8th International IFAC Symposium on
Dynamics and Control of Process Systems
Preprints Vol.2, June 6-8, 2007, Cancún, Mexico

COMPUTATIONAL DESIGN OF THE TWO-LEVEL CONTROL FOR THE SINGULARLY PERTURBED SYSTEM

Alexandrov V.V. [*] **Maribel Reyes. R.** [**]
W. Fermin Guerrero S. [***]

[*] *Faculty of Physics and Mathematics, Autonomous Univesity of Puebla, Mexico, Email: valex@fcfm.buap.mx*
[**] *(·*), Email: maribelrr@gmail.com*
[***] *(·*), Email: willi@fcfm.buap.mx*

Abstract: We present a new algorithm of the two-level control for a singularly perturbed system. If the mathematical model of the controllable dynamic system has a singular perturbation, it can be reduce with the help of hierarchical controls that can be obtained with Tikhonov's theorem. This is the stabilization of the adjoining system around an equilibrium point and to stabilize the desired movement through the use of a simplified system. In this paper, we present the application of the stabilization and the control algorithm of an aeronautics problem.

Keywords: Control algorithms, Dynamic stability, Stabilization

1. INTRODUCTION

In this article we present a method that makes possible the simplification and control of a dynamic controllable system. This method is based on the use of Tikhonov's theorem (Tikhonov, 1952) in order to determine the controls that are necessary for the stabilization of the system.

We propose design of two controls, the first one is u_1 wich depends on the fast and slow variables. Control u_2 realizes the desired movement of the simplified system. In this way, the two controls will be dependent on the slow variables. Thus, u_1 is known as the first-level control, and u_2 is called the second-level control. We present here the application of this method for the vertical launching of an automatic airplane for those whose task consists in taking the airplane in flight up to a height Y_k. The criterion used for the realization of the desired movement of the simplified system consists in the optimization of a functional J based on the Pontryagin maximum principle and Kelley's necessary condition (Afanasiev *et al*, 1996; Pontryagin *et al*, 1962).

2. DESCRIPTION OF THE SINGULARLY-PERTURBED SYSTEM WITH TWO CONTROLS

Let us suppose that the controllable system can be represented in the operating form

$$F(x, u_1, u_2, T) = 0 \tag{1}$$

where x is the matrix $(1 \times n)$ of the coordinates; u_1, u_2 are the controls, and T is the real time. After the transformation $T = T_* t$, $x = (y, z)S$, (where t is the time without dimension, T_* is the time scale, S is the matrix $(n \times n)$, (y, z) is the matrix $(1 \times n)$ of $(m + l)$ coordinates without dimensions) the system (1) has the form

$$\frac{dy}{dt} = f(y, z, u_2, t), \quad y(t_0) = y^0, \quad (2)$$

$$\varepsilon\frac{dz}{dt} = \varphi(y, z, u_1), \quad z(0) = z^0 \quad (3)$$

$$\textit{and} \quad 0 < \varepsilon = \textit{constant} \ll 1, \quad t \; \epsilon \; [t_0, t_k].$$

The coordinates $z_i(t)$ $(i = 1 \dots l)$ can change very fast $(\frac{dz}{dt} \simeq \frac{1}{\varepsilon})$ and for that reason we can name them fast coordinates (Kokotovic *et al.*, 1999). The coordinates $y_j(t)(j = 1, \dots, m)$ we can name slow coordinates (m+l=n).

Let us suppose that the functions $f(y, z, u_2, t)$, $\varphi(y, z, u_1)$ are analytical functions of its arguments, which belong to an opened region R of Euclides space R^{n+3}. In the space R^n it is given smooth variety M

$$f_0(y) = 0. \quad (4)$$

We are going to design the controls - to reach this variety by the time $(t_1 - t_0) \leq (t_k - t_0)$. The process derived by this way should be stable with the estimation $\nu(t)$ in the interval $[t_0, t_1]$ (Alexandrov *et al.*, 2005):

$$\mid x(t) - x^0(t) \mid \leq \nu(t) \mid x(t_0) - x^0(t_0) \mid, \quad (5)$$

where

$$\sup_{t_0 \leq t \leq t_1} \nu(t) < \infty, \; \nu(t_1) < 1 \quad (6)$$

3. DESIGN OF THE CONTROL

We can present a new algorithm to solve the problem (2), (3), (4) approximately. This algorithm has five steps (A,B,C,D,E) and an annex.

Step A Control design of the additional subsystem by the fast time.

A1 We can present the subsystem (3) by the fast time $\tau = \frac{t}{\varepsilon}$ (or computer time) in the following form:

$$\frac{dz}{d\tau} = \varphi(y, z, u_1) \quad (7)$$

where now all slow coordinates $y_j(t)$ $(j = 1, \dots, m)$ are fixed (Novozhilov, 1997). We search the control u_1 as one combination of the controls:

$$u_1 = u_1^0(y) + \Delta u_1(z, y), \quad (8)$$

where $u_1^0(y)$ is main control and $\Delta u_1(z, y)$ is an additional control. Let us suppose, that the analytic function $u_1^0(y)$ exists when $z^0 = \varphi^{-1}(y, u_1^0(y))$ is the unique solution of the equation (9):

$$\varphi(y, z, u_1^0(y)) = 0.. \quad (9)$$

For this reason we can say that the additional subsystem has an unique, critical point (or equilibrium point) i.e. unique stationary process. This is to say we used the main control $u_1^0(y)$ to obtain this process.

A2 We search the additional control for the stabilization of the stationary process $z^0(y)$ in the following form:

$$u_1 = K(z - z^0)(y) = K\Delta z, \quad (10)$$

where $K = -\frac{1}{r}b^T\mathcal{L}_0$ with $b = \frac{\partial\varphi(y, \; z^o(y), \; u_1^0(y))}{\partial u_1}$ and $\mathcal{L}_0$ $(\mathcal{L}_0^T = \mathcal{L}_0 > 0)$ is the solution of the Rikkati equation:

$$\frac{1}{r}\mathcal{L}\mathbf{b}\mathbf{b}^\mathbf{T}\mathcal{L} - (\mathcal{L}\mathbf{A} + \mathbf{A}^T\mathcal{L}) - \mathbf{Q} = 0, \quad (11)$$

where $A = \frac{\partial\varphi(y, \; z^0(y), \; u_1^0(y))}{\partial z}$.

In order to obtain the equation (11) we used the functional $J_1 = \int_0^{\tau_k} \left[(\Delta\mathbf{z})^T \mathbf{Q} (\Delta\mathbf{z}) + r(\Delta u_1)^2\right] d\tau$, when $\tau_k = \frac{t_k}{\varepsilon} \to \infty$ $(\varepsilon \to 0)$ with the application of the Kalman's theorem (Afanasiev *et al.*, 1996), when $det(b(y), A(y)b(y), \dots, A^{l-1}(y)b(y)) \neq 0, y\epsilon R.$

In compliance with the theorem on stability by the first approach, we have the affirmation that the stationary process is asymptotically stable.

Step B Let us suppose that the initial condition $z^0(\tau_0)$ of the subsystem (7) belongs to the attraction region of the critical point $z^0(y)$. Then we can say that all the conditions of Tikhonovs theorem (Novozhilov, 1997) on reduction of the original system (2), (3) to the simplified system are fulfilled, if the control $u_2(t)$ (yet it is unknown) is an analytical function. According to Tikhonov's theorem (when $\varepsilon = 0$),we have the simplified system

$$\begin{aligned} &\dot{\tilde{y}} = f(\tilde{y}, \tilde{z}, u_2(t), t), \quad \tilde{y}(t_0) = \tilde{y}^0, \\ &0 = \varphi(\tilde{y}, \tilde{z}, u_1), \\ &u_1 = u_1^0(y) - \frac{1}{r}b^T(\tilde{y})\mathcal{L}_0(\tilde{y})(\tilde{z} - z^0(y^0)). \end{aligned} \quad (12)$$

Step C We rewrite the simplified system in the shorter form:

$$\dot{\tilde{y}} = f(\tilde{y}, \tilde{z}, u_2(t), t), \quad \tilde{y}(t_0) = \tilde{y}^0. \quad (13)$$

We solve the problem of the search for the control as a programmed optimal control:

$$J_2(u) = \varphi_0(\tilde{y}(t_1)) \to min_u, \quad (14)$$

where t_1 is the first moment, when $\tilde{y}(t_1)\epsilon M = \{f_0(\tilde{y}) = 0\}$. After this we must approximate the control $u_2^0(t)$ to fulfill the last condition of Tikhonov's theorem $(u_2^0(t) \to \tilde{u}_2^0(t))$.

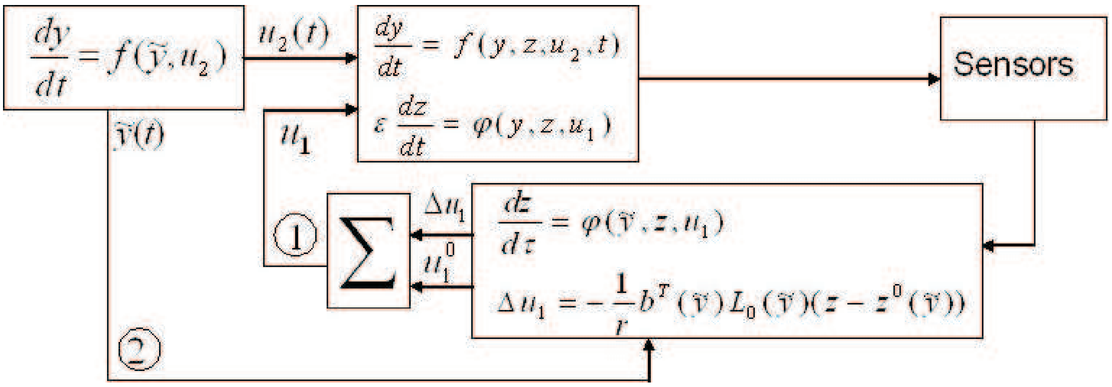

Fig. 1. Scheme of the two level control for the singularly perturbed system.

Step D In this step, we can detail the function $u_1^0(y)$ in such a way, when the process $\tilde{y}^0(t)$ that corresponds to the the control $\tilde{u}^0(t)$ is stable in the interval $[t_0, t_1]$ with desired wished estimation $(u_1^0(y) \rightarrow u_1^0(\tilde{y}^0))$ (Alexandrov *et al.*, 2005).

Finally we have both controls (figure 1):

$$u_1 = u_1^0(\tilde{y}^0) - \frac{1}{r} b^T(\tilde{y}^0) L_0(\tilde{y})(\tilde{z} - z^0(\tilde{y}^0)) \quad -control\ of\ first\ level-, \tag{15}$$

$$u_2 = \tilde{u}_2^0(t) \ - \ control\ of\ second\ level- \tag{16}$$

Step E The affirmations of Tikhonov's theorem are the asymptotical equalities:

$$\lim_{\varepsilon \to 0} y\,(t, \varepsilon) = \tilde{y}\,(t)\,, \quad t\ \epsilon\ [t_0, t'], \tag{17}$$

$$\lim_{\varepsilon \to 0} z\,(t, \varepsilon) = \tilde{z}\,(t)\,, \quad t\ \epsilon\ (t_0, t']. \tag{18}$$

Because of the reality $\varepsilon \equiv constant \neq 0$, it is necessary to return to the original system with the synthesis of both controls (15), (16).

Annex In fact, frequently we do not have exact and complete information of the $\Delta z = z - z_0$. In this situation also, this algorithm with some modifications can be applied.

4. STATEMENT OF THE PROBLEM OF VERTICAL LAUNCHING OF AN AIRPLANE

For this point, we analyze the application of the algorithm described in section 3, in aeronautics. We consider the following desired movement: vertical launching of an automatic airplane to a fixed height Y_k in such a way that the fuel-consumption rate is minimized. The optimal control to achieve the vertical launching will be determinated with the application of the theorems: Pontryagin Maximum Principle and the theorem of H. J. Kelley (Afanasiev *et al.*, 1996). Also, we want to stabilize this desired movement. Let us suppose that the automatic airplane is launched vertically with an initial perturbation, with a trajectory angle $\theta_1 \neq 0$ and with an inclination angle $\varphi_1 \neq 0$. Traditionally the equations for the longitudinal movement are, (Novozhilov, 1997):

$$\begin{aligned} M\dot{V} &= -Mg\sin\theta_1 + P^T\cos\alpha - \frac{1}{2}PV^2Sc_x \\ MV\dot{\theta}_1 &= -Mg\cos\theta_1 + P^T\sin\alpha + \frac{1}{2}PV^2Sc_y \\ I_z\dot{\Omega} &= \frac{1}{2}\rho SV^2 b_a\left(m_z^{\alpha}\alpha + m_z^{\delta_z}\delta_z\right) \\ \dot{\varphi}_1 &= \Omega_z;\ \dot{M} = -U \quad ,\ \alpha = \varphi_1 - \theta_1 \\ \dot{M} &= -U \quad ;\ \dot{Y} = V\cos(\theta_1)\,; \dot{X} = V\cos(\theta_1) \end{aligned} \tag{19}$$

Here M is the mass of the airplane, $X$and Y are the coordinates of the center of mass, V corresponds to the speed of the center of mass; U denotes the speed of decrease of mass due to the fuel-consumption, P is the push force, and we suppose that it is given as: $P = \mu U$, where μ is the speed, relative to the airplane, with which the gases escape due to the combustion of fuel; ρ is the air density; θ_1, α, φ_1 and δ are the angles of trajectory, of attack, of inclination, and of the elevator deflection; I, S and b_a are the moment of inertia, the characteristic area, and the longitude characteristic; c_x, c_y, m_z^{α} and m_z^{δ} are the aerodynamic characteristics of the airplane.

The equations (19) that describe the movement are normalized, introducing the following dimensionless quantities:

$$T = T_* t, \quad M = M_* m, \quad V = V_* v, \tag{20}$$

$$Y = Y_* y, \quad U = U_* u, \quad \Omega = \Omega_* \omega. \tag{21}$$

The values of M_*, V_*, U_*, Y_* are assumed to be equal to the maximum values that correspond to the type of airplane. We will also assume that the characteristic values of the aerodynamic forces and propulsion forces are of the order of magnitude of the weight of the airplane:

$$\frac{3}{2}P_*V_*^2S_*/2 = \mu U_* = 3M_*g. \quad (22)$$

The value of Ω_* is estimated by the simplified equation $I\frac{d^2\varphi_1}{dT^2} = \frac{1}{2}P_*V_*^2Sb_am_z^\alpha\varphi_1$, where $m_z^\alpha \sim 1$. The temporal constant for this oscillating element can be considered with the following expression:

$$T_1^2 = \frac{I_*}{P_*V_*^2Sb_a/2} = \frac{M_*r_*^2}{M_*gb_a};\ asi\ \Omega_* = \frac{1}{T_1} \quad (23)$$

We also denote: $T_2 = \frac{V_*}{g}$; $T_3 = \frac{Y_*}{V_*}$. Now, let us consider that: $Y_* = 4km$, $M_* = 430kg$ (structures and fuel) y $U_* = 6.8\ \frac{kg}{s}$. With these data, some characteristics of the flight are seen (Novozhilov, 1997) in table 1.

Table 1

$V_*, \frac{m}{s}$	300	$T_3 = \frac{4000}{300} =$	13.333
T_1, s	0.5	$\frac{M_*}{T_2U_*} = \frac{450}{30*6.8} =$	2.205
T_2, s	30	$\frac{T_3}{T_2} = \frac{13.333}{30} =$	0.4443
T_*, s	30	$\varepsilon = \frac{T_1}{T_2} =$	$0.5*10^{-1}$

With such characteristic values, the system (19) becomes a singular perturbed system by the small parameter ε:

$$\begin{aligned} \varepsilon\frac{d\varphi_1}{dt} &= \omega \\ \varepsilon\frac{d\omega}{dt} &= \{m_z^\alpha(\varphi_1-\theta_1) + m_z^\delta\delta\}v^2 \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{dv}{dt} &= -\cos(\theta_1) + \frac{3u}{m}\cos(\varphi_1-\theta_1) - \frac{2v^2c_x}{m} \\ \frac{d\theta}{dt} &= \frac{1}{v}\sin(\theta_1) + \frac{3u\sin(\varphi_1-\theta_1)}{mv} + \frac{2v^2c_y(\varphi_1-\theta_1)}{mv} \\ \frac{dm}{dt} &= -\frac{1}{2.2059}u;\quad \frac{dy}{dt} = \frac{1}{0.4443}v\cos(\theta_1). \end{aligned} \quad (25)$$

It can be observed of (24)-(25) that the controls of the first and second level are δ y u respectively.

4.1 Application of the algorithm for the simplified subsystem

Step A. The control δ is given as: $\delta = \delta^0 + \Delta\delta$, where δ^0 corresponds to the main control and $\Delta\delta = \delta - \delta^o$ corresponds to the additional control.

4.1.1. ***A1*** *Determination of principal control* δ^0

We consider the adjoining sub-system ($\tau = \frac{t}{\varepsilon}$) and assume that $\Delta\delta = 0$. Then δ_0 is the control that realizes the equilibrium point of the adjoining system. Therefore, for $v \neq 0$ and for all t, with $0 < t \leq 1$, we have: $\delta^0 = -\frac{m_z^\alpha}{m_z^\delta}(\varphi_{10} - \theta_1)$.

We can designate the control δ_0 in the next form:

$$\delta_0 = \kappa\theta_1 \quad (26)$$

where κ is a constant to be determined in the next section (**Step D**), and the equilibrium point of the additional system is unique, and it is given by:

$$z^0(\delta^0(\theta_1)) = \left(\varphi_1^0 = \left(1 - \frac{m_z^\delta}{m_z^\alpha}\kappa\right)\theta_1, \omega^0 = 0\right)^T \quad (27)$$

4.1.2. ***A2*** *Determination of additional control* $\Delta\delta$

Considering that we have complete information of φ_1 and ω, the additional control is obtained by expressing the additional system in terms of deviations and proposing a control of the form $\Delta\delta = -K(\Delta\varphi_1, \Delta\omega)^T$ that minimizes the functional J_1 of the 3 section, where $\Delta\varphi = \varphi_1 - \varphi_1^0$, $\Delta\omega = \omega - \omega^0$.

For the particular case in which

$$m_z^\alpha = 1.85;\quad m_z^\delta = 1.6;\quad v = 1;\quad r = 2;$$

$Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; the optimal gains K are obtained:

$$K = \left(\, k_1 = -0.1991\ \ k_2 = -0.8654 \,\right) \quad (28)$$

Hence, δ is obtained:

$$\delta = \kappa\theta_1 - k_1\left[\varphi_1 - \left(1 - \frac{m_z^\delta}{m_z^\alpha}\kappa\right)\theta_1\right] - k_2\omega, \quad (29)$$

4.2 ***Step B.*** *Reduction to Simplified System*

Supposing that we can determine a control of the second level u in analytic form, and given the control δ, satisfying the conditions of Tikhonov's theorem, then we can reduce to the simplified system. Taking $\varepsilon = 0$ and substituting the root of the resulting algebraic equations, we obtain:

$$\begin{aligned} \frac{d\bar{v}}{dt} &= -\cos(\bar{\theta}_1) + \frac{3u}{\bar{m}}\cos\left(\frac{m_z^\delta}{m_z^\alpha}\kappa\bar{\theta}_1\right) - \frac{2\bar{v}^2c_x}{\bar{m}}, \\ \frac{d\bar{\theta}_1}{dt} &= \frac{\sin(\bar{\theta})}{\bar{v}} - \frac{3u\sin\left(\frac{m_z^\delta}{m_z^\alpha}\kappa\bar{\theta}_1\right)}{\bar{m}\bar{v}} - \frac{2\bar{v}^2c_y\left(\frac{m_z^\delta}{m_z^\alpha}\kappa\bar{\theta}_1\right)}{\bar{m}\bar{v}}, \\ \frac{d\bar{m}}{dt} &= \frac{1}{2.2059}u;\quad \frac{d\bar{y}}{dt} = \frac{1}{0.443}\bar{v}\cos(\bar{\theta}_1) \end{aligned} \quad (30)$$

The control u to be determined by the system (30).

4.3 **Step C**. Algorithm for the Launching

For the system (30), let us consider the following desired movement:

$\bar{\theta}_1(t) \equiv 0; v(t) \geq 0; \bar{m}(t) > 0; y(t) \geq 0 \, \forall t \in \left[0, t_k^0\right]$
$y\left(t_k^0\right) = y_k = 1.$

Moreover, we minimize the functional J_2, where

$$J_2(u) = \varphi_o\left(\mathbf{y}\left(t_k^0\right)\right) = \bar{m}_0 - \bar{m}\left(t_k^0\right) \longrightarrow \min_u . \quad (31)$$

with $\bar{m}_0$ such that $\bar{m}(t=0) = \bar{m}_0$.

Thus, substituting $\theta_1(t) \equiv 0$ in the equations (30), we obtained:

$$\frac{d\bar{y}}{dt} = \frac{1}{0.443}\bar{v}; \quad \frac{d\bar{m}}{dt} = \frac{1}{2.2059}u$$
$$\frac{d\bar{v}}{dt} = -1 + \frac{3u}{\bar{m}} - \frac{2\bar{v}^2 c_x}{\bar{m}}, \quad (32)$$

where the control u is subject to the restriction $0 \leq u \leq u_{max}$. The problem of determining of the optimal control u, is a problem that was solved with the help of two fundamental results of the optimal control theory: the Pontryaguin maximum principle and Kelley's necessary condition. The application of these two principles produce one algorithm for the optimal control u^0 given for:

$$u = u^0 = \left\{ \begin{array}{c} u_{\max} \quad si \quad t \in \left[0, \hat{t1}\right] \\ u_{opt.especial} \quad si \quad t \in \left[\hat{t1}, \tilde{t1}\right] \\ u = 0 \quad si \quad t \in \left[\tilde{t1}, t_k^0\right] \end{array} \right\} \quad (33)$$

In order to use the control (33) according to Tikhonov's theorem, it is necessary that $u^0(t)$ is analytical of the time interval $[0, t_k^0]$.

4.4 **Step D.** Stability Conditions of the Simplified System

To determine the control completely δ, now we must obtain κ from the simplified system equation (30) with the condition that $\theta_1 \neq 0$:

$$\frac{d\bar{v}}{dt} = -1 + \frac{3u}{\bar{m}} - \frac{\bar{v}^2 c_x}{m}; \quad \frac{d\bar{m}}{dt} = \frac{1}{2.2059}u;$$
$$\frac{d\bar{y}}{dt} = \frac{1}{0.443}\bar{v};$$
$$\frac{d\bar{\theta}_1}{dt} = \frac{1}{\bar{v}}\bar{\theta}_1 - \frac{3u\frac{m_z^\delta}{m_z^\alpha}\kappa\bar{\theta}_1}{\bar{m}\bar{v}} - \frac{\bar{v}^2 c_y \frac{m_z^\delta}{m_z^\alpha}\kappa\bar{\theta}_1}{\bar{m}\bar{v}} \quad (34)$$

The constant κ is obtained when the condition of the vertical launching is stable with estimation (5). This is satisfied if the next condition is accomplished:

$$\kappa > \frac{m_z^\alpha \bar{m}(t)}{m_z^\delta\left(3u(t) + \bar{v}^2(t)\, c_y\right)} = F_1(t). \quad (35)$$

With the condition (35) for parameter κ, the control is totally certain. Therefore, the problem of determining the control that stabilizes the desired movement has been resolved by applying the control algorithm described in section 3.

4.5 Numerical Results (**Step E**)

4.5.1 Considering that the objective is achieved at height $Y_k = 7.2\ Km$, the control $u^0(t)$ has three regimes as in (30), but it is a continuous function at the intervals, then we approximate $u^0(t)$ by an analytic curve $u(t)$ (Figure 2). Taking $c_x = 0 \cdot 23$, the numeric simulation of the system (32) determine $v(t_k) = 0$ when $Y_k(T_k) = 7.2\ Km$ (Reyes *et al.*, 2006).

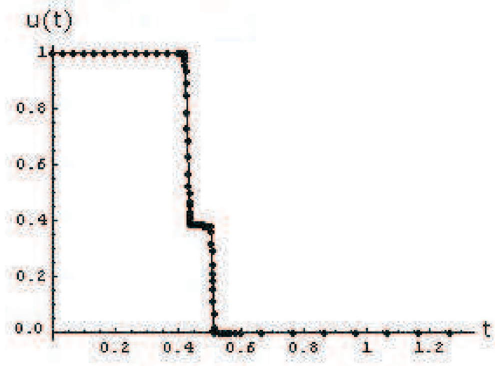

Fig. 2. The control of the second level $u(t)$ minimizes the fuel consumption for the airplane.

We finish the launching when the height is 6 Km. Then the velocity is $V(t_1) \neq 0$, the flight can be continued.

With the control $u(t)$ (Fig. 2), we can determine the constant κ if we graph the function $F_1(t)$ (Eq. 35). The graphics of $F_1(t)$ can be seen in Figure 3. According to condition (35), it should be taken $\kappa > 1$, also the numerical solution of (34) shows that with $\kappa \geq 0.7$, the desired solution $\theta_1 \equiv 0$ is stable with estimation (5) (Reyes *et al.*, 2006).

4.5.2 Now when the controls are completely certain, we can return to the dimensional system (according to transformation (20)-(21) to check that the controls stabilize the desired movement.

We do the numeric simulation for system (19) with $V(0) = 1 \times 10^{-6}\ m/s$; $X(0) = 0\ m$, $Y(0) = 0\ m$, $M(0) = 410\ Kg$, $\theta_1(0) = 0.1\ rad$; $\varphi_1(0) = 0.1\ rad$; $\Omega(0) = 0\ rad/s$;, and the values for the other parameters are: $c_y = 8$; $m_z^\delta = 1.85$;

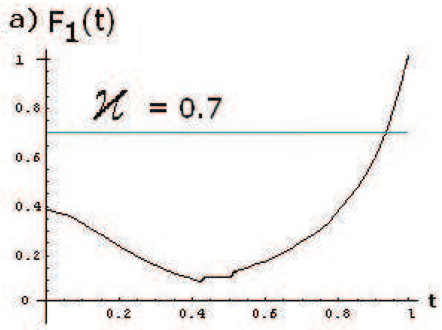

Fig. 3. The Graph of $F_1(t)$ show the condition for κ.

$m_z^\alpha = 1.6$; $\kappa = 0.88$; $S = 0.16\ m^2$; $g = 9.81\ m/s^2$; $\rho = 1.22\ kg/m^3$; $\mu = 2041.6\ m/s$; $U_{max} = 6.8574\ Kg/s$; $I = 300\ Kg \cdot m^2$. In figure 4 we only show the slow variables: $V(T)$, $X(T)$ and $\theta_1(T)$; in figure (5) we show the fast variable $\Omega(T)$:

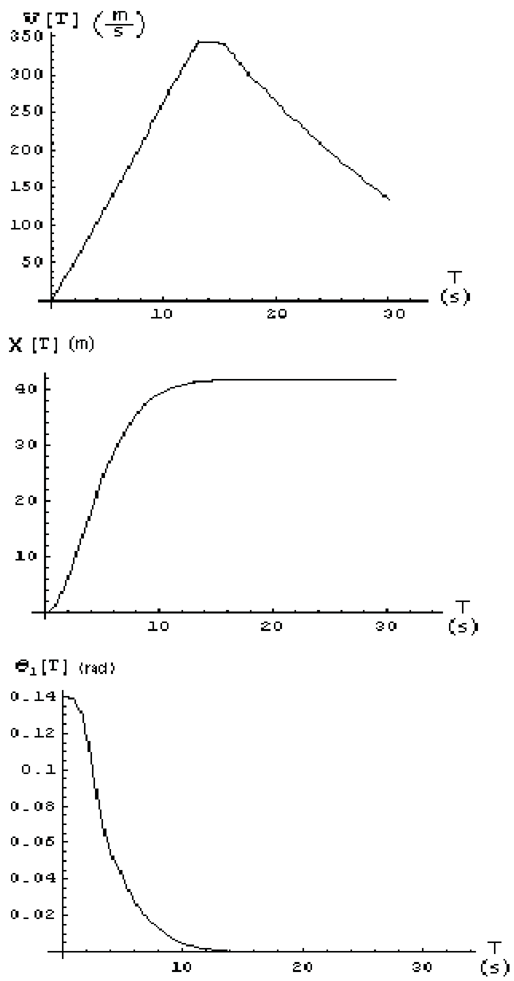

Fig. 4. The slow variables show that the movement desired is achieved.

In the figure 4 can be seen that the airplane has deviated 42 meters on the horizontal direction.

Finally, we can say that the problem has been solved. With the help of Tikhonov's theorem, using the additional subsystem and the simplified system, two controls have been found.

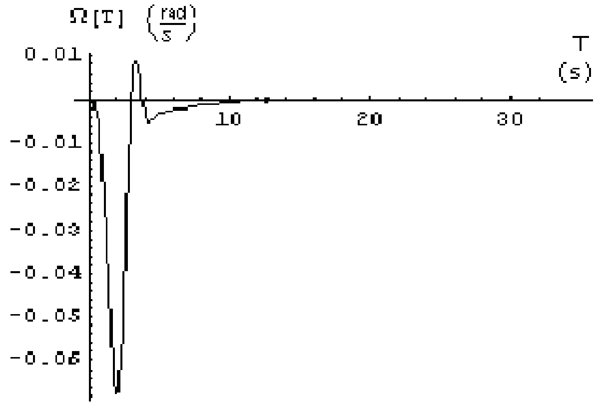

Fig. 5. The fast Variable Ω decrease quickly, then the movement desired is achieved.

Conclusion:

In this paper, we formulated and applied a new algorithm for a singularly perturbed and controllable system, which was constructed using Tikhonov's theorem. This theorem makes possible the formation of two control levels.

Agreements:
We thank the support of the Department of Automatic Control (CINVESTAV), México.

REFERENCES

Afanasiev V. N., V. R. Nosov, V. B. Kolmanovskii (1996). *Mathematical theory of Control Systems Design.* Kluwer Kluwer Academic Publishers, Netherlands.

Alexandrov V. V., V. G. Boltianskii, S. S. Lemak, N. A. Parushnikov, V. M. Tikhomirov (2005). *Motion Control.* Fismatlit. Russia.

Kokotovic P., K. K. Hassan, J. O'reilly (1999). *Singular Perturbation Methods in Control, Analysis and Design.* SIAM, U.S. A.

Novozhilov I.V (1997). *Fractional Analysis, Methods of Motion Decomposition.* Birkhauser, Boston.

Pontryagin L.S., V. G. Boltyanskii, R. V. Gamkrelidze, E. F. Mishchenko (1962). *The Mathematical Theory of Optimal Processes.* Interscience Publishers, U.S. A.

Reyes R. M. (2006). *Master Thesis: Lanzamiento y Estabilización suboptimales de un avión automático.* Dirigided by V. V. Alexandrov and W. F. Guerrero, F. C. F. M (B. U. A. P).

Tikhonov A. N. (1952). *Systems of Differential Equations Containing Small Parameters by Derivates. Matem. sbornic*, **31**, 575-586.