CONTROL OF MULTI-PHASE BATCH PROCESSES: FORMULATION AND CHALLENGE

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Abstract: This paper deals with control of multi-phase batch processes. The process in each cycle is formulated as a switched system with internally forced switching instants. In this formulation, there are two kinds of switching sequence: dynamics-switching-sequence and control-switching-sequence. The control problem is transformed into finding of the control-switching-sequence and the control signal between any two consequential points in the sequence. This problem is proposed to be decomposed into two subtasks: determining the control-switching-sequence by detecting the dynamics-switching-sequence and designing the control law by using iterative learning scheme. Challenges within this framework are discussed, and some possible solutions to these challenges are suggested. *Copyright* ©2007 *IFAC*

Keywords: multi-phase batch process, switched system, iterative learning control (ILC), challenge

1. INTRODUCTION

Being preferred manufacturing choice for lowvolume and high-value products, batch processes occupy an important position in the chemical industry, especially in manufacturing specialty chemicals, pharmaceutical products and polymers. Most discrete-time batch processes are carried out in a sequence of discrete steps. In (Undey and Cinar, 2002), two definitions were introduced: "Steps occurring in a single processing unit as succession of events caused by operational or phenomenological (chemical reactions, microbial activities, etc.) regimes are called phases. Steps occurring in different processing units and performing different unit operations are called stages." Hence, the difference between "multi-stage batch process" and "multi-phase batch process" is clear: the multi-stage batch process is multi-unit, while the multi-phase batch process is single-unit.

As pointed in (Gu and Bahri, 2002), a batch process control system can be refined into four levels: planning, scheduling, supervision and coordination, and local control. There are many literatures about multi-stage processes, and all of them are about planning (Grunow *et al.*, 2002), scheduling (Maravelias, 2006) and supervision and coordination (Gonnet and Chiotti, 1999). In all these works, the local control is assumed to be given. For the multi-stage process, designing local control is not the emphasis and nodus.

However, the situation for multi-phase batch processes is totally different. Since there is only one unit, there is limited planning, scheduling or su-



Fig. 1. Illustration of an injection molding cycle

pervision problem. While the multi-dynamics and the switching time have significant effect on the end-product quality, designing local control for multi-phase batch processes is very difficult. To the best knowledge of the authors, there is no reported works on this problem.

In fact, there are many industry processes being multi-phase, such as injection molding (Gao et al., 2001), fermentation (Undey et al., 2000) and sequencing batch reactor (Azwar *et al.*, 2006). As a multi-phase batch process, injection molding typically operates sequentially in phases as illustrated in Figure 1, among which, filling and packing-holding are the most important phases to product quality such as weight and dimension. The transition from filling to packing in each cycle, referred to as the V/P transfer, has a significant effect on the control performance and product quality. When and how to perform the V/P transfer, is also an important issue in a successful molding. In this paper, a mathematic formulation of the multi-phase batch processes is given, and the challenges and possible solutions for controller design are presented.

2. MOTIVATION: INJECTION MOLDING PROCESS

Injection molding, a major plastic processing technique for converting thermoplastic into all types of products, is a typical batch process. In each cycle, the polymer melt is injected under pressure to occupy the mold cavity. The melt flow pattern in a simple rectangular mold is illustrated as in Figure 2. Melt is injected through the gate and fills the mold cavity from the left to the right, as shown in the figure. The contours indicate successive flow front positions at different filling times in the spreading plane.

The time when the cavity is filled is a critical point. For convenience, this time is named *filled-time*. Before and after the filled-time, the process dynamics has a significant variation: the cavity pressure increases gradually before the filled-time;



Fig. 2. Melt flow in a single-gate rectangular mold

while the pressure rises rapidly after the filledtime. Hence, the injection molding process in each cycle can be described as a hybrid system, and this kind of batch processes is also called multi-phase batch process.

To deal with this dynamics variation, a two-phase control algorithm, which include filling and packing, was widely used in practice. In the filling phase, the injection velocity needs to be controlled to achieve a uniform mold filling, while in the packing phase, the nozzle pressure needs to be maintained to compensate for the material shrinkage. The transition from filling to packing in each cycle, referred to as the V/P transfer, has a significant effect on the control performance. The ideal transfer time should equal to the filledtime. In practice, however, we do not know the filled-time a prior. Early transfer results in a short shot, whereas late transfer causes flash. Accurate detection of the filled-time is important for the proper operation of V/P transfer. By utilizing the process characteristics that the nozzle pressure significantly increases at the end of filling, a patent based on fuzzy system technique has been successfully developed at HKUST to timely control the V/P transfer (Gao and Zhou, 2001).

As pointed out earlier, this multi-phase phenomenon is common in chemical and bio- processes. Hence, studying this class of processes theoretically and systematically is important and interesting. In summary, this class of processes has the following character. There is a partition of the batch duration in disjoint segments. In different segment, the process dynamics is different, so the control objective is different, and the control law is also different; the segment is called phase. A batch process with single unit and multi-phase is called multi-phase batch process. In the following section, a mathematical formulation will be given for the multi-phase batch process.

3. PROBLEM FORMULATION

Consider the following batch process:

$$\begin{cases} \Delta x_k(t) = f(x_k(t), u_k(t), t) \\ y_k(t) = h(x_k(t), u_k(t), t) \end{cases}, 0 \leqslant t \leqslant T_k \quad (1)$$

where operator Δ denotes, respectively, derivative for continuous systems and difference for discrete systems, t denotes time, k denotes batch index, $x_k(t) \in \mathbb{R}^n, y_k(t) \in \mathbb{R}^l$ and $u_k(t) \in \mathbb{R}^m$ represent, respectively, the state, output and input of the process at time t in the kth batch run, $f(\cdot, \cdot, \cdot) :$ $\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^+ \to \mathbb{R}^n$ and $h(\cdot, \cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^+ \to \mathbb{R}^l$ represent the system dynamics. T_k is the duration of the kth batch.

Assume there are P phases in each cycle, and then the process can be described as the following hybrid system:

$$\begin{cases} \Delta x_k(t) = f_{q(t)}(x_k(t), u_k(t), t) \\ y_k(t) = h_{q(t)}(x_k(t), u_k(t), t) \end{cases}, \quad (2)$$

$$q(t+) = \delta(q(t-), x_k(t)) \tag{3}$$

where $q(t) \in \{1, 2, \dots, P\}$ represents the discrete mode and $\delta(\cdot)$ is the deterministic mode transition function. For most industry processes, the sequence of discrete mode is fixed and known, so the mode transition function can be rewritten as

$$q(t+) = \begin{cases} q(t-)+1, & if \ x_k(t+) \in \Omega_{q(t-)+1} \\ q(t-), & other \end{cases}$$
(4)
$$q(0) = 1$$

where Ω_i $(i = 1, 2, \dots, P)$ is the switching region for phase *i*. According to switching law (4), the batch process in each cycle can be divided into P phases: $[T_k^{i-1}, T_k^i]$ is called phase *i* $(i = 1, 2, \dots, P)$, where $T_k^0 = 0$ and $T_k^P = T_k$, and T_k^i is called switching time or switching instant. In phase *i*, the process has the following dynamics,

$$f = f_i, h = h_i, \quad t \in \left[T_k^i, T_k^{i+1}\right) \tag{5}$$

where $\{f_i, h_i\}_{i=1,\dots,P}$ is a given sequence of active subsystems. For convenience, $x_k(t)$ is also denoted as $x_k^i(t)$, if $t \in [T_k^{i-1}, T_k^i)$. Obviously, we have

$$x_k^{i+1}(T_k^i) = x_k^i(T_k^i)$$
(6)

In phase *i*, the control objective is such that output $y_k^i = h_i$ tracking the reference y^i . Define $e_k^i = y_k^i - y^i$, then the control objective is such that

$$\lim_{t \to \infty} e_k^i(t) = 0 \tag{7}$$

Remark 1: Equation (1) can describe continuoustime batch processes as well as discrete-time batch processes. The control objective may be different in different phases. Taking injection molding as an example, the injection velocity and nozzle pressure should be controlled to follow certain profiles in the filling and packing phases, respectively. Similarly, the different control objectives are formulated as tracking different references in this paper. Of course, this is a simplified formulation. **Remark 2:** In (7), $t \to \infty$ is fictitious as a phase of a batch has limited duration. If subsystem $\{f_i, h_i\}_{i=1,\dots,P}$ is autonomous, however, this equation might be validated by using stability theories; in other case, equation (7) can be substituted by the following criteria

$$\min \sum_{t=T_{k}^{i}}^{T_{k}^{i+1}-} \left\| e_{k}^{i}(t) \right\|$$
(8)

The sequence of the switching times is defined as $\sigma_k = \{T_k^1, T_k^2, \cdots, T_k^P\}$. For convenience, σ_k is named dynamics-switching-sequence. Corresponding, there is another sequence of times when control switch, denoted as $\bar{\sigma}_k = \{\bar{T}_k^1, \bar{T}_k^2, \cdots, \bar{T}_k^P\}$ and named control-switching-sequence. If all the states are measurable and all the switching regions are known, σ_k is totally known. Then we can choose $\bar{\sigma}_k = \sigma_k$. In practice, however, the online measurement of all states is often not possible or switching region Ω_j is unknown. Therefore, we should estimate σ_k firstly. Assume $\hat{\sigma}_k$ is estimation of σ_k , then the control-switching-sequence can be chosen as

$$\bar{\sigma}_k = \hat{\sigma}_k \tag{9}$$

If we regard $\bar{\sigma}_k$ as a discrete input, then the overall control input to the system is a pair $(\bar{\sigma}_k, U_k)$, where,

$$U_{k} = \left[u_{k}(0) \ u_{k}(1) \ \cdots \ u_{k}(T_{k}) \right]^{T}$$
(10)

so the control objective is such that

$$\lim \, \bar{\sigma}_k = \sigma_k, \tag{11}$$

and

$$\lim_{k \to \infty} e_k^i(t) = 0 \tag{12}$$

4. TWO-SUBTASK DECOMPOSITION

In section 3, the multi-phase batch process in each cycle is formulated as a switched system with internally forced switching time; then control of multi-phase batch process is described as a hybrid control problem. To solve this problem, we should accomplish two subtasks. The first subtask is to obtain the control switching sequence $\hat{\sigma}_k$ in each batch. While the second subtask is to design control law U_k from cycle to cycle.

4.1 First Subtask

Estimating the dynamic switching sequence equals identifying the current mode of the hybrid process. The literature on this problem is rich. A wellknown approach for the estimation of the current mode is the computation of the so-called currentlocation tree, described in (Caines *et al.*, 1988), that gives the subset of locations the system can be in at the current time. Balluchi *et al.* (2002) proposed a hybrid observer with two parts: a location observer to identify the current mode and a continuous observer to estimate the continuous states. Wang *et al.* (2006) extended the design methods proposed in (Balluchi *et al.*, 2002) to the hybrid system with unknown disturbances and faults.

Since the subsystem sequence is assumed to be fixed in this article, estimating the dynamic switching sequence may be easier than the cases considered in the above references. And these methods proposed in the above references provide us many choices.

4.2 Second Subtask

Since the switching time have been chosen in the first subtask, we can design the control law for each phase respectively. Since typical batch processes are repetitive in nature, iterative learning control (ILC) is widely used in control batch processes (Lee and Lee, 2003; Shi *et al.*, 2005). For phase i, design an ILC law with the following form

$$u_k(t) = u_{k-1}(t) + r_k(t), \quad t \in \left[\bar{T}_k^{i-1}, \bar{T}_k^i\right]$$
(13)

where $r_k(t) \in \mathbb{R}^m$ is referred as the updating law of the ILC. The objective for ILC design is to determine updating law $r_k(t)$ such that not only (12) holds, but also

$$\lim_{k \to \infty} e_k^i(t) = 0 \tag{14}$$

holds. There are also many methods to design $r_k(t)$, see (Lee and Lee, 2003).

5. CHALLENGE AND SOLUTION

In this section, the difficulties with the twosubtask decomposition are discussed and some possible methods to conquer these difficulties are also suggested.

5.1 Switching Delay

As illustrated in Section 4.1, there are many methods to estimate the dynamic-switching-sequence. Then we can choose the control-switching-sequence as (9), but this scheme can not implemented in practice. To estimate the dynamic-switchingsequence, an online monitoring scheme should be used to detect the variation of the dynamics. Assume that a dynamics variation is detected at t^* , in other words $\hat{T}_k^i = t^*$, then we note that control law $u_k(t^*)$ has already outputed to the process. Therefore, the fastest switching time for control law is $\bar{T}_k^i = t^* + 1$. Of course, a reasonable estimation satisfies $\hat{T}_k^i \ge T_k^i$, so it always be $\bar{T}_k^i > \hat{T}_k^i \ge T_k^i$. This is called *switching delay*.

A possible solution to this question is choosing the control-switching-sequence as

$$\bar{\sigma}_k = \hat{\sigma}_{k-1} \tag{15}$$

A suitable ILC designed for the process should satisfy the following equation

$$\lim_{k \to \infty} r_k(t) = 0 \tag{16}$$

Hence, there exists K > 0 such that

$$\sigma_k = \sigma_K, \ k > K \tag{17}$$

If we can design a detection scheme such that

$$\lim_{k \to \infty} \hat{\sigma}_k = \sigma_k \tag{18}$$

Then, from (15), (17) and (18), we obtain that (11) holds. Therefore, scheme (15) could conquer the switching delay problem in some cases. This scheme has another advantage: since the switching times of a batch have been chosen before this batch begins, designing the control law simultaneously for the whole batch, as discussed in Section 5.3, will become much easier.

In fact, due to unavoidable uncertainties, exact estimation for the dynamic-switching-sequence is impossible. Therefore, mode-mismatch might occur. If the boundaries of uncertainties are known, a robust controller and a robust monitoring module can be designed for each phase based on these boundaries. In the mode-mismatched period, the robust controller can ensure the system's stability if the difference of two modes is less than the boundary; while, the mode-mismatch can be detected if the difference exceeds the boundary, then the right controller will be used.

5.2 ILC with Variable Duration

In general, in order to use ILC, the duration of each cycle is required to be uniform. For the multiphase batch processes, since the switching time is variable in cycle direction, the duration of each phase is different from cycle to cycle. Therefore, how to design ILC in variable duration is an open problem.

If the duration of phase *i* in batch *k* is less than that in batch *k*-1, in other words, $\bar{T}_k^i - \bar{T}_k^{i-1} < \bar{T}_{k-1}^i - \bar{T}_{k-1}^{i-1}$, it is easy to design the ILC law. In the following section, we mainly discuss how to design the ILC law when $\bar{T}_k^i - \bar{T}_k^{i-1} > \bar{T}_{k-1}^i - \bar{T}_{k-1}^{i-1}$. Two methods could be used to deal with this situation: retime-from-left and retime-from-right. They are illustrated in Figure 3 (a) and (b) respectively.



Fig. 3. Updating for ILC: (a) retime-from-left; (b) retime-from-right

Using retime-from-left scheme, equation (13) becomes the following form:

 $u_{k}(t) = u_{k-1}(t - \bar{T}_{k}^{i-1} + \bar{T}_{k-1}^{i-1}) + r_{k}^{i}(t), t \in [\bar{T}_{k}^{i-1}, \bar{T}_{k}^{i}]$ (19) For $t \in [\bar{T}_{k}^{i-1} + (\bar{T}_{k-1}^{i} - \bar{T}_{k-1}^{i-1}), \bar{T}_{k}^{i})$, we should obtain the predicted value $\hat{u}_{k-1}(t)$ by using the model of the process in phase *i* firstly, then replace $u_{k-1}(t)$ in equation (19) by $\hat{u}_{k-1}(t)$. Similarly, retime-from-right scheme can also be defined.

5.3 Jump of Control Signal in Switching Time

Since the control law in each phase is updated respectively, there may be jumps of control signal in the switching times, see Figure 4. This discontinuousness can affect the transient performance and cause serious performance degradation. To deal with this problem, people have tried many methods. In (Simon et al., 2000), two simple and widely used reconfiguration techniques, the state zeroing and state preserving methods were surveyed, and the novel output fitting method was proposed. Due to designing the whole control law in one batch simultaneously, model predictive control may be a promising tool to conquer this difficulty, as shown in (Bemporad and Morari, 1999; Bemporad et al., 2000). Since all these method is provided to smooth the control signal near the switching times, like the red line in Figure 4, they can be called *smoothing operation*.





6. CONCLUSION

Designing controller for multi-phase batch processes has been considered in this paper. Firstly, the multi-phase batch process in each cycle is formulated as a switched system with internally forced switching instants, and the decision variables are the control-switching-sequence and the continuous control law. Secondly, the problem is decomposed into two subproblems. Finally, some challenges within this frame have been discussed and possible methods to conquer these challenges have been presented.

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