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MPC RELEVANT IDENTIFICATION USING GENERALIZED ORTHONORMAL BASIS FILTERS

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Abstract: In MPC relevant identification, it is necessary to identify models that are suited for multi-step ahead predictions. This can be achieved by minimizing the multi-step ahead prediction error in the identification stage. This work aims at the development of a methodology for identification of MPC relevant models based on Generalized orthonormal basis filters (GOBF). Specifically, ARX models parameterized using GOBF are identified. The efficacy of the proposed modeling technique is demonstrated by carrying out simulation studies on the benchmark Shell control problem. The relative quality of the obtained models is evaluated through closed-loop performance with MPC.

Keywords: Model Predictive Control, MPC relevant identification, Generalized orthonormal basis filters, ARX models

1. INTRODUCTION

Model predictive control (MPC) has been widely used in the process industry over the last two decades for controlling key unit operations in chemical plants. As a consequence, there has been significant research activity in the process control community with the aim of improving the analysis and synthesis of MPC controllers (Morari and Lee, 1999). MPC determines the optimal input moves by solving an optimization problem in which, the objective function makes use of predicted outputs over a finite horizon. Hence, MPC requires models that are capable of providing accurate multi-step ahead predictions rather than one step ahead predictions (Shook et al, 1992). In reality, the process and noise model structures are not known accurately and moreover most of the processes encountered are nonlinear. To guarantee satisfactory closed-loop performance in such a scenario, it is advantageous to use models that are "tuned" for multi-step ahead predictions. This is facilitated by employing an identification criterion based on the multi-step ahead prediction error.

Recently, considerable amount of work has been done in the area of MPC relevant identification or MRI. Huang and Wang (1999) have discussed MRI in the context of data prefiltering and have shown that the problem of multi-step ahead prediction error minimization can be reduced to that of filtered one-step ahead prediction error minimization. Shreesha and Gudi (2004) have proposed methods to design control relevant prefilters for MISO model identification from closed loop data. The use of multiple models for multi-step ahead predictions has been proposed by Rossiter and Kouvaritakis (2001). Gopaluni et al. (2003) proposed the use of an identification criterion that is based on the multi-step ahead prediction error which is equivalent to minimizing a pre-filtered one-step ahead prediction error. The prefilter is based on the noise model and hence the noise model plays an important role in determining the quality of the multi-step ahead predictions. Gopaluni et al. (2004) have discussed the influence of the noise model (and hence the data prefilter) on the quality of multistep ahead predictions from the bias distribution viewpoint.

In recent years there has been growing interest in the use of orthonormal basis filters (OBF) for representing process dynamics (Van den Hof and Ninness, 2005; Patwardhan and Shah, 2005; Patwardhan et al., 2006). The orthogonal filter approximations provide a simple and elegant method of representing open loop stable systems. In fact, these models can be looked upon as a compact (parsimonious in parameters) representation of convolution type models, which have been widely used in MPC schemes. Moreover, if some *a priori* information about system dynamics is available, then the resulting parameter estimation problem can be solved analytically using linear regression. From an industrial practice viewpoint as well, GOBFs can elegantly represent large scale multivariable system dynamics because the underlying representation is in state-space form, and are therefore more suited than other parameterizations. Patwardhan and Shah (2005) proposed a method of identification of multivariable state observers using GOBF parameterization. They have also shown that the parameter estimation problem can be formulated as two nested optimization problems in which, the GOBF poles are estimated using a nonlinear iterative search and the GOBF expansion coefficients are obtained analytically.

In this work, we propose a method to identify MPC relevant models based on GOBF parameterization. We specifically propose the use of a GOBF-ARX structure, the parameters of which are determined by minimizing the multistep ahead prediction error. We employ the nested optimization formulation mentioned above in order to reduce the dimensionality of the parameter estimation problem.

The paper is organized as follows. We begin by discussing MPC relevant identification in Section 1. In Section 2, we present the GOBF-ARX structure. The proposed identification technique is discussed in Section3. We then demonstrate the efficacy of the proposed identification method by carrying out simulation studies on the Shell Benchmark Control Problem in Section 4. The main conclusions reached are presented in Section 5.

2. MPC RELEVANT IDENTIFICATION

Gopaluni et. al. (2002) propose control relevant methodology as " The philosophy of identification by minimizing an objective function that is commensurate with the control objective function ". MPC requires models that can provide accurate multi-step ahead predictions. Hence, in the context of MPC, control relevant identification necessarily implies identification of models based on the minimization of multistep ahead prediction errors. This can be achieved by relating the objective function used for model identification with the MPC objective function.

If the true process is described by,

$$y(t) = G(q)u(t) + H(q)e(t) \tag{1}$$

where G and H represent the true process and noise dynamics respectively and $e(t)$ is white noise with variance σ_e^2. Then the p step ahead optimal predictor is given by (Ljung, 1999),

$$\widehat{y}(t+p \mid t) = \widehat{W}_p \widehat{G} u(t+p) + (1-\widehat{W}_p) y(t+p) \tag{2}$$

$$\widehat{W}_p = \widehat{F}_p \widehat{H}^{-1}; \quad \widehat{F}_p = \sum_{i=0}^{p-1} \widehat{h}(i) q^{-1}; \quad \widehat{F}_1 = 0. \tag{3}$$

$\widehat{h}(i)$ being the impulse response coefficients of the estimated noise model $\widehat{H}$.

Eq.2 implies that the predicted multistep ahead output is influenced by the estimated noise model. Further, the influence of $\widehat{W}_p$ on the deterministic as well as the stochastic models increases with an increase in p as can be seen from Eq.3.

2.0.1. MPC Relevant Identification Objective Function MPC involves the solution of an optimization problem that employs the following objective function,

$$J_{mpc} = \sum_{k=1}^{p} [e(t+k \mid t)]^2 \tag{4}$$

$$where, e(t+k \mid t) = r(t+k) - [\widehat{y}(t+k \mid t) + d(t)] \tag{5}$$

$d(t)$ being the difference between process output and predicted output of the model at instant t

and is given as $d(t) = y(t) - \widehat{y}(t \mid t-1)$.Thus the objective function becomes,

$$J_{mpc} = \sum_{k=1}^{p} [r(t+k) - \widehat{y}(t+k \mid t) + d(t)]^2. \quad (6)$$

Readjusting the terms,

$$J_{mpc} = \sum_{k=1}^{p} \{([r(t+k) - \widehat{y}(t+k \mid t)]^2 + [d(t)]^2 + 2[r(t+k) - \widehat{y}(t+k \mid t)][d(t)]\} \quad (7)$$

where, r is the reference trajectory and p is the prediction horizon. This objective function consists of quadratic and linear terms of predicted output $\widehat{y}$ and reference trajectory r. The linear terms, weighted less than the quadratic terms, can be neglected from the objective function. Thus, the objective function is governed mainly by the quadratic error between predicted output and reference trajectory. This is the motivation behind the following proposed objective function which minimizes the similar quadratic error between desired model output and actual model output. The objective function minimizes p-step ahead prediction error as follows.

$$J_{pstep} = \sum_{t=1}^{N-p} \sum_{k=1}^{p} [y(t+k) - \widehat{y}(t+k \mid t)]^2 \quad (8)$$

where N is the data length. The prediction error is thus accumulated over the prediction horizon at each instant. Also, note that the objective function depends explicitly on the prediction horizon p.

3. THE GOBF-ARX STRUCTURE

For a SISO system represented by a strictly proper stable transfer function $G(q)$,

$$y(q) = G(q)u(q) \quad (9)$$

there exists a unique generalized Fourier series expansion of $G(q)$ such that,

$$G(q) = \sum_{i=1}^{\infty} c_i F_i(q) \quad (10)$$

where c_i and $F_i(q)$ are the Fourier coefficients and basis filters which are strictly proper stable transfer functions in q.

Now, consider a SISO ARX structure,

$$y(k) = \widehat{y}(k|k-1) + e(k) \quad (11)$$

$$\widehat{y}(k|k-1) = -\left[\sum_{i=1}^{n_a} a_i q^{-i}\right] y(k) + \left[\sum_{i=1}^{n_b} b_i q^{-i}\right] u(k) \quad (12)$$

where $e(k)$ represents a zero mean Gaussian white noise sequence. Since orthonormal basis $\{q^{-i}\}$ has shorter memory, we propose to use GOBF to parameterize the ARX model as follows,

$$\widehat{y}(k|k-1) = \left[\sum_{i=1}^{n_y} c_{y,i} F_{y,i}(q, \xi_y)\right] y(k) + \left[\sum_{i=1}^{n_u} c_{u,i} F_{u,i}(q, \xi_u)\right] u(k) \quad (13)$$

where the GOBFs $F(q, \xi)$ and expansion coefficient c_i are defined as in Patwardhan and Shah (2005).

The structure in Eq.13 has two advantages. Firstly, it retains the advantages of the conventional ARX structure. Secondly, the GOBF parameterization provides a compact representation (parsimonious in parameters) of the ARX structure. This measure can significantly reduce the number of parameters to be identified, and, as a consequence, time of identification experiments can be significantly reduced.

4. GOBF BASED MPC RELEVANT IDENTIFICATION

The state realization of the proposed MISO GOBF-ARX model in Eq.13 can be expressed as (Srinivasarao et al., 2005),

$$X(k+1) = \Phi X(k) + \Gamma u(k) + K y(k) \quad (14)$$

$$y(k) = C X(k) + e(k) \quad (15)$$

where $X(k) \,\epsilon\, R^n$ represents the state vector, $u(k) \epsilon\, R^m$ represents the input vector, $y(k) \epsilon\, R^m$ represents the output vector, C represents linear static map relating states with the outputs and $e(k)$ represents a white noise sequence. K represents the steady state Kalman gain. Rearranging above equations,

$$X(k+1) = \Psi X(k) + \Gamma u(k) + K e(k)$$

$$y(k) = C X(k) + e(k)$$

$$\Psi = [\Phi + KC]$$

As mentioned earlier, the parameter estimation problem can be posed as a two step optimization problem. The parameters to be estimated are the filter poles ξ and filter expansion coefficients C_i. The algorithm starts with an initial guess of filter poles ξ obtained from the a priori knowledge of the system.

(1) From an initial guess of the filter poles, $\widehat{\xi}$'s, the Ψ and Γ matrices are formed which are then used to compute the vector of filter

expansion coefficients, C using Eq.14 and Eq.15 as,

$$C_{opt} = \arg \begin{matrix} \min \\ C \end{matrix} \frac{1}{N} \sum_{k=1}^{N} \varepsilon_c(k, C)^2 \tag{16}$$

which is a linear least squares problem. The error ε_c is 1-step ahead prediction error.

(2) The estimated coefficients C_{opt} along with the filter poles $\widehat{\xi}$'s are then used for generating the multistep ahead predictions as,

$$\hat{X}(k+1) = \Psi X(k) + \Gamma u(k) + K e(k)$$
$$\hat{y}(k+j \mid k) = C[\Psi^{j-1}\hat{X}(K+1) + \sum_{i=0}^{j-2} \Psi^i \Gamma u[k+j-(i+1)] \tag{17}$$

where $j = 2, 3, .., p$. Using these predictions, the multi-step ahead prediction error, $\varepsilon(k + j \mid k)$, is generated, which is then minimized to obtain the filter poles,

$$\widehat{\xi}_{opt} = \arg \begin{matrix} \min \\ \xi \end{matrix} \left[\sum_{k=1}^{N-p} \sum_{j=1}^{p} \varepsilon(k+j \mid k)^2 \right] \tag{18}$$

where,

$$\varepsilon(k+j \mid k) = y(k+j) - \hat{y}(k+j \mid k)$$

It can be seen from Eq.18 above that at each instant, the prediction errors are computed for $j = 1, 2...p$ and their squares are summed up. This process is repeated for all k. The accumulated errors corresponding to all k's are added-up and this sum is minimized. This can be viewed as the multi-step ahead prediction error being minimized in a moving horizon fashion. The above mentioned two step optimization procedure is explained with the help of flowchart shown in Figure.1

5. ILLUSTRATIVE EXAMPLE

The efficacy of the proposed multistep ahead prediction algorithm is demonstrated by carrying out modeling studies on Shell Control Problem. The Shell Control Problem is a benchmark problem proposed at the Shell Process Control Workshop and involves control of a heavy oil fractionator system characterized by large time delays in each input output pair. The heavy oil fractionator has three product draws, three side circulating loops and a gaseous feed stream. The system consists of seven measured outputs, three manipulated inputs and two unmeasured disturbances. Product specifications for top and side draws are determined by economic considerations. There is no product specification on bottom draw, however, there is an operating constraint on the bottom

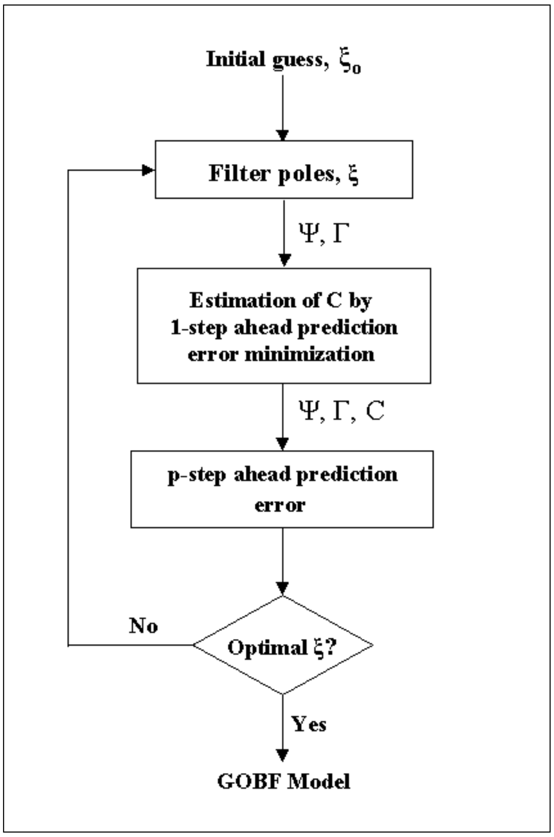

Fig. 1. Flowchart for two step optimization

reflux temperature. Top draw, side draw and bottoms reflux duty can be used as manipulated variables to control the column while heat duties on the two other side loops (upper reflux duty and intermediate reflux duty) act as unmeasured disturbances to the column. Since the controlled outputs of interest are top end point, side end point and bottoms reflux temperature, in this work we consider a subsystem consisting of only these three outputs. Further, the process dynamics are simulated under following assumptions

- Manipulated inputs are piecewise constant
- Disturbances entering the plant can be adequately represented using piecewise constant functions

Under these assumptions, a discrete dynamic model of the form

$$\widehat{\mathbf{y}}(z) = G_p(z)\mathbf{u}(z) + G_d(z)\mathbf{d}(z) \tag{19}$$

is developed with sampling time (T) equal to 2 minutes. A minimal order state space realization of (19) of the form

$$\mathbf{X}(k+1) = \mathbf{A}\mathbf{X}(k) + \mathbf{B}_u\mathbf{u}(k) + \mathbf{B}_d\mathbf{d}(k) \tag{20}$$
$$\widehat{\mathbf{y}}(k) = \mathbf{C}\mathbf{X}(k) \tag{21}$$

with 51 state variables is used for simulation of process behavior. The stationary unmeasured disturbances $d(z)$ are assumed to be generated by the following stochastic process

$$\mathbf{x}_w(k+1) = \mathbf{A}_w\mathbf{x}_w(k) + \mathbf{B}_w\mathbf{w}(k) \quad (22)$$

$$\mathbf{d}(k) = \mathbf{C}_w\mathbf{x}_w(k) + \mathbf{D}_w\mathbf{w}(k) \quad (23)$$

$$\mathbf{A}_w = \mathbf{C}_w = 0.95\,I \quad ;\mathbf{B}_w = \mathbf{D}_w = I \quad (24)$$

or equivalently by

$$\mathbf{d}(z) = \begin{bmatrix} \frac{z}{z-0.95} & 0 \\ 0 & \frac{z}{z-0.95} \end{bmatrix} \mathbf{w}(z) \quad (25)$$

where $\mathbf{w} \in R^2$ is a zero mean normally distributed white noise process with $\sigma_{w1} = \sigma_{w2} = 0.0075$. In addition, the measured outputs are assumed to be corrupted with measurement noise

$$\mathbf{y}(k) = \widehat{\mathbf{y}}(k) + \boldsymbol{\upsilon}(k) \quad (26)$$

where $\boldsymbol{\upsilon} \in R^3$ represents zero mean normally distributed white noise process with $\sigma_{\upsilon i} = 0.005$ for $i = 1, 2, 3$.

In order to carry out system identification, a low frequency signal (maximum value 0.15 Hz) random binary signals with amplitude 0.08 were simultaneously introduced in all the manipulated inputs and 1000 data points were collected. The estimates of signal to noise ratios between each input and disturbance in each output are given by Eq.27,

$$\widehat{S}_{NR} = \left(\frac{\widehat{\sigma}^2_{u_i}}{\widehat{\sigma}^2_{\upsilon_j}}\right) = \begin{bmatrix} 8.81 & 9.00 & 9.14 \\ 3.64 & 3.70 & 3.76 \\ 6.88 & 7.00 & 7.11 \end{bmatrix} \quad (27)$$

The proposed identification methodology was tested on the simulation data for p =1,2,5,10 and 20.

5.1 Comparison of Predictive Ability

The models obtained were compared for their open-loop prediction capability based on the percentage prediction errors (PPE),

$$PPE = \frac{\sum_{k=1}^{N}[y(k) - \hat{y}(k)]^2}{\sum_{k=1}^{N}[y(k) - \bar{y}(k)]^2} \times 100 \quad (28)$$

where, $\bar{y}$ in above equation represents the mean value of measurements $y(k)$and $\hat{y}(k)$ is the predicted value of $y(k)$. Table 1 compares the predictive ability of the models identified with different horizons in the identification step, for the accuracy related to a 20 step ahead prediction.

The predictive ability of the models is better for larger horizons in the identification step, as evidenced by lower PPE values.

Table 1. PPE values for 20 step ahead predictions with 1, 5, 10, 20 step models

P	Y1	Y2	Y3
1 Step	13.57	21.85	54.67
2 Step	14.18	8.24	16.98
5 Step	2.73	3.76	2.19
10 Step	3.51	3.67	1.78
20 Step	2.85	4.96	1.69

5.2 Comparison of Closed-Loop Performance

The effect of the horizon used in the identification step on the performance of MPC was studied. The prediction and control horizons in the controller were fixed at 40 and 5 respectively. The other tuning parameters and constraints are taken as given in Patwardhan et.al.(2005).Firstly, a setpoint change of 0.5 units was introduced in the third output. The closed-loop performances of MPC was evaluated using two models: 1. Model A – This model was identified with a prediction horizon of $p = 1$ during the identification step (Eq.18) and 2. Model B – In this case p was set to 10 during the identification step. Since model A is identified with a smaller prediction horizon, it is expected (see discussion related to Eq.2 and Eq.3) to show good fidelity and provide tight control at high frequencies. Likewise, model B is expected to match the plant dynamics well at low frequencies and perform well for slowly moving targets. The closed-loop performance of MPC when the above models were employed are compared in Figure 2. It is clearly seen that the response of MPC with model B is faster and the third output, Y3, settles to its new setpoint earlier than the performance obtained with model A. Also, outputs Y1 and Y2 settle down to their setpoints rapidly in case of model B. These results verify the relative superiority of model B for low frequency targets as compared with model A.

Next, the performance of MPC in presence of fast changing disturbances was evaluated. Step type disturbances of magnitude ± 0.4 were introduced in the process. The disturbance magnitude was switched between $+0.4$ and -0.4, every 40 instants. Figure 3 shows the regulatory performance of MPC with the above models. It is evident that MPC based on model A is able to reject the high frequency disturbances rapidly as compared to that with the performance obtained with model B. This is again an expected result in view of the relatively small mismatch of model A at high frequencies.

The setpoint change applied in the first case above can be viewed as a low frequency input to the closed-loop system. The superior performance of MPC with the model B as compared to the model A indicates that the models obtained with larger prediction horizons in the identification objective

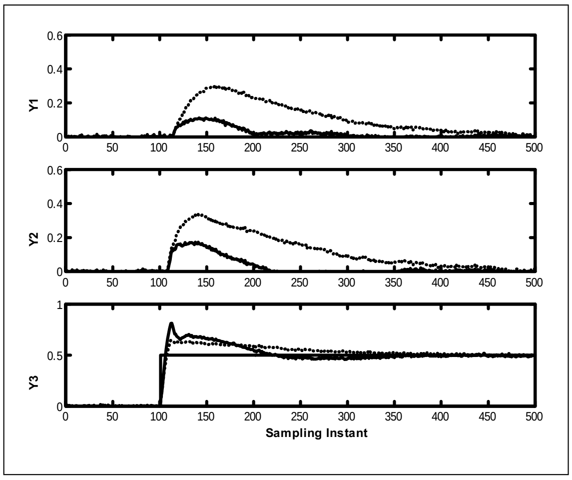

Fig. 2. Comparison of closed-loop performances of Model A (...) and Model B (—) for setpoint change in output 3

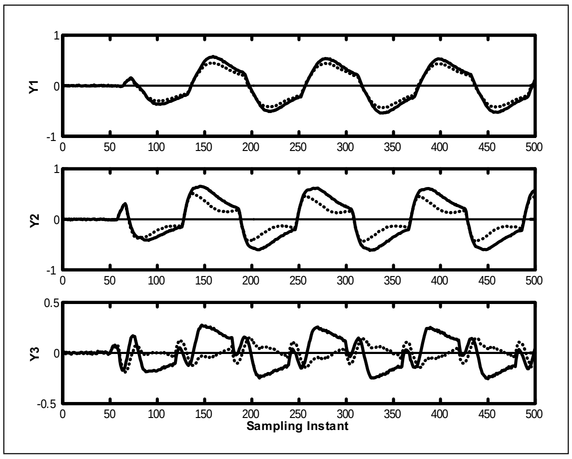

Fig. 3. Comparison of closed-loop performances of Model A (...) and Model B (—) for fast changing disturbances

have better response in the low-frequency region. On the other hand, from the results in Figure 3, we can conclude that models obtained by minimizing the 1-step ahead prediction error are more fidel at high frequencies as compared to those obtained by minimizing the multi-step ahead prediction error.

6. CONCLUSIONS

An approach for MPC relevant identification that is based on the use of GOBF parameterization for the identification of multivariable dynamics, has been proposed. The strategy is relevant for industrial practice considering that processes can be affected by targets and disturbances that could be both slow and fast. Since, model-plant mismatch is inevitable, the model that would be used in the controller needs to have high fidelity in the expected region of interest. The MPC relevant methodology proposed here is shown to accomodate these requirements by appropriately using different horizons P during the identification step. A large value of P in the identification objective results in a reduced bias in low frequency region as such the value of P could be a suitable handle to shape the bias in model estimate. The proposed methodology can be expected to provide improved control due to this reduced bias in th low to mid frequency region. Closed loop simulation results involving the benchmark Shell Control Problem demonstrate the efficacy of the proposed approach.

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