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STABLE INDIRECT ADAPTIVE FUZZY CONTROL BASED ON TAKAGI-SUGENO MODEL

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Abstract: This paper presents an indirect adaptive fuzzy control scheme for nonlinear uncertain stable plants with unmeasurable states. A discrete-time T-S fuzzy model is employed as a dynamic model of an unknown plant. Based on this model, a feedback linearization controller is designed and applied to both the model and the plant. Parameters of the model are updated on-line to allow for partially unknown and time-varying plants. Stability analysis shows that the adaptive controller guarantees the boundedness of all the closed-loop signals and achieves bounded tracking error. In the ideal case where there is no modelling error and the signal for parameter learning is persistently exciting, perfect tracking is ensured. The effectiveness of the method is verified by simulation examples. Copyright © 2007 IFAC

Keywords: ***nonlinear control, adaptive control, fuzzy control,.***

1. INTRODUCTION

Adaptive control has become very popular in many fields of control engineering and attracted much attention in developing advanced applications. Adaptive control is based on feedback of signals in a controlled system for control adaptation to effectively handle system uncertainties (Tao, 2003). The adaptive control can be divided into two classes: direct adaptive control and indirect adaptive control. The former tunes the parameters of the controller while the latter updates the parameters of the model. This paper focuses on indirect adaptive control, which means the T-S fuzzy model is adapted on-line. Design of provably stable indirect adaptive fuzzy controller posed more difficult theoretical questions. Some of the early adaptive control schemes using continuous-time T-S fuzzy model (Takagi &Sugeno, 1985) were presented in the papers (Wang, 1994; Spooner 1996; Yoo, 1998) with stability results. Wang's control law is composed of a certainty equivalence control and a supervisory controller which is used to force the tracking error not to exceed certain error ball. Spooner and Passino's approach consists of a certainty equivalence control term, a bounding control term used to restrict the plant output trajectory, and a sliding mode control term to compensate the approximation errors. Their parameter adaptation algorithms are chosen to make the derivative of a certain Lyapunov equation nonpositive. Recent adaptive fuzzy control schemes for continuous-time system can be found in the papers (Golea，2003; Park & Cho, 2004). In Park and Cho (2004), an adaptive law for updating parameters is designed based the Lyapunov theory and with the on-line parameter estimator, any type of fuzzy controllers works adaptively to the parameter perturbations.

All the above adaptive control schemes assume that full states of the controlled plant are available for measurement. Adaptive output feedback control for nonlinear systems when full states are not measurable remains one of the outstanding problems. Kulawski and Brdys (2000) present an adaptive

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control scheme for uncertain nonlinear plants with unmeasurable states based on dynamic neural networks. In their approach, a recurrent neural network is employed as a dynamical model of the plant and the control law is designed by using the states of the neural network instead of the plant. Our proposal in this paper is inspired by the above consideration and applies it to discrete-time adaptive fuzzy control system. A discrete-time T-S fuzzy model is trained to approximate the input-output behaviour of an unknown continuous-time plant by using I/O data from the plant. Then an appropriate control law is calculated completely based on the model, using the model structure, parameters and states. The control input is applied to both the plant and the model. The parameters of the model are updated on-line by recursive least square estimation (RLSE) method, which is fast and is able to ensure the boundedness of parameters (Tao, 2003). With this approach, at least the model can be stabilized and provides bounded control input without extra supervisory controller. The sufficient conditions for the convergence of tracking error are derived for stable plant.

The paper is organized as follows. The control algorithm including parameter adaptation law is presented in Section 2. Stability of the closed-loop system and convergence of the tracking error are proven in Section 3. Section 4 shows the simulation results on control of a single-link robot manipulator with parameter disturbance. Finally, Section 5 concludes the paper.

2. CONTROL ALGORITHM

2.1 Modelling for adaptive control

It has been shown that T-S fuzzy systems with piecewise linear rule consequents are universal approximators to approximate any continuous nonlinear system to an arbitrary accuracy (Ying, 1998 & 1999) if all fuzzy membership functions are Gaussian and the t-norm is the multiplication. In this paper, the T-S fuzzy model is used to express a real nonlinear plant. That is, the nonlinear plant can be represented by the following fuzzy rules:

R^i : If $x_1(k)$ is M_1^i and $\cdots$ and $x_n(k)$ is M_n^i, then

$$x_n^i(k+1) = \theta_0^i + \theta_1^i x_1(k) + \cdots + \theta_n^i x_n(k) + \theta_{n+1}^i u(k) \tag{1}$$

where R^i ($i = 1, 2, \cdots, N$) denotes the ith fuzzy rule and N is the number of rules. $x_1(k), \cdots, x_n(k)$ are the premise variables which are the past plant output measurements and $u(k)$ is the control input. M_j^i ($i = 1, \cdots, n$) are fuzzy sets which are described by Gaussian functions

$$M_j^i(x_j \mid c_j^i, \sigma_j^i) = \exp\left\{-\frac{(x_j - c_j^i)^2}{2(\sigma_j^i)^2}\right\} \tag{2}$$

The model parameter values θ_j^i, c_j^i and σ_j^i, $i = 1, \cdots, N$, $j = 1, \cdots, n$ are unknown. The T-S fuzzy model approximates a nonlinear system by combining a group of local affine models. The premise of a fuzzy rule defines a local operating region and each consequent describes a local input-output relation in the local operating region. Let $\boldsymbol{x}(k) = [x_1(k), x_2(k), \cdots, x_n(k)]^T = [y(k-n+1), \cdots, y(k-1), y(k)]^T$, then the overall T-S fuzzy model is calculated as

$$\begin{aligned} x_n(k+1) &= \sum_{i=1}^{N} \bar{\omega}^i(\boldsymbol{x} \mid \boldsymbol{c}^i, \boldsymbol{\sigma}^i)\boldsymbol{\theta}^i \begin{bmatrix} 1 & \boldsymbol{x}^T(k) & u(k) \end{bmatrix}^T \\ &= \Psi^T(\boldsymbol{x}(k) \mid \boldsymbol{c}, \boldsymbol{\sigma})\Theta \end{aligned} \tag{3}$$

$$y(k) = x_n(k)$$

where

$$\begin{aligned} &\omega^i(\boldsymbol{x} \mid \boldsymbol{c}^i, \boldsymbol{\sigma}^i) = \prod_{j=1}^{n} M_j^i(x_j \mid c_j^i, \sigma_j^i), \\ &\bar{\omega}^i(\boldsymbol{x} \mid \boldsymbol{c}, \boldsymbol{\sigma}) = \omega^i(\boldsymbol{x} \mid \boldsymbol{c}^i, \boldsymbol{\sigma}^i) \Big/ \sum_{i=1}^{N} \omega^i(\boldsymbol{x} \mid \boldsymbol{c}, \boldsymbol{\sigma}), \\ &\boldsymbol{c} = \left\{c_j^i \mid i = 1, \cdots, N; j = 1, \cdots, n\right\} \\ &\boldsymbol{\sigma} = \left\{\sigma_j^i \mid i = 1, \cdots, N; j = 1, \cdots, n\right\} \end{aligned} \tag{4}$$

$$\boldsymbol{\theta}^i = \begin{bmatrix} \theta_0^i & \cdots & \theta_{n+1}^i \end{bmatrix}^T, \Theta = \left[\left(\boldsymbol{\theta}^1\right)^T \quad \cdots \quad \left(\boldsymbol{\theta}^N\right)^T \right]^T,$$

$$\boldsymbol{x}_e(k) = \begin{bmatrix} 1 & \boldsymbol{x}^T(k) & u(k) \end{bmatrix}^T,$$

$$\Psi^T(\boldsymbol{x} \mid \boldsymbol{c}, \boldsymbol{\sigma}) = \begin{bmatrix} \bar{\omega}^1 \boldsymbol{x}_e^T & \bar{\omega}^2 \boldsymbol{x}_e^T & \cdots & \bar{\omega}^N \boldsymbol{x}_e^T \end{bmatrix}^T.$$

An assumption is made here.

Assumption 1. The modelling error is negligible and there exist parameter values $(\boldsymbol{c}, \boldsymbol{\sigma})$ and Θ that make the model (3) to become a perfect representation of the real plant**.**

From now on, the nonlinear plant is completely represented by (3). Since all the parameters in (3) are unknown, we define an estimation model as

$$\begin{aligned} \hat{x}_n(k+1) &= \sum_{i=1}^{N} \bar{\omega}^i(\hat{\boldsymbol{x}} \mid \hat{\boldsymbol{c}}^i, \hat{\boldsymbol{\sigma}}^i)\hat{\boldsymbol{\theta}}^i \begin{bmatrix} 1 & \hat{\boldsymbol{x}}^T(k) & u(k) \end{bmatrix}^T \\ &= \hat{\Psi}^T(\hat{\boldsymbol{x}}(k) \mid \hat{\boldsymbol{c}}(k), \hat{\boldsymbol{\sigma}}(k))\hat{\Theta}(k) \end{aligned} \tag{5}$$

$$\hat{y}(k) = \hat{x}_n(k)$$

where $(\hat{\boldsymbol{c}}(k), \hat{\boldsymbol{\sigma}}(k))$ and $\hat{\Theta}(k)$ are the estimates of $(\boldsymbol{c}, \boldsymbol{\sigma})$ and Θ at time k. $\hat{\boldsymbol{x}}(k) \in \mathbb{R}^n$ is the model state vector.

2.2 Control synthesis

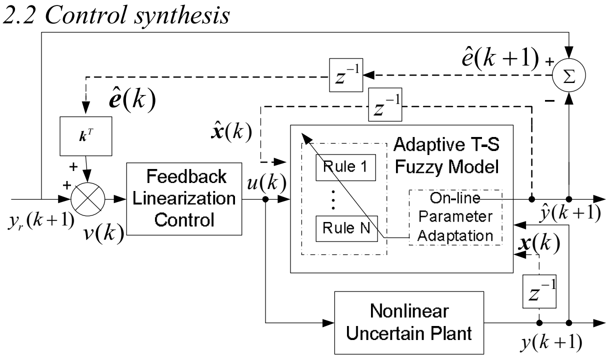

Fig. 1 Adaptive control based on T-S fuzzy model

The control objective is to make the output of the nonlinear plant (3) tracking a specified reference trajectory $y_r \in \mathbb{R}$.

Assumption 2. The reference $y_r(k)$ satisfies

$$\|\mathbf{y}_r(k)\| \le U \tag{6}$$

where $\mathbf{y}_r(k)=\left[y_r(k-n+1) \quad \cdots \quad y_r(k)\right]^T$ and U is a known bound.

The overall control structure is shown in Fig. 1. It consists of three parts: a nonlinear uncertain plant, an adaptive T-S fuzzy model and a model based feedback linearization controller.

The control law is designed based on the model (5) whose evolution of output can be expressed as

$$\begin{aligned}\hat{y}(k+1)&=\sum_{i=1}^{N}\bar{\varpi}^i(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{c}}^i,\hat{\boldsymbol{\sigma}}^i)\hat{\boldsymbol{\theta}}^i(k)\left[1 \quad \hat{\boldsymbol{x}}^T(k) \quad u(k)\right]^T\\&=\sum_{i=1}^{N}\bar{\varpi}^i(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{c}}^i,\hat{\boldsymbol{\sigma}}^i)\hat{\boldsymbol{\theta}}_f^i(k)\left[1 \quad \hat{\boldsymbol{x}}^T(k)\right]^T\\&\quad+\sum_{i=1}^{N}\bar{\varpi}^i(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{c}}^i,\hat{\boldsymbol{\sigma}}^i)\hat{\boldsymbol{\theta}}_g^i(k)u(k)\\&=\hat{f}(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{p}}(k))+\hat{g}(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{p}}(k))u(k)\end{aligned}\tag{7}$$

where

$$\hat{\boldsymbol{\theta}}_f^i(k)=\left[\hat{\theta}_0^i(k) \quad \hat{\theta}_1^i(k) \quad \cdots \quad \hat{\theta}_n^i(k)\right]^T,$$
$$\hat{\boldsymbol{\theta}}_g^i(k)=\hat{\theta}_{n+1}^i(k), \quad \hat{\boldsymbol{p}}(k)=\left\{\hat{\boldsymbol{c}}(k),\hat{\boldsymbol{\sigma}}(k),\Theta(k)\right\}$$
$$\hat{f}(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{p}}(k))=\sum_{i=1}^{N}\bar{\varpi}^i(\hat{\boldsymbol{x}}(k))\hat{\boldsymbol{\theta}}_f^i(k)\left[1 \quad \hat{\boldsymbol{x}}^T(k)\right]^T,\tag{8}$$
$$\hat{g}(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{p}}(k))=\sum_{i=1}^{N}\bar{\varpi}^i(\hat{\boldsymbol{x}}(k))\hat{\boldsymbol{\theta}}_g^i(k).$$

To apply feedback linearization technique to design the control law $\hat{g}(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{p}}(k))$ is required to be invertible.

Assumption 3. $\left|\hat{g}(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{p}}(k))\right|>\varepsilon$ where ε is a small real positive number, which implies the relative degree of the T-S fuzzy model is equal to one.

With the assumption 3, the control law can be calculated as

$$u(k)=\frac{1}{\hat{g}(\hat{\boldsymbol{x}}\,|\,\hat{\boldsymbol{p}}(k))}[-\hat{f}(\hat{\boldsymbol{x}}\,|\,\hat{\boldsymbol{p}}(k))+v(k)]\tag{9}$$

where $v(k)$ is a new input chosen as

$$v(k)=y_r(k+1)+\boldsymbol{k}^T\hat{\boldsymbol{e}}(k\,|\,\hat{\boldsymbol{p}}(k))\tag{10}$$

where $\hat{\boldsymbol{e}}(k\,|\,\hat{\boldsymbol{p}}(k))=\left[\hat{e}(k-n+1\,|\,\hat{\boldsymbol{p}}(k-n+1)) \quad \cdots \quad \hat{e}(k\,|\,\hat{\boldsymbol{p}}(k))\right]^T$ and $\hat{e}(k\,|\,\hat{\boldsymbol{p}}(k))$ is the model tracking error

$$\hat{e}(k\,|\,\hat{\boldsymbol{p}}(k))=y_r(k)-\hat{y}(k\,|\,\hat{\boldsymbol{p}}(k))\tag{11}$$

and $\boldsymbol{k}=(k_n,\cdots,k_1)^T\in\mathbb{R}$ be such that the zeros of the polynomial $h(z)=z^n+k_1z^{n-1}+\cdots+k_n$ lie within the unit circle centered at the origin of the z plane.

The control law (9) applied to (7) results in it being decoupled and linear with respect to the new input $v(k)$

$$\hat{y}(k+1)=v(k)\tag{12}$$

2.3 Online parameter adaptation

The initial T-S fuzzy model can be obtained by either off-line or on-line identification methods. In this paper, the initial model is obtained by the on-line identification method proposed in ((Qi and Brdys, 2005). The initial model is good enough for the model based controller to produce proper control signal in the beginning of closed-loop control. However, the plant may have external disturbances and parameters disturbances so that the initial model needs to be updated. Therefore, the model should be adaptive when plant changes. The parameters of the model can be adjusted on-line to provide an adaptive model.

For fixed parameters $(\hat{\boldsymbol{c}}(k),\hat{\boldsymbol{\sigma}}(k))$, Θ can be estimated by recursive least square estimation (RLSE).

$$\hat{\Theta}(k+1)=\hat{\Theta}(k)-\frac{S(k)\Psi(k)\varepsilon(k+1)}{1+\Psi^T(k)S(k)\Psi(k)}\tag{13}$$

$$S(k+1)=S(k)-\frac{S(k)\Psi(k)\Psi^T(k)S(k)}{1+\Psi^T(k)S(k)\Psi(k)}\tag{14}$$

where $\Psi(k)$ denotes $\Psi(\boldsymbol{x}\,|\,\hat{\boldsymbol{c}}(k),\hat{\boldsymbol{\sigma}}(k))$ and $\varepsilon(k+1)=\hat{y}(k+1)-y(k+1)=\Psi^T(k)\hat{\Theta}(k)-y(k+1)$.

The initial conditions are chosen as

$$\hat{\Theta}(0)=\left[\underline{\hat{\theta}}_1^T(0) \quad \underline{\hat{\theta}}_2^T(0) \quad \cdots \quad \underline{\hat{\theta}}_N^T(0)\right]^T, \quad S(0)=\Omega I$$

where $\underline{\hat{\theta}}_1^T(0) \quad \underline{\hat{\theta}}_2^T(0) \quad \cdots \quad \underline{\hat{\theta}}_N^T(0)$ are the parameters of the initial T-S model and Ω is a large positive constant. Large Ω can speed up parameter convergence, but too large may cause instability. It needs to be chosen properly during on-line adaptation.

3. STABILITY ANALYSIS

3.1 Convergence of model tracking error

Applying (9) to (7) , we get

$$\hat{y}(k+1)=y_r(k+1)+\boldsymbol{k}^T\hat{\boldsymbol{e}}(k\,|\,\hat{\boldsymbol{p}}(k))\tag{15}$$

Since $\hat{e}(k+1\,|\,\hat{\boldsymbol{p}}(k))=y_r(k+1)-\hat{y}(k+1\,|\,\hat{\boldsymbol{p}}(k+1))$, we have the following error dynamics

$$\begin{aligned}\hat{e}(k+1\,|\,\hat{\boldsymbol{p}}(k+1))&=-k_n\hat{e}(k-n+1\,|\,\hat{\boldsymbol{p}}(k-n+1))\\&\quad-\cdots-k_2\hat{e}(k-1\,|\,\hat{\boldsymbol{p}}(k-1))-k_1\hat{e}(k\,|\,\hat{\boldsymbol{p}}(k))\end{aligned}\tag{16}$$

which can be formulated in matrix form

$$\hat{\mathbf{e}}(k+1\,|\,\hat{\boldsymbol{p}}(k+1))=\Lambda\hat{\mathbf{e}}(k\,|\,\hat{\boldsymbol{p}}(k))\tag{17}$$

where

$$\Lambda=\begin{bmatrix}0 & 1 & 0 & 0 & \cdots & 0 & 0\\0 & 0 & 1 & 0 & \cdots & 0 & 0\\\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\0 & 0 & 0 & 0 & \cdots & 0 & 1\\-k_n & -k_{n-1} & \cdots & \cdots & \cdots & \cdots & -k_1\end{bmatrix},\tag{18}$$

Since Λ is a stable matrix ($\left|zI-\Lambda\right|=z^n+k_1z^{n-1}+\cdots+k_n$ is stable), the system (17) is asymptotically stable, which implies $\hat{\boldsymbol{e}}(k\,|\,\hat{\boldsymbol{p}}(k))\to 0$ when $k\to\infty$. The model tracking error will converge to zero.

3.2 Boundedness of parameters and control signal

Assumption 4. In order to deal with the T-S model bring linear in parameters when performing tracking

error boundedness analysis, we shall assume that the premise parameters of the model $(\hat{\boldsymbol{c}},\hat{\boldsymbol{\sigma}})$ are the same as the plant parameters $(\boldsymbol{c},\boldsymbol{\sigma})$.
However, updates of all paramters will be carried during the control simulation studies.

Lemma 1. *The adaptive algorithm (13)-(14) guarantees that* $\tilde{\Theta}(k)$ *and* $\hat{\Theta}(k)$ *are bounded.*

Lemma 2. *If the signal* $\Psi(k)$ *is persistently exciting, then the least-squares algorithm (13)-(14) ensures that* $\lim_{k\to\infty}\|\tilde{\Theta}(k)\|=0$.

These follow from standard properties of the least-square estimation methods applied to linearly parameterized models.

Applying (9) to (7) yields

$$\hat{y}(k+1)=y_r(k+1)+\boldsymbol{k}^T\hat{\boldsymbol{e}}(k) \tag{19}$$

From assumption 2, $y_r(k+1)$ is bounded and since $\hat{\boldsymbol{e}}(k)$ is bounded. Thus, equation (19) implies the boundedness of $\hat{y}(k+1)$. Since $\hat{\boldsymbol{x}}(k)$ is a vector consisting of past outputs, $\hat{\boldsymbol{x}}(k)$ is bounded.
Bounded $\hat{\boldsymbol{x}}(k)$, bounded parameters $\Theta(k)$ and bounded $\bar{\omega}^i(\hat{\boldsymbol{x}}(k))$ imply bounded $\hat{f}(\hat{\boldsymbol{x}}(k))$ and bounded $\hat{g}(\hat{\boldsymbol{x}}(k))$ in (8). Thus, from (9) we obtain the boundedness of $u(k)$.

3.3 Boundedness of plant tracking error

From (3), we have the plant output evolution.

$$\begin{aligned} y(k+1)&=\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x}(k))\boldsymbol{\theta}^i\left[1\quad \boldsymbol{x}^T(k)\quad u(k)\right]^T\\ &=\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}_f^i\left[1\quad \boldsymbol{x}^T(k)\right]^T+\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}_g^i(k)u(k)\\ &=f(\boldsymbol{x}(k))+g(\boldsymbol{x}(k))u(k) \end{aligned} \tag{20}$$

Applying (9) to (20) and after some manipulations, we have

$$\begin{aligned} y(k+1)&=f(\boldsymbol{x}(k))+g(\boldsymbol{x}(k))u(k)\\ &=f(\boldsymbol{x})+g(\boldsymbol{x})u(k)-(\hat{f}(\hat{\boldsymbol{x}})+\hat{g}(\hat{\boldsymbol{x}})u(k))\\ &\quad+y_r(k+1)+\boldsymbol{k}^T\hat{\boldsymbol{e}}(k) \end{aligned} \tag{21}$$

Moving $y_r(k+1)$ to the left side of (21) and after some manipulation, we get

$$\begin{aligned} e(k+1)&=\hat{\Psi}^T(k)\hat{\Theta}(k)-\Psi^T(k)\Theta-\boldsymbol{k}^T\hat{\boldsymbol{e}}(k)\\ &=\hat{\Psi}^T(k)\tilde{\Theta}(k)+\tilde{\Psi}^T(k)\Theta-\boldsymbol{k}^T\hat{\boldsymbol{e}}(k) \end{aligned} \tag{22}$$

where $\Psi(k)$ denotes $\Psi(\boldsymbol{x}(k)\,|\,\boldsymbol{c},\boldsymbol{\sigma})$ and $\hat{\Psi}(k)$ denotes $\hat{\Psi}(\hat{\boldsymbol{x}}(k)\,|\,\hat{\boldsymbol{c}}(k),\hat{\boldsymbol{\sigma}}(k))$.
Considering

$$\begin{aligned} \tilde{\Psi}^T(k)\Theta&=\hat{\Psi}^T(k)\Theta-\Psi^T(k)\Theta\\ &=\sum_{i=1}^{N}\bar{\omega}^i(\hat{\boldsymbol{x}})\boldsymbol{\theta}^{iT}\hat{\boldsymbol{x}}_e(k)-\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}^{iT}\boldsymbol{x}_e(k)\\ &=\sum_{i=1}^{N}\left(\bar{\omega}^i(\hat{\boldsymbol{x}})-\bar{\omega}^i(\boldsymbol{x})\right)\boldsymbol{\theta}^{iT}\hat{\boldsymbol{x}}_e(k)\\ &\quad-\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}_x^{iT}\hat{\boldsymbol{e}}(k)+\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}_x^{iT}\boldsymbol{e}(k) \end{aligned} \tag{23}$$

where $\boldsymbol{\theta}_x^i=\left[\theta_1^i\quad \theta_2^i\quad \cdots\quad \theta_n^i\right]^T$.
Substituting (23) into (22), the plant tracking error dynamics can be obtained as follows

$$\begin{aligned} e(k+1)&=\hat{\Psi}^T(k)\tilde{\Theta}(k)+\sum_{i=1}^{N}\left(\bar{\omega}^i(\hat{\boldsymbol{x}})-\bar{\omega}^i(\boldsymbol{x})\right)\boldsymbol{\theta}^{iT}\hat{\boldsymbol{x}}_e(k)\\ &\quad-\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}_x^{iT}\hat{\boldsymbol{e}}(k)+\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}_x^{iT}\boldsymbol{e}(k)-\boldsymbol{k}^T\hat{\boldsymbol{e}}(k)\\ &=\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}_x^{iT}\boldsymbol{e}(k)+\Gamma(k) \end{aligned} \tag{24}$$

where

$$\begin{aligned} \Gamma(k)&=\hat{\Psi}^T(k)\tilde{\Theta}(k)-\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})\boldsymbol{\theta}_x^{iT}\hat{\boldsymbol{e}}(k)-\boldsymbol{k}^T\hat{\boldsymbol{e}}(k)\\ &\quad+\sum_{i=1}^{N}\left(\bar{\omega}^i(\hat{\boldsymbol{x}})-\bar{\omega}^i(\boldsymbol{x})\right)\boldsymbol{\theta}^{iT}\hat{\boldsymbol{x}}_e(k) \end{aligned} \tag{25}$$

The error dynamics (24) can also be written into a matrix form

$$\boldsymbol{e}(k+1)=\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})A^i\boldsymbol{e}(k)+\bar{\Gamma}(k) \tag{26}$$

where

$$A^i=\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0\\ 0 & 0 & 1 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & 0 & 1\\ \theta_1^i & \theta_2^i & \cdots & \cdots & \cdots & \cdots & \theta_n^i \end{bmatrix},\quad \bar{\Gamma}(k)=\begin{bmatrix}\boldsymbol{0}\\ \Gamma(k)\end{bmatrix}. \tag{27}$$

Assumption 5. There exists a positive constant a and a common positive definite symmetric matrix P for all the matrices A^i ($i=1,\cdots,N$) such that

$$A^{iT}PA^i-P\le -aI<0 \qquad \text{for } i=1,\cdots,N \tag{28}$$

Theorem 1. *Consider the closed-loop system consisting of plant* (3)*, controller* (9) *and parameter adaptation law (13)-(14). If the assumptions 1-5 hold, then*

(i) *the plant tracking error* $\boldsymbol{e}(k)$ *is bounded;*

(ii) *if in addition, the signal* $\Psi(k)$ *is persistently exciting,* $\boldsymbol{e}(k)$ *converges to zero.*

Proof. Consider the following possible Lyapunov function

$$V(\boldsymbol{e}(k))=\boldsymbol{e}(k)^TP\boldsymbol{e}(k) \tag{29}$$

Then the difference of the Lyapunov function along the plant tracking error trajectory (26) is calculated as

$$\begin{aligned} \Delta V&=\boldsymbol{e}(k+1)^TP\boldsymbol{e}(k+1)-\boldsymbol{e}(k)^TP\boldsymbol{e}(k)\\ &=\sum_{i=1}^{N}(\bar{\omega}^i(\boldsymbol{x}))^2\boldsymbol{e}(k)^T(A^{iT}PA^i-P)\boldsymbol{e}(k)+\\ &\quad+\sum_{i=1}^{N}\sum_{j=i+1}^{N}\bar{\omega}^i\bar{\omega}^j\boldsymbol{e}(k)^T(A^{iT}PA^j+A^{jT}PA^i-2P)\boldsymbol{e}(k)\\ &\quad+2\bar{\Gamma}(k)^TP\sum_{i=1}^{N}\bar{\omega}^i(\boldsymbol{x})A^i\boldsymbol{e}(k)+\bar{\Gamma}(k)^TP\bar{\Gamma}(k) \end{aligned} \tag{30}$$

With assumption 5, we have

$$A^{iT}PA^{j}+A^{jT}PA^{i}-2P\le -2aI \qquad \text{for all } i \text{ and } j \quad (31)$$

Applying (31) in (30), the following inequality can be obtained

$$\begin{aligned}\Delta V &\le -\boldsymbol{e}(k)^{T}\left(\sum_{i=1}^{N}\bar{\varpi}^{i}(\boldsymbol{x})\sum_{j=1}^{N}\bar{\varpi}^{j}(\boldsymbol{x})a\right)\boldsymbol{e}(k)\\ &\quad +2\bar{\Gamma}(k)^{T}P\sum_{i=1}^{N}\bar{\varpi}^{i}(\boldsymbol{x})A^{i}\boldsymbol{e}(k)+\bar{\Gamma}(k)^{T}P\bar{\Gamma}(k) \qquad (32)\\ &\le -\frac{1}{2}a\left\|\boldsymbol{e}(k)\right\|^{2}+\left(\frac{2M^{2}}{a}+\lambda_{\max}(P)\right)\left\|\bar{\Gamma}(k)\right\|^{2}\end{aligned}$$

where

$$M=\lambda_{\max}(P)\cdot\max_{i}\left\{\left\|A^{i}\right\|,i=1,\cdots N\right\}\ge\left\|P\right\|\left(\sum_{i=1}^{N}\bar{\varpi}^{i}(\underline{x})\left\|A^{i}\right\|\right)$$

Consider

$$\begin{aligned}&\left\|\bar{\Gamma}(k)\right\|=\left\|\Gamma(k)\right\|\\ &\le\left\|\hat{\Psi}^{T}(k)\right\|\left\|\tilde{\Theta}(k)\right\|+\left(\sum_{i=1}^{N}\left\|\bar{\varpi}^{i}(\boldsymbol{x})\boldsymbol{\theta}_{x}^{iT}\right\|+\left\|\boldsymbol{k}^{T}\right\|\right)\left\|\hat{\boldsymbol{e}}(k)\right\| \quad (33)\\ &+\sum_{i=1}^{N}\left\|\bar{\varpi}^{i}(\hat{\boldsymbol{x}})-\bar{\varpi}^{i}(\boldsymbol{x})\right\|\left\|\boldsymbol{\theta}^{i}\right\|\left\|\hat{\boldsymbol{x}}_{e}(k)\right\|\end{aligned}$$

Since all the elements in (33) are bounded, $\left\|\bar{\Gamma}(k)\right\|$ is bounded. Hence there exists positive constant ρ_{Γ} such that

$$\left\|\bar{\Gamma}(k)\right\|\le\rho_{\Gamma} \qquad (34)$$

We can conclude outside the ball

$$\left\{\boldsymbol{e}(k):\left\|\boldsymbol{e}(k)\right\|<\varepsilon=\frac{\sqrt{4M^{2}+2a\lambda_{\min}(P)}}{a}\rho_{\Gamma}\right\} \quad (35)$$

the Lyapunov function difference can be bounded by

$$\Delta V(\boldsymbol{e}(k))\le 0 \qquad \forall\left\|\boldsymbol{e}(k)\right\|\ge\varepsilon \qquad (36)$$

The hypothesis (i) has now been proven.

With Lemma 2, if the signal $\Psi(k)$ is persistently exciting, then the least-squares algorithm (13)-(14) ensures that $\lim_{k\to\infty}\left\|\tilde{\Theta}(k)\right\|\to 0$. With the assumption 4, $\lim_{k\to\infty}\left\|\tilde{\Theta}(k)\right\|\to 0$ means the model become a perfect representation of the plant, which will make the model state $\boldsymbol{x}(k)$ become the same as the plant state $\boldsymbol{x}(k)$. Thus we have $\lim_{k\to\infty}\left\|\bar{\varpi}^{i}(\hat{\boldsymbol{x}})-\bar{\varpi}^{i}(\boldsymbol{x})\right\|=0$. With the control law (9), we have proven $\lim_{k\to\infty}\left\|\hat{\boldsymbol{e}}(k)\right\|\to 0$. Therefore, when $k\to\infty$, $\left\|\bar{\Gamma}(k)\right\|$ in (33) converges to zero and (32) becomes

$$\Delta V(\boldsymbol{e}(k))\le-\frac{1}{2}a\left\|\boldsymbol{e}(k)\right\|^{2} \qquad k\to\infty \qquad (37)$$

we obtain $\boldsymbol{e}(k)\to 0$ as $k\to\infty$. The hypothesis (ii) and the Theorem 1 have now been proven.

5. SIMULATION RESULTS

In this example, the validity and effectiveness of the proposed control scheme are illustrated through the tracking control of a single-link robot arm described by

$$\begin{aligned}\dot{q}_{1}(t)&=q_{2}(t)\\ \dot{q}_{2}(t)&=-\alpha_{1}\sin[q_{1}(t)]-\alpha_{2}q_{2}(t)+bu(t)\end{aligned} \qquad (38)$$

where $y(t)=q_{1}(t)$ is the arm position which is the measured output, $q_{2}(t)$ is the angular velocity which is *unmeasurable* and $u(t)$ is the input torque. The parameters α_{1} and α_{2} depend on the mass and length of the arm. The values of α_{1}, α_{2} and b are chosen as $\alpha_{1}=\alpha_{2}=b=1$. The premise variables are selected as $x_{1}(kT)=y[(k-1)T]$ and $x_{2}(kT)=y(kT)$. The sampling time $T=0.01\text{sec}$. The control objective is to make q_{1} track a bounded reference signal. The initial T-S fuzzy model for closed-loop control is an input-output model obtained by the on-line identification method proposed in Qi and Brdys (2005) which follows the structure of (38).

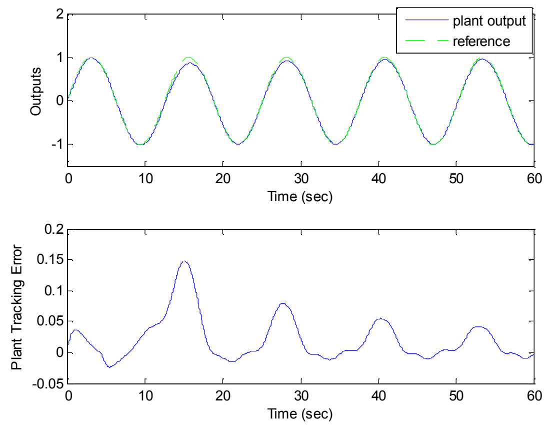

Fig. 2. Plant tracking response with parameter disturbance (α_{2} changes from 1 to 1.5 at $t=5s$)

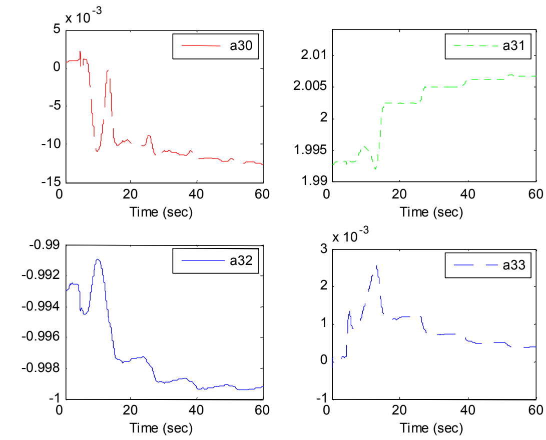

Fig. 3. Consequent parameter adaptation of rule 3

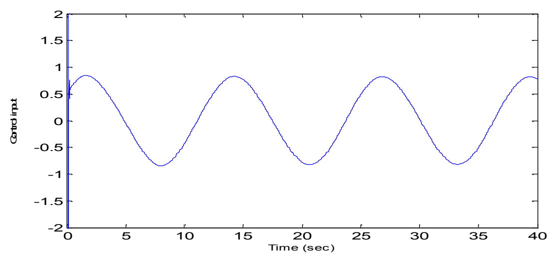

Fig. 4. Control signal

Fig. 2 illustrates the plant tracking response with disturbance and tracking error and Fig. 4 shows the control signal. The plant parameter α_{2} changes from 1 to 1.5 at $t=5s$, which causes the tracking error to increase after $t = 5s$. However, the controller is able to adapt to the change and regain good tracking performance. The tracking error decreases gradually as the model is learning to become a good approximation of the plant. Parameter adaptation of rule 3 is shown in Fig. 3. The learning rate Ω is chosen as $\Omega = 1.0 \times 10^6$.

The reference trajectory in Fig. 5 is obtained by passing a piecewise constant through a first-order stable linear filter. A disturbance in α_1 was introduced at $t = 44s$. As a result, it can be seen that the tracking error increases after $44s$. The parameter adaptations are shown in Fig. 6.The control law can produce control signal that adapts to the change and regain good tracking performance.

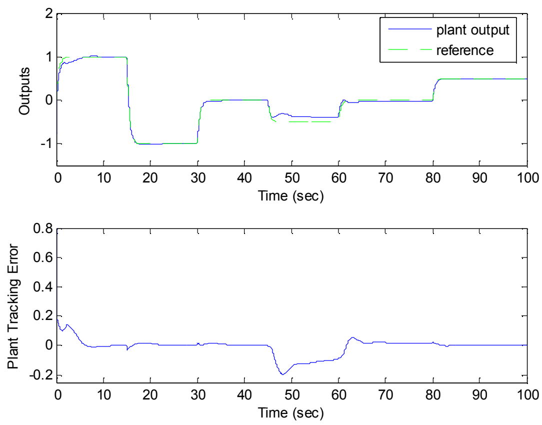

Fig. 5. Plant tracking response with parameter disturbance (α_1 changes from 1 to 1.5 at $t = 44s$)

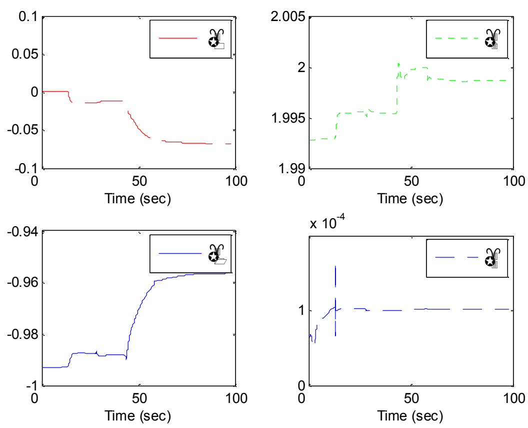

Fig. 6. Consequent parameter adaptation of rule 2

6. CONCLUSION

The indirect adaptive fuzzy control algorithm for uncertain nonlinear stable plants is presented in this paper. A discrete-time T-S fuzzy model is employed as a dynamical model of an unknown continuous-time plant with unmeasurable states. By proper training, the T-S fuzzy model is able to achieve input-output behaviour close to the plant and provide useful information about the states of the plant. A feedback linearization control is calculated entirely based on the model, using the model structure, parameters and states, while the parameters of the model are updated on-line by RLSE method. Sufficient conditions stability and tracking error convergence are derived for stable nonlinear plant. It should be noted here that although the error boundedness or the closed-loop global stability are proven under the assumption that the premise parameters of the model are the same as in the plant, a local stability or the error boundedness are achieved if the initial parameter values are close enough to the plant parameter values. Simulation examples verify the effectiveness, adaptation and tracking performance of the proposed control scheme.

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