GAIN AND PHASE MARGINS ITERATIVE CONTROLLER TUNING

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Abstract: In this paper it is presented a procedure for closed loop controller tuning using relay experiments. The experiments are used to evaluate gain and phase margins. The controller redesign is performed by minimizing a frequency domain criterion based on gain and phase margins in addition to crossover frequency. The procedure may be repeated iteratively. Simulation examples illustrate the properties of the design scheme.

Keywords: PI controllers, relay, closed-loop identification, gain and phase margins.

1. INTRODUCTION

In many plants, process control usually is implemented through many control levels. The PID controller is often used in regulatory levels to provide the process robust stability and fast response to load disturbances (Skogestad and I.Postlethwaite, 2005). In most systems, a simple PI controller is sufficient to handle the regulatory level functions. Frequently this controllers need to be redesigned under operation due to poor performance.

In this context, techniques for identification and controller redesign using closed-loop data have become very attractive. The closed-loop identification doesn't cause stops in system operation unlike open-loop identification. Other reasons which can be listed are demands on safety in process operation, unstable processes and restrictions in production. Its has also been argued that in closed loop it is possible to obtain representative restricted complexity process models in an interesting frequency range which can be used to redesign controllers such as PI and PID (Albertos and A.Salas, 2002).

The redesigned controller specifications may be expressed in terms of gain and phase margins that are classical measures of robustness and together with the crossover frequency represent the time performance of the closed-loop as well. Several gain and phase margin tuning methods have been proposed in literature. Some are based on graphical methods which are not suitable for PID autotuning, or simplified equations using approximation what do not guarantee that the specification will be achieved. There are techniques based on numerical methods also as the one presented in (A. Karimi and R.Longchamp, 2003). Its major drawback is that is not suitable for non-minimum phase systems. Other methods are based on simple models as the one presented in (W.K. Ho and L.S.Cao, 1995).

Model based tuning techniques that rely only on simple dynamics may have poor performance when the process has a too complex dynamics. For example, decoupled process that usually results in complicated diagonal elements with non-minimum phase behavior (Nordfeldt and T.Hägglund, 2006).

The use of good information provided by specific relay experiments together either with accurate simple models or special experiments on interesting frequencies can overcome those problems.

In this paper is proposed a method for iterative controller evaluation and autotuning based on the knowledge of Gain and Phase Margins and crossover frequency. It is established a frequency criterion that is optimized using a gradient method. The numerical problem is solved using closed loop frequency information together with either restricted complexity models accurate on the interesting frequency range using constrained optimization for identification or estimation through specific relay based experiments. Open loop experiments are not necessary and the proposed method can be applied to a large number of processes types including non-minimum phase and time delay dynamics.

The paper is organized as follows. Initially the tuning technique will be presented. After that the used experiments and relevant information obtained will be described. Following it will be explained how experimental data can be used to solve the problem using representative simple models or frequency estimation. Finally simulation examples illustrate the effectiveness of the proposed method.

2. PROBLEM STATEMENT

Consider the closed loop shown in Fig. 1. The process transfer function is given by $G(s)$ while the controller is $C(s) = K_p(1 + \frac{1}{T_i s})$. The closed loop transfer function from the reference signal $r(t)$ to the process output $y(t)$ is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{L(s)}{1 + L(s)} \tag{1}$$

where $L(s) = G(s)C(s)$ is the *Loop Gain Transfer Function.*

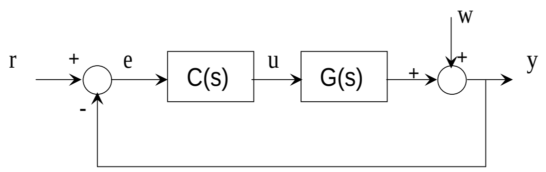

Fig. 1. The Closed Loop.

The crossover and critical frequencies are ω_g and ω_c. The Phase Margin (ϕ_m) is the frequency point where

$$\angle L(j\omega_g) = -\pi + \phi_m \text{ and } |L(j\omega_g)| = 1.$$

The Gain Margin (A_m) is defined as

$$\angle L(j\omega_c) = -\pi \text{ and } |L(j\omega_c)| = \frac{1}{A_m}.$$

The problem statement is: Given a closed loop system, evaluate robustness and performance through Gain and Phase Margins plus the crossover frequency estimation using closed loop experiments. If necessary, redesign the controller to iteratively and safely match the desired specifications.

3. THE CONTROLLER REDESIGN PROCEDURE

The controller redesign is based on optimization of a frequency criterion that is defined as follows:

$$J(\rho) = \left[\left(\frac{\omega_g - \omega_d}{\omega_d} \right)^2 + \left(\frac{\phi_m - \phi_d}{\phi d} \right)^2 + \left(\frac{K_u - K_d}{K_d} \right)^2 \right]$$

where $\rho = [K_p; \frac{K_p}{T_i}]$ is the controller parameter vector, ω_d and ω_g are the desired and measured crossover frequencies, ϕ_m and ϕ_d are the measured and desired phase margins, K_u is the loop gain magnitude at the critical frequency and K_d is the inverse of the desired gain margin A_d.

The controller parameters are obtained using a gradient based optimization technique, the iterative Newton's formula

$$\rho_{i+1} = \rho_i - \gamma_i R^{-1} J'(\rho_i).$$

To solve this numerical problem is necessary compute the Gradient and the Hessian,$J'(\rho_i)$ and R respectively.

The algorithm will converge if the Hessian exists and is positive definite, even if the gradient is approximated. The choice of the parameter γ is very important to convergence (Luenberger, 1965) and brings to an iterative tuning strategy.

The gradient is given by

$$J'(\rho) = \left(\frac{\omega_g - \omega_d}{\omega_d^2} \right) \frac{\partial \omega_g}{\partial \rho} + \left(\frac{\phi_m - \phi_d}{\phi^2 d} \right) \frac{\partial \phi_m}{\partial \rho} + \left(\frac{K_u - K_d}{K_d^2} \right) \frac{\partial K_u}{\partial \rho}$$

and Hessian can be computed as

$$R = \frac{1}{\omega_d^2} \frac{\partial \omega_g}{\partial \rho} (\frac{\partial \omega_g}{\partial \rho})^T + \frac{1}{\phi_d^2} \frac{\partial \phi_m}{\partial \rho} (\frac{\partial \phi_m}{\partial \rho})^T + \frac{1}{K_d^2} \frac{\partial K_u}{\partial \rho} (\frac{\partial K_u}{\partial \rho})^T$$

where the second order derivatives have been suppressed to avoid a non-positive definite Hessian.

The problem solution requires the computation of some derivatives what is made using the frequency response functions features at the critical and crossover frequencies.

This problem is solved in (A. Karimi and R.Longchamp, 2003) approximating the derivatives to permit the use of Bode's Integrals. In this paper derivative approximations are not used which makes the procedure applicable to a larger number of processes types.

The derivative $\frac{\partial \omega_g}{\partial \rho}$ is computed observing that between optimization iterations the magnitude of the loop gain is one at the crossover frequency

$$\frac{\partial |L(j\omega_g)|}{\partial \rho} = 0$$

what results in

$$\frac{\partial \omega_g}{\partial \rho} = -\frac{|G(j\omega_g)| \frac{\partial |C(j\omega_g)|}{\partial \rho}}{\left(|C(j\omega_g)| \frac{\partial |G(j\omega)|}{\partial \omega}|_{\omega_g} + |G(j\omega_g)| \frac{\partial |C(j\omega)|}{\partial \omega}|_{\omega_g}\right)}.$$

For the Phase Margin derivative $\frac{\partial \phi_m}{\partial \rho}$ it follows that at ω_g

$$\frac{\partial \angle L(j\omega_g)}{\partial \rho} = \frac{\partial \angle C(j\omega_g)}{\partial \rho} + \\ + \left(\frac{\partial \angle C(j\omega)}{\partial \omega}|_{\omega_g} + \frac{\partial \angle G(j\omega)}{\partial \omega}|_{\omega_g}\right) \frac{\partial \omega_g}{\partial \rho}.$$

Finally, the computation of $\frac{\partial K_u}{\partial \rho}$ uses

$$\frac{\partial |L(j\omega_c)|}{\partial \rho} = |G(j\omega_c)| \frac{\partial |C(j\omega_c)|}{\partial \rho} + \\ \left(|C(j\omega_c)| \frac{\partial |G(j\omega)|}{\partial \omega}|_{\omega_c} + |G(j\omega_c)| \frac{\partial |C(j\omega)|}{\partial \omega}|_{\omega_c}\right) \frac{\partial \omega_c}{\partial \rho}$$

and

$$\frac{\partial \omega_c}{\partial \rho} = -\left(\frac{\partial \angle C(j\omega)}{\partial \omega}|_{\omega_c} + \frac{\partial \angle G(j\omega)}{\partial \omega}|_{\omega_c}\right)^{-1} \frac{\partial \angle C(j\omega_c)}{\partial \rho}$$

where was used the fact that between iterations at the critical frequency

$$\frac{\partial \angle L(j\omega_c)}{\partial \rho} = 0.$$

The solution presented here can be easily extended to PID controllers.

4. THE RELAY EXPERIMENTS

The necessary information for controller optimization is estimated using the following Gain Margin and Phase Margin experiments. This experiments are applied to closed loop systems.

4.1 Gain Margin Experiment

The standard relay test is used to estimate the critical point and frequency. It can be shown (see (Schei, 1994)) that if this relay test is applied to a closed loop system, with transfer function $T(s)$, the limit cycle occurs at the closed loop critical frequency and the gain margin can be computed from the loop gain

$$L(j\omega_c) = G(j\omega_c) C(j\omega_c) \cong \frac{m}{1-m}.$$

m is the magnitude of $T(s)$ at the critical frequency.

4.2 Phase Margin Experiment

A general relay procedure to estimate the frequency point for which a given transfer function has a desired gain is presented in (Arruda and P.R.Barros, 2003). If the loop-gain is under test, the feedback structure is presented in Fig. 2.

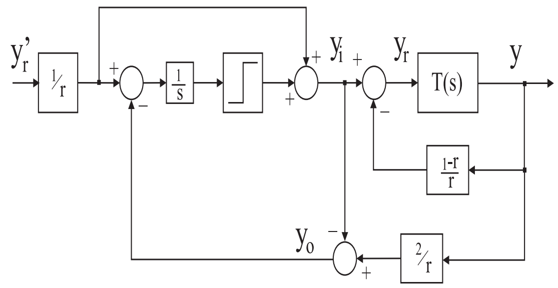

Fig. 2. Loop Gain Transfer Function Estimation.

This procedure allows the estimation of the frequency at which the loop transfer function magnitude is close to r. Selecting $r = 1$, the current gain crossover frequency ω_g and the phase margin can be estimated. In this case the scheme reduces to the one presented in (Schei, 1992).

5. PARAMETERS ESTIMATION

The information obtained through the Relay Experiments are used to estimate the necessary parameters for computation.

Two approaches are proposed in this paper using either good approximate models close to the critical and crossover frequencies or frequency estimation through relay based generated excitations. Both are suitable for any sort of process given the relay feedback systems develops limit cycle.

The frequency response is estimated using the DFT on the reference y_r and output signals y, computing the closed loop gain T_i and then recovering the loop gain (L_i) using the loop equations

$$L_i(j\omega) = \frac{T_i(j\omega)}{1 - T_i(j\omega)}.$$

5.1 Model-Based Procedure

In this paper the used models are first-order plus dead-time (FOPDT) continuous-time represented by

$$G(s) = \frac{b}{s+a} e^{-\theta s}.$$

The model identification is performed using closed loop time-frequency data to estimate two continuous-time models. The time data is discrete in time and frequency domain equalities constraints are used to accurate the model close to the crossover frequency and the critical frequency, i.e. $\hat{G}(j\hat{\omega}_g)$ and

$\hat{G}(j\hat{\omega}_c)$ respectively. The procedure solves a time least-squares problem subjected to a constraint in frequency. The available data is discrete-time and the constraint is obtained through the process frequency response on the first harmonic of the relay experiments signals. Details of the technique can be found in (G. Acioli Jr and P.R.Barros, 2006).

The derivatives calculated using the model are given by

$$\frac{\partial \angle G(j\omega)}{\partial w} = \frac{a}{\omega^2 + a^2} - \theta$$

and

$$\frac{\partial |G(j\omega)|}{\partial w} = \frac{-b\omega}{(\omega^2 + a^2)^{1.5}}$$

It should be noted that no approximations are necessary. An advantage of the use of models is to provide a good guess to project parameters specifications.

5.2 Estimation-Based Procedure

An alternative approach estimating the derivatives with respect to frequency is also considered here.

To do that, ie to estimate the frequency responses at additional frequencies, two square wave excitation signals can be generated based on the critical and crossover frequency estimated using the relay experiments similarly to the procedure applied in (Berger and Barros, 2005). Let us choose frequencies $1.1\omega_c$ and $1.1\omega_g$. This procedure will need 4 experiments instead of 2. The derivatives are estimated as follows:

$$\frac{\partial \angle G(j\omega)}{\partial w} \simeq \frac{\angle G(j1.1\omega) - \angle G(j\omega)}{1.1\omega - \omega}$$

and

$$\frac{\partial |G(j\omega)|}{\partial w} \simeq \frac{|G(j1.1\omega)| - |G(j\omega)|}{1.1\omega - \omega}$$

An advantage of this approach is no need for a process model. However, more experiments are necessary what it is time consuming.

6. SIMULATION EXAMPLES

In this section three representative simulation examples are shown which illustrate the use of the technique. The noise power applied during identification is 0.001 and the DFTs are computed evaluating just one period of the signals. The closed loop time response is simulated applying a step of magnitude 1 on the setpoint and after that step disturbance of magnitude 0.1 on the process output. The crossover frequency is measured in rad/s and the phase margin in degrees. At the first two example it is used $\gamma_i = 1$.

6.1 Example 1

The process is given by

$$G(s) = \frac{(-0.3s + 1)(0.08s + 1)}{(2s + 1)(s + 1)(0.04s + 1)(0.2s + 1)(0.05s + 1)^3}.$$

and the initial controller designed using the Ziegler-Nichols Step Response Method (Ziegler and N.B.Nichols, 1942) is

$$C_i(s) = 2.69(1 + \frac{1}{3.90s}).$$

Initially $A_m = 1.67$ and $\phi_m = 32$. In this case, the estimated crossover frequency was $\hat{\omega}_g = 0.87$.

The estimated model using Gain Margin experiment data is

$$G_{mg}(s) = \frac{0.1729}{s - 0.5782} e^{-1.1253s}.$$

The estimated model using Phase Margin experiment data is

$$G_{mf}(s) = \frac{0.3196}{s - 0.3056} e^{-0.4815s}.$$

It should be observed that ARX models estimated in closed-loop may be unstable even if the process is stable (Albertos and A.Salas, 2002). It is desired $A_d = 2$, $\phi_d = 50$ and $\hat{\omega}_d = 1.4\hat{\omega}_g$. As a model is known ($G_{mf}$), the desired crossover frequency can be chosen considering the performance that can be achieved without sacrificing robustness (Albertos and A.Salas, 2002). The redesigned controller using the model-based procedure is

$$C_{rm}(s) = 1.8(1 + \frac{1}{4.47s}).$$

Alternatively, the computations can be executed using only frequency estimate data. The redesigned controller using the estimation based procedure is

$$C_{re}(s) = 1.91(1 + \frac{1}{3.29}).$$

In table 1 the frequency response points and its derivatives using frequency and models estimation are compared with the actual ones.

Table 1.

	Process	*Model*	*Estimated*
$\lvert G(j\omega_c)\rvert$	0.2112	0.0786	0.2138
$\frac{\partial \lvert G(j\omega)\rvert}{\partial w}\rvert_{\omega_c}$	−0.2239	−0.0724	−0.2388
$\frac{\partial \angle G(j\omega)}{\partial w}\rvert_{\omega_c}$	−1.1336	−1.3881	−0.9400
$\lvert G(j\omega_g)\rvert$	0.3810	0.3739	0.3750
$\frac{\partial \lvert G(j\omega)\rvert}{\partial w}\rvert_{\omega_g}$	−0.4996	−0.3529	−0.4061
$\frac{\partial \angle G(j\omega)}{\partial w}\rvert_{\omega_g}$	−1.6405	−0.8389	−1.0973

It can be seen that the derivatives obtained using the models are not accurate close to ω_c. The results obtained using the model-based procedure are closer to the Phase Margin specification because the gradient at the critical frequency is smaller and not accurate. This can be noted at the Nyquist plots also (Fig.3 and Fig.4).

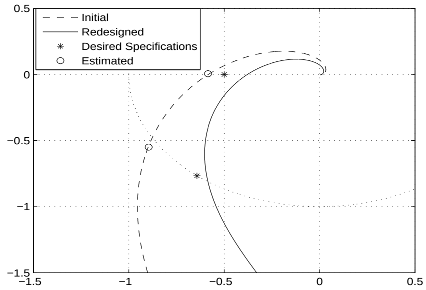

Fig. 3. Nyquist Diagram for the Model-Based Tuning (Example 1)

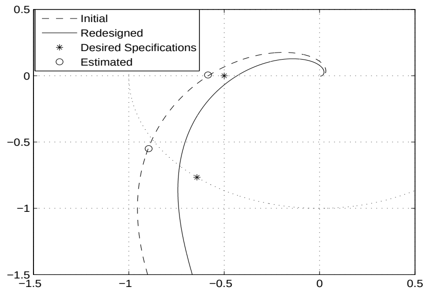

Fig. 4. Nyquist Diagram for the Estimation-Based Tuning (Example 1)

The time response is shown in Fig.5 and Fig.6. The technique has improved the system stability margin and time response using both estimation procedures. The estimation-based procedure has presented better results. Therefore, in the following examples, only it will be shown.

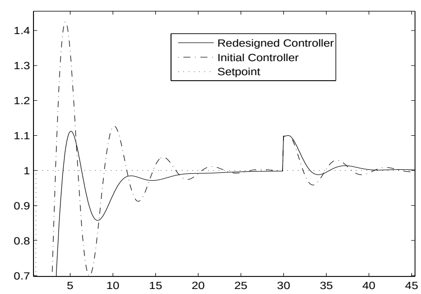

Fig. 5. Time Response for the Model-Based Tuning (Example 1)

6.2 Example 2

The process is given by

$$G(s) = \frac{(6s+1)(3s+1)}{(10s+1)(8s+1)(s+1)} e^{-0.3s}.$$

and the initial controller is

$$C_i(s) = 5.94(1 + \frac{1}{6.4s}).$$

It was designed using the SIMC-PI Rules(Skogestad and I.Postlethwaite, 2005). The initial actual Gain

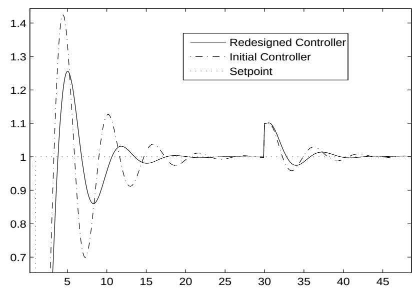

Fig. 6. Time Response for the Estimation-Based Tuning (Example 1)

and Phase Margins are respectively 4.23 and 93.5, the estimated ones are 5.65 and 100. The estimated crossover frequency was $\hat{\omega}_g = 0.64$. It is desired improve system response. The new specification are $A_d = 3.5$, $\phi_d = 80$ and $\hat{\omega}_d = \hat{\omega}_g$. In this case the tuning procedure is repeated iteratively, two iterations are used to show the convergence of the method. The final controller is

$$C_{re}(s) = 6.53(1 + \frac{1}{3.81s}).$$

The final crossover frequency is 0.82. The results are shown in Fig.7 and Fig. 8. It can be noted that performance and stability have been improved.

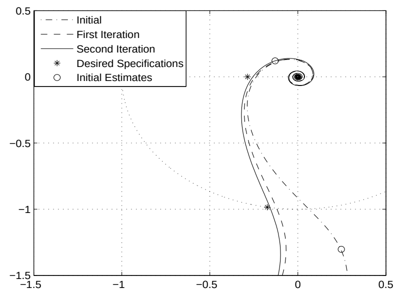

Fig. 7. Nyquist for the Estimation-Based Tuning (Example 2)

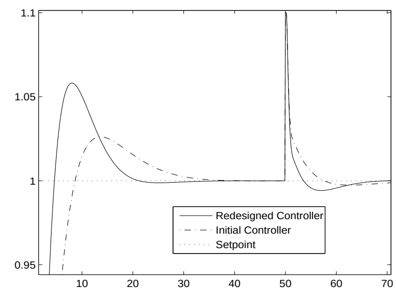

Fig. 8. Time Response for the Estimation-Based Tuning (Example 2)

6.3 Example 3

The process is given by

$$G(s) = \frac{1}{(s+1)^8}.$$

and the initial controller designed using the Ziegler-Nichols Step Response Method is

$$C_i(s) = 1.365(1 + \frac{1}{12.41s}).$$

Initially $A_m = 1.24$, $\phi_m = 38.6$ and $\hat{\omega}_g = 0.2805$. It should be noted that the both stability margins are too small what turns the tuning critical. It is desired to improved system robustness safely. Therefore, the iterative tuning procedure will not move too far the frequency points at each the first iteration. The new specification are $A_d = 2$, $\phi_d = 50$ and $\hat{\omega}_d = \hat{\omega}_g$. The procedures takes three iterations and it is used $\gamma = 0.5$ at the first iteration and $\gamma = 1$ in the last two iterations. The final controller is

$$C_{re}(s) = 0.37(1 + \frac{1}{1.95s})$$

and the obtained margins are $A_m = 1.54$ and $\phi_m = 46.5$. The results for the estimation based procedure are shown in Fig. 9 and Fig.10.

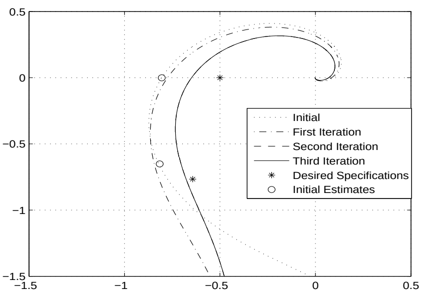

Fig. 9. Nyquist for the Estimation-Based Tuning (Example 3)

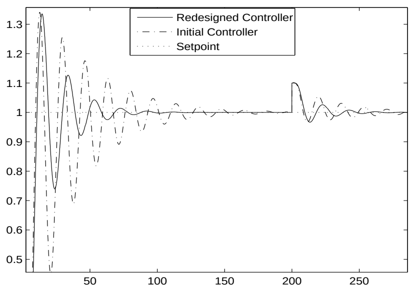

Fig. 10. Time Response for the Estimation-Based Tuning (Example 3)

In this example was shown how the procedure can be applied to safely approach the desired specification. The final controller safely achieved a better trade-off between desired stability specification and time response.

7. CONCLUSIONS

In this paper a controller evaluation and redesign technique was presented. The closed loop is evaluated using relay experiments then an optimization technique is applied to a frequency criterion. This numerical problem is solved using two approaches: identifying simple models using constrained optimization or frequency response estimation using relay information based experiments. It was shown how the technique can safely approach specifications. The procedures can be applied to a large number of industrial processes. Simulation examples illustrate its effectiveness.

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