NEURAL MODELLING AND SLIDING MODE CONTROL OF BIODEGRADATION PROCESS IN A ROTATING BIOREACTROR

Ieroham S. Baruch¹, Carlos R. Mariaca-Gaspar¹, and Josefina Barrera-Cortes²

(1) Dept. of Automatic Control,
(2) Dept. of Biotechnology and Bioengineering, CINVESTAV-IPN, Mexico

Abstract: A new Recurrent Neural Network Model (RNNM) has been applied for measurement data filtering and parameters plus state estimation of hydrocarbons biodegradation process, contained in polluted slurry, treated in a rotating bioreactor. The pattern used for RNNM back-propagation learning is composed by six input variables and three output variables. The total time of learning is 200 epochs of 76 iterations each and the Mean Squared Error reached is below 1.25%. Then the RNNM is simplified and used to design a sliding mode control the two-input two-output high order nonlinear plant. The MSE% of control reached 1% at the end of the process. *Copyright* © 2007 IFAC

Keywords: adaptive control, backpropagation algorithms, biotechnology, identification, learning algorithms, neural network models, state estimation, sliding mode control.

1. INTRODUCTION

Recent advances in understanding of the working principles of artificial Neural Networks (NN) has given a tremendous boost to identification and control tools of nonlinear systems, (Narendra, and Parthasarathy, 1990). The main network property namely the ability to approximate complex nonlinear relationships without prior knowledge of the model structure makes them a very attractive alternative to the classical modelling and control techniques. This property has been proved by the universal approximation theorem (see Haykin, 1999). Among several possible network architectures the ones most widely used are the Feedforward NN (FFNN) and Recurrent NN (RNN). In a FFNN the signals are transmitted only in one direction, starting from the input layer, subsequently through the hidden layers to the output layer, which requires applying a tap delayed global feedbacks and a tap delayed inputs to achieve a nonlinear autoregressive moving average neural dynamic plant model. A RNN has local feedback connections to some of the previous layers. Such a structure is suitable

alternative to the FFNN when the task is to model dynamic systems, and the universal approximation theorem has been proved for RNN too. The preference given to RNN identification with respect to the classical methods of process identification is clearly demonstrated in the solution of the "biasvariance dilemma" (see Haykin, 1999). In (Boskovic, and Narendra, 1995) a comparative study of linear, nonlinear and neural-network-based adaptive controllers for a class of fed-batch baker's and brewer's yeast fermentation is done. The paper proposed to use the method of neural identification control, given in (Narendra, and Parthasarathy, 1990), and applied FFNNs (multilayer perceptron and radial basis functions NN). The proposed control gives a good approximation of the nonlinear plants dynamics, better with respect to the other methods of control, but the applied static NNs have a great complexity, and the plant order (and plant structure, especially for MIMO plants) has to be known. The application of RNNs could avoid these problems and could reduce significantly the size of the applied NNs. Furthermore, in biotechnology there exists a great variety of processes with incomplete

information where analytical models description is missing. One of them is the hydrocarbon degradation process in rotating drum. The control and manipulation of the hydrocarbon removal by a biostimulation process is a complex task in itself due to the great variety of native micro-organisms involved. Considering the lack of knowledge about each micro-organism's metabolism and their interactions, it is of our concern to develop a neural model that might correlate on the time the behaviour of some variables such as residual hydrocarbon concentration, soil viscosity, evaporated water, temperature, velocity of agitation, etc. at which occurs that biodegradation process.

In some early papers, (see Baruch, et al., 2004), the state-space approach is applied to design a RNN, defining a Jordan canonical two or three layer Recurrent Trainable Neural Network (RTNN) model and a Backpropagation (BP) algorithm of its learning. This RNN model is a parametric one and it serves as state and parameter estimator, which permits to use the estimated states and parameters directly for process control. In (Baruch et al., 2004) this general RTNN approach is applied in an indirect neural control scheme for identification and control of continuous wastewater treatment fermentation bioprocess, where unfortunately the plant and measurement noises affected the control precision. In the proposed paper we go ahead extending the topology of this RNN with local and global feedbacks. So we obtain a topology with built in output filter, capable to decrease measurement noise and to correlate different process measurements in order to obtain a complete process neural model learned by the BP algorithm. Then this RNNM is simplified and could be used for different control system design methods so to achieve the control objectives using the available process measurements.

2. DESCRIPTION OF THE BIO-DEGRADATION PROCESS IN A ROTATING DRUM

Biological treatment is attractive as a potentially low-cost technology, which converts toxic organic contaminants into CO₂ and biomass. Since the 70's, this technology has been applied for the hydrocarbon degradation, and today, it is considered as the best alternative to cleanup polluted soils. For this bioprocess, one challenge is to provide enough O₂ and nutrients to enable rapid conversion of contaminants by either indigenous microorganisms or inoculated species. Another challenge is to achieve efficient contact between the active microorganisms and the contaminants, which may represented a problem with in-situ treatment. An attractive alternative to overcome this problem is to apply a biological treatment in slurry phase using Horizontal Rotating Drum (HRD) (see Fig. 1). The HRD can effectively mix heterogeneous blends of a wide range of particle sizes, and high solid concentration (more than 60 %), (Gray et al., 1993). The HRD operated with oxygen supply or aeration.



Fig. 1. Schematic diagram of a rotating drum system.

Independently of the type of HRD operation (open or close), the insufficiency of water decreased the efficiency of hydrocarbon degradation in HRD favouring the formation of hydrocarbon balls (Alexander, 1999; Cookson, 1995; Gray *et al.*, 1993). So one objective of the process control is to maintain the humidity at 60%, which is the maximal solid concentration determined as the best for hydrocarbon removal from polluted soils treated in open rotating slurry bioreactors (Manilla-Pérez *et al.*, 2004).

Nowadays, semi empirical models, based on the Monod equation, have been developed to predict micro-organism growth as a function of available contaminants concentration. However, as the application of such models requires experimental work for calculating the kinetics parameters involved, so an alternative modelling technique is required. The RNNM offers many advantages as the possibility to approximate complex non linear high order multivariable processes, as the biodegradation process is (Caudill and Butler, 1995).

The bioremediation of polluted soils selected for modelling purpose was carried out by bio-stimulation in slurry phase using an open HRD. A silt loam (sand 36.5%w/w, silt 62.5% w/w and clay 1% w/w) soil of an average diameter of 15 µm, particle diameter in the range 2 - 75 μ m, was used (see Cantelaube *et al*, 1997; Rodriguez et al, 2004). The soil was contaminated with 50000 ppm of crude oil collected from a contaminated zone located near from a petroleum refinery. The slurry was prepared with 40% weight of soil (715 g) and 60% weight of a mineral solution (formula in kg·m⁻³: $(NH_4)_2SO_4$, 19; KH₂PO₄, 1.7; MgSO₄, 1; CaCl₂·2H₂O, 0.005; FeCl₃·6H₂O, 0.0025; yeast extract, 0.59; tergitol -0.5%), (for more details see Manilla-Pérez, et al., 2004). The slurry was added to a HRD of 4 litters (13 cm diameter by 30 cm long), which was opened, on the flat faces, for a natural air supply (see Fig. 1). The drum was operated during 19 days at a fix turning in the interval 3.5-20 RPM. During this time, the reactor was daily weight in order to replace the water lost, so to maintain constant the water concentration. Samples were removed each day for analysis of residual hydrocarbons, pH, water concentration and slurry viscosity. The hydrocarbon concentration was determined by an infrared spectrometer; the pH was measured with a Beckman Φ potentiometer; water concentration was calculated by difference of two sequence data of the drum weight; finally, slurry viscosity was measured with an AND Vibro-viscometer SV-10 (MED BY A&D LTD). The biodegradation process was repeated at a

different turning value (3.5, 5, 7.5, 10, 15, 20 RPM) in order to vary the oxygen available into the HRD.

3. LEARNING PATTERN

The learning pattern (see Fig. 2) used for RNNM training is composed by six input variables and three output variables. In order to avoid saturation problems in the RNNM training, the variables of the learning pattern are normalized in the interval 0-1. The measured variables are: Residual Hydrocarbon Concentration (RH), Evaporated Water (EW); Soil Viscosity (VISC), Added Water (AW); Temperature (T); Velocity of Agitation (VA). The RNNM outputs are: OUT (RH, EW, VISC).

Depending on the available measurements and the control objectives, this model could be simplified, where the input- output pattern is chosen as: ILP (RH, EW, AW, VA); OLP (RH, EW). This reduced model will be used for sliding model control system design.

4. RNNM TOPOLOGY AND LEARNING

In the present paper, a modified version of the RNN (Baruch et al., 2004) is proposed to be used as a model of a degradation process in rotating drums. This RNN model has been used to predict a Bt fermentation process (Valdez-Castro et al., 2003), and biodegradation in biopile system (Dela Torre Sanchez et al., 2004). Block-diagrams of the RNN topology and its adjoint, are given on Fig. 3, Fig. 4. Following Fig. 3, and Fig. 4, we could derive the Backpropagation algorithm of its learning based on the RNN topology using the diagrammatic method of (Wan and Beaufays, 1996). Both block-diagrams and algorithms are given in vector-matricial form as:



Fig. 2. The learning pattern.



Fig.3. Block diagram of the RNNM.



Fig.4. Block diagram of the adjoint RNNM.

| $X(k+1) = A_1X(k) + BU(k) - DY(k);$ | (1) |
|--|------|
| $Z(k) = \Gamma [X(k)]; Z_1(k) = C Z(k)$ | (2) |
| $V(k+1) = Z_1(k) + A_2V(k);$ | (3) |
| $Y(k) = \Phi[V(k)]$ | (4) |
| $A_1 = block-diag (A_1,i), A_1,i < 1;$ | (5) |
| $A_2 = block-diag (A_2,i), A_2,i < 1$ | (6) |
| $W(k+1) = W(k) + \eta \Delta W(k) + \alpha \Delta W_{ij}(k-1)$ | (7) |
| E(k) = T(k) - Y(k) | (8) |
| $R_1 = E(k) [1-Y^2(k)]$ | (9) |
| $\Delta C(k) = R_1 Z^T(k)$ | (10) |
| $\Delta A_2(k) = R_1 V^T(k)$ | (11) |
| $R = C^{T}(k) E(k) [1-X^{2}(k)]$ | (12) |
| $\Delta B(k) = R U^{T}(k)$ | (13) |
| $\Delta D(k) = R Y^{T}(k)$ | (14) |
| $\Delta A_1(k) = R X^T(k-1)$ | (15) |
| $Vec(\Lambda A_{a}(k)) = R_{1} V(k)$ | (16) |
| $Vec(\Delta A_1(k)) = R \circ X(k-1)$ | (17) |
| | |

Where: Y, X, U are output, state and input vectors with dimensions l, n, (l+m), respectively; here $U^{T}=[U_{1};T]$, where U_{1} is the real plant input vector with dimension m; T is the plant output vector with dimension l, considered as a RNN reference; A1, A2 are (nxn) and (lxl)- local feedback block-diagonal weight matrices respectively, defined by (5), (6); $B=[B_1;B_2]$ and C are [nx(l+m)] and (lxn)- weight matrices, where B_1 corresponds to U_1 and B_2 corresponds to T; D is a (nxl) global output closed loop matrix; $\Gamma[.]$, $\Phi[.]$ are vector-valued tanh activation functions; W is a general weight, denoting each weight matrix (C, D, A1, A2, B) in the RNNM model, to be updated; ΔW (ΔC , ΔD , ΔA_1 , ΔA_2 , ΔB), is the weight correction of W; η , α are learning rate parameters; ΔC , ΔA_2 are weight corrections of the (lxn) learned matrix C and the (lxl) learned diagonal matrix A₂; R₁ and R are auxiliary vector variables; ΔB , ΔD are weight corrections of the [nx(1+m)]learned matrix B and the (nxl) learned matrix D; the diagonals of the matrices A1, A2 are denoted by Vec(.) and equations (16), (17) represents its learning as an element-by-element vector products. The stability of the RNNM is assured by the restricted activation functions and by the local stability bound conditions, given by equations (5), (6). As it could be seen in Fig. 3, the first part of the RNNM, given by (1), (2) (without the global feedback entry) represents the plant model, and the second part, given by (3), (4) represents an output filter part. The complete RNN structure is in fact a full order state observer (Kalman – like filter) where the balance between the reference and feedback parts ($B_2T \rightarrow DY$) is achieved during the learning, when $(E \rightarrow 0)$. The RNN was trained with four sets of experimental data. An experimental data set, not included in the training process, is used for RNN generalization. Due to the high scale variation of the variables, used in the training process, the experimental data were normalized in the range 0-1.

A simplified version of this RNN model, containing only the plant part of the model is used as a base for the design of a Sliding Mode Control (SMC). In the following part it will be show that the indirect adaptive neural control (see Baruch et al., 2004) could be derived as an SMC defining the Sliding Surface (SS) with respect to the plant output.

5. SLIDING MODE CONTROL SYSTEM DESIGN

Let us suppose that the identified nonlinear plant is given by the equations (1), (2) omitting the DY(k) part. The block diagram of the control scheme is shown on Fig. 5. It contains identification and state estimation RNNM and a SMC. The stable nonlinear plant is identified by a RNNM with topology, given by equations (1)-(6) learned by the stable BPlearning algorithm, given by equations (7)-(17), where the identification error (8) tends to zero. The linearization of the activation functions of the simplified identification RNNM (1), (2), leads to the following local linear plant model:

$$X(k+1) = A_1X(k) + BU(k)$$
 (18)

$$Z(k) = F X(k); F = C \Gamma'$$
(19)

Where Γ ' is the derivative of the activation function and l = m, is supposed. Let us define the following SS as an output tracking error function:

$$\sum_{k=1}^{p} \sum_{i=1}^{p} \sum_{k=1}^{p} \sum_{i=1}^{p} \sum_{$$

Where: S(.) is the Sliding Surface Error Function (SSEF); E(.) is the systems output tracking error; γi are parameters of the desired SSEF; p is the order of the SSEF. The tracking error is defined as:

$$E(k) = R(k) - Z(k)$$
(21)

Where R(k), Z(k) are l-dimensional reference and output vectors. The objective of the sliding mode control systems design is to find a control action which maintains the systems error on the sliding surface which assure that the output tracking error reaches zero in p steps, where p < n. So, the control objective is fulfilled if:

$$S(k+1) = 0$$
 (22)

The iteration of the error (21) gives:

$$E(k+1) = R(k+1) - Z(k+1)$$
(23)

Now, let us to iterate (19) and to substitute (18) in it so to obtain the input/output local plant model, which yields:



Fig. 5. Block diagram of the closed-loop system.

$$Z(k+1) = F X(k+1) = F [AX(k) + BU(k)]$$
(24)

From (20), (22), and (23), it is easy to obtain:

$$R(k+1) - Z(k+1) + \sum_{i=1}^{p} \gamma_i E(k-i+1) = 0$$
 (25)

The substitution of (24) in (25) gives:

$$R(k+1) - FAX(k) - FBU(k) + \sum_{i=1}^{p} F(k-i+1) = 0$$
 (26)

As the local approximation plant model (18), (19), is controllable, observable and stable, (see Baruch et al., 2004), the matrix A_1 is diagonal, and l = m, the matrix product (FB) is nonsingular, and the plant states X(k) are smooth non-increasing functions. Now, from (26) it is easy to obtain the equivalent control capable to lead the system to the sliding surface which yields:

$$U_{eq}(k) = (FB)^{-1} \left[-FAX(k) + R(k+1) + \sum_{i=1}^{p} \gamma_i E(k-i+1) \right]$$
(27)

Following (Young et al., 1999), the SMC avoiding chattering is taken using a saturation function instead of sign one. So the SMC takes the form:

$$U^{*}(k) = \begin{cases} U_{eq}(k), & \text{if } ||U_{eq}(k)|| < Uo \\ -Uo |U_{eq}(k)/||U_{eq}(k)||, & \text{if } ||U_{eq}(k)|| \ge Uo. \end{cases}$$
(28)

The SMC substituted the multi-input multi-output coupled high order dynamics of the linearized plant with desired decoupled low order one.

6. EXPERIMENTAL AND SIMULATION RESULTS

6.1 Identification and States Estimation Using Experimental Data.

The described above learning algorithm is applied simultaneously to 4 fermentation kinetic data, represented by its input/output learning data patterns, and containing 19 points each (one per day). The total time of learning is 200 epochs, where the epoch size, corresponding to the number of data, is 76 iterations. After each epoch of training, the 4 sets are interchanged in an arbitrary manner from one epoch to another. The learning is stopped when the MSE% of learning reached values below 1.5%, the MSE% of generalization reached valued below 2%. and the relationship $|\Delta W_{ii}(k)|/|W_{ii}(k)| *100\%$ reached values below or equal of 2% for all updated parameters. Graphical results of RNNM training are given in Fig. 6 for the last epoch of learning. In the graphics, the output variables of the RNNM are compared with the experimental data. The Fig. 6 a, b, c compared the 4 kinetics experimental data with those, issued by the RNNM. The output process data of 76 points are the

hydrocarbon residual, the water requirements and the soil viscosity. The Fig. 6d represents the evolution of the mean squared error of approximation for whole learning process of 200 epochs. An unknown set of kinetic data, containing 19 points and repeated 4 times, so to maintain the same 76-points epoch size, is used as a validation (generalization) set, and these results are given on Fig. 7. The obtained graphical results of RNNM training and generalization shows a good convergence with an MSE% below 1.5% for the training and 2% for the generalization.

6.2 Sliding Mode Control Simulation Using the Identification Results.

The simplified process model has been used to design a SMC system. The RNNM particular model has two inputs (AW, VA), two outputs (EW, RH) and nine states. The SSEF is chosen as a first order one (p=1) with parameters Uo=1, γ =0.07, l=m=2. The control variables (AW, VA) are given on Figure 8 for 76 points. The graphical simulation results of the controlled system outputs (EW, RH), and the MSE%, also for 76 points, are given on Fig. 9. The two system set points (continuous line) are compared with the plant outputs (EW, RH) (data point line) and are plotted subsequently for four sets of set point data. The obtained MSE% of control at the end of the process is below 1%.

The behaviour of the control system in the presence of 5% white Gaussian noise added to the plant output has been studied accumulating some statistics of the final MSE% (ξ_{av}) for multiple run of the control program, which results are given on Table 1 for 20 runs. The mean average cost for all runs (ϵ) of control, the standard deviation (σ) with respect to the mean value and the deviation (Δ) are given by the following formulas, where the mean and standard deviation values of the process control output variables are also computed as:

$$\varepsilon = \frac{1}{n} \sum_{k=1}^{n} \xi_{av_k}; \sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \Delta_i^2}; \qquad (29)$$

$$\Delta = \xi_{av} - \varepsilon; \varepsilon = 0.6663 \%; \sigma = 0.0593 \%$$
(30)

Where k is the run number and n is equal to 20.

7. CONCLUSIONS

This paper proposes a new full order observer-filter RNNM with closed loop topology for state and parameter estimation and measurement noise filtering of hydrocarbon degradation process carried out in a rotating drum system. The proposed RNNM has six inputs, three outputs and nine neurons in the hidden layer, with global and local feedbacks. The BP learning algorithm is derived using the diagrammatic method and the adjoint RNNM.



Fig. 6. Graphical results of RNNM learning. a) EW; b) RH; c) VISC; d) MSE%.



Fig. 7. Graphical results of RNNM generalization. a)EW; b) RH; c) VISC; d) MSE%.



Fig. 8. Graphical results of SMC. a)VA; b) AW.



Fig. 9. Graphical results of SMC. a) EW; b) RH; c) MSE%.

| Table 1: Final means | squared | error | (%) |) of | <u>control</u> |
|--------------------------|-----------|--------|-----|------|----------------|
| (ξ_{av}) for 20 runs | of the co | ontrol | pro | gra | m |

| No | MSE of Control | No | MSE of Control |
|----|-------------------|----|-------------------|
| 1 | 0.6434 | 11 | 0.6602 |
| 2 | 0.6577 | 12 | 0.7759 |
| 3 | 0.7669 | 13 | 0.7732 |
| 4 | 0.6805 | 14 | 0.6566 |
| 5 | 0.6662 | 15 | 0.6408 |
| 6 | 0.5757 | 16 | 0.6481 |
| 7 | 0.5835 | 17 | 0.6061 |
| 8 | 0.7043 | 18 | 0.7240 |
| 9 | 0.7040 | 19 | 0.6514 |
| 10 | 0.6350 | 20 | 0.5725 |

Then the obtained RNNM is simplified and used to design a SMC. The experimental and simulation identification and control results obtained exhibit a good convergence and precise reference tracking. The MSE% of the RNNM learning and generalization is below 1.5% (2% for generalization) and the MSE% of control is below 1%. The results could be improved augmenting the number of measurement points per fermentation and augmenting the number of fermentations per epoch.

ACKNOWLEDGEMENTS

This work is partially supported by the Project SEMARNAT 2002-C01-0154 of CONACYT, Mexico. The Ph. D student Carlos-Roman Mariaca-Gaspar is thankful to CONACYT, Mexico for the fellowship received during his study in CINVESTAV-IPN, Mexico.

REFERENCES

- Alexander, M. (1999). *Biodegradation and bioremediation*. Academic Press, New York.
- Baruch, I.S., J. Perez-Medina, and J. Barrera-Cortes (2004). An indirect adaptive neural control of

wastewater treatment bioprocess. In: *Proc. of the First International Meeting on Environmental Biotechnology and Engineering, 11MEBE,* (H. Poggy-Varaldo. (Ed.)).CINVESTAV-IPN, Mexico City, Mexico, Sept. 6-8, ISBN 970-94112-0-9, [1MEBE155.pdf].

- Boskovic, J. D., Narendra K. S. (1995). Comparison of linear, nonlinear and neural-network-based adaptive controllers for a class of fed-batch fermentation processes. *Automatica*, **31**: 817-840.
- Cantelaube F., D. Bideau, and S. Roux (1997). Kinetics of segregation of granular media in a two-dimensional rotating drum. *Powder Technology*, **93**, 1-11.
- Caudill, M., and Ch. Butler (1995). Understanding neural networks: computer exploration, 2, 78-107. MIT Press, Cambridge, London.
- Cookson, J.T. (1995). *Bioremediation engineering: design and application*. Mc Graw-Hill, Inc. New York.
- De La Torre Sanchez, R., I.S. Baruch, B. Chavez-Gomez, E. Rios-Leal, and J. Barrera-Cortes (2004). Neural model of a biodegradation process profiles in biopile systems. In: Proc. of the First International Meeting on Environmental Biotechnology and Engineering, 11MEBE, (H. Poggy-Varaldo. (Ed.)).CINVESTAV-IPN, Mexico City, Mexico, Sept. 6-8, ISBN 970-94112-0-9, [1MEBE021.pdf].
- Gray, M.R., B. Mehta, and J.H. Masliyah (1993). Liquidside mass transfer coefficients for liquids and slurries in a rotating drum. *Chemical Engineering Science*, 48 (19), 3442- 3446.
- Gray, M.R., D.K. Banerjee, Ph.M. Fedorak, A. Hashimoto, J.H. Masliyah, and M.A. Pickard (1994). Biological remediation of anthracene-contaminated soil in rotating bioreactors. *Applied Microbiology and Biotechnology*, 40, 933-940.
- Haykin, S. (1999). Neural networks, a comprehensive foundation (second edition). Prentice-Hall, Upper Saddle River, New Jersey 07458, section 2.13, 84-89; section 4.13, 208-213.
- Manilla-Pérez E., H.M Poggi-Varaldo, B. Chávez- Gómez, F. Esparza-García, and J. Barrera-Cortés (2004). Evaluation of a rotatory drum used for bioremediation of a hydrocarbon contaminated soil (in Spanish), *Interciencia*, **29** (9), 515-520.
- Narendra, K.S., and K. Parthasarathy (1990). Identification and control of dynamic systems using neural networks. *IEEE Trans. on Neural Networks*, 1 (1), 4-27.
- Rodríguez-Meza, M.A., J. Santa-Cruz Medina, H.M. Poggi-Varaldo, B. Chávez Gómez, and J. Barrera-Cortés (2006). Effect of the ratio length to diameter on the bioremediation of a polluted soil treated in opening rotating drums. In: Proc. of the Second International Meeting Environmental on Biotechnology and Engineering, 2IMEBE, (H. Poggy-Varaldo. (Ed.)).CINVESTAV-IPN, Mexico City, Mexico, Sept. 26-29, ISBN 970-95106-0-6 [2MEBE032.pdf]
- Valdez-Castro, L., I.S. Baruch, and J. Barrera-Cortes (2003). Neural networks applied to the prediction of fed-batch fermentation kinetics of Bacillus thringiensis. *Bioprocess and Biosystems Engineering*, Springer-Verlag, 25, 229-233.
- Wan, E., and F. Beaufays (1996). Diagrammatic method for deriving and relating temporal neural networks algorithms. *Neural Computations*, 8, 182-201.
- Young, K.D., V.I. Utkin, and U. Ozguner (1999). A control engineer's guide to sliding mode control, *IEEE Trans.* on Control Systems Technology, **7** (3), 328-342.