are shown in Fig. 4. Based on the two LTI models, two corresponding input-output mappings of the nonlinearity are thus estimated from the test of multi-step signal. These mappings have been fitted with multivariable polynomials as shown in Fig. 5. The outputs of original Hammerstein system and two identified models to random input are simulated as shown in Fig. 6. As mentioned previously, although these two representations of Hammerstein model are quite different, both of them can produce very similar outputs with that of original system.

7. CONCLUSION

In this paper, a new method has been presented to identify and model MIMO Hammerstein systems. By the proposed special test signals, the identifications of LTI subsystem and nonlinearity are separated. Because the proposed method is a nonparametric one, the parameterization of system is not required in advance and thus the representation of model is not unique. The exact model representation can be obtained by incorporating a priori knowledge of the LTI system. In addition, engineers can have the flexibility of modelling Hammerstein systems to meet their demands by adjusting a few parameters. Simulation results have shown that accurate model can be identified by the proposed method and different model representations can be used to describe or predict the behaviour of the system.

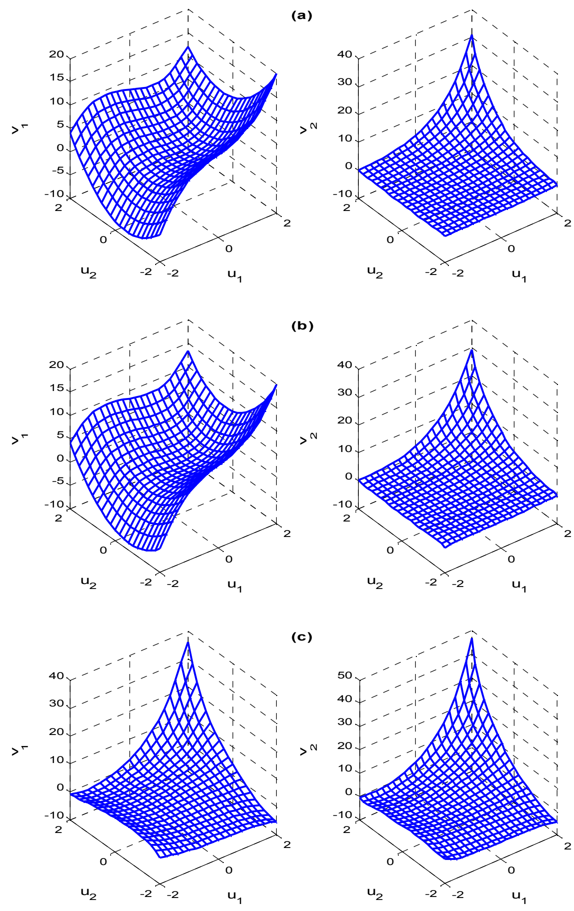

Fig. 5. Plot of nonlinearity. (a) True process (b) Model 1 (c) Model 2

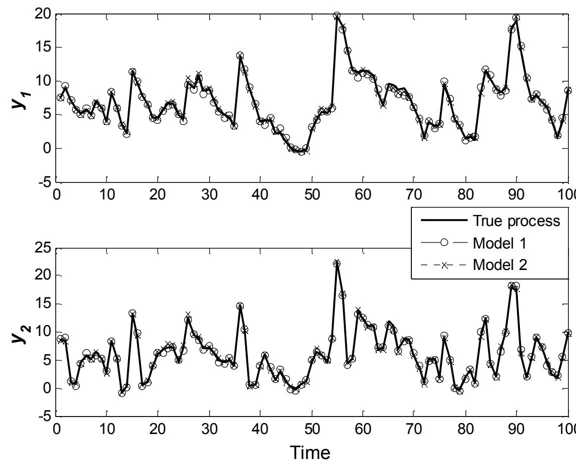

Fig. 6. Process and model outputs to random input.

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