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SIMULTANEOUS PROCESS AND CONTROL DESIGN OF DYNAMIC SYSTEMS UNDER UNCERTAINTY

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Abstract: In this paper, a new methodology to integrate process design and control for systems under uncertainty is proposed. Instead of using dynamic optimizations to estimate the system's maximum variability, process stability and process constraints, this methodology applies a robust control approach to calculate bounds on these conditions. To illustrate the methodology the design of a mixing tank process is considered.

Keywords: systems design, robust control, systems identification.

1. INTRODUCTION

In the last decades, the necessity to integrate process design and control in one single step has been widely recognized; i.e., develop a feasibility, flexibility and controllability analysis of the process simultaneously. Due to its complexity, several methodologies had been reported to address this problem. One set of methodologies had made use of controllability measures combined with an economic index to assess the simultaneous design (e.g. Luyben and Floudas, 1994; Alhammadi and Romagnoli, 2004). Most of these methods use steady state models or linear dynamic models to represent the systems behaviour; therefore, the application of these methods to highly nonlinear systems is limited by this fact. Also, the indices used to measure the closed loop performance do not directly represent an economic cost of the process variability. This is the major drawback for these methods.

The most recent methodologies address this problem by using a single performance index objective function (e.g. Mohideen et al., 1996; Swartz, 2000; Kookos and Perkins, 2001; Chawankul et al., 2005). Most of these methods use a rigorous nonlinear dynamic model of the system within a dynamic optimization problem to assess the system's closed loop variability. Consequently, this calculation would require an intensive computational effort. Therefore, these methodologies can only be realistically applied to chemical processes with a small number of process units. A more comprehensive review of the current methodologies that address the integration of design and control problem can be found in Seferlis and Georgiadis (2004) and Sakizlis et al. (2004).

This paper presents an approach to the integration of process design and control problem that does not require dynamic optimization. The proposed methodology uses robust control analysis tools to address the design of dynamic systems under uncertainty.

2. METHODOLOGY

The conceptual mathematical formulation followed to achieve the integration of design and control is explained in this section.

2.1 Cost Function

In the present work, the objective function is formulated as the combination of the steady state cost, i.e. the capital and operating costs, and a

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variability cost, which is directly related to the dynamic performance of the system to be designed. This can be mathematically expressed as:

$$O.F. = CC\left(\mathbf{d}, \mathbf{u}, \phi^d\right) + OP\left(\mathbf{d}, \mathbf{u}, \phi^d\right) + VC\left(\mathbf{d}, \mathbf{u}, \lambda, \phi^d\right) \quad (1)$$

Where the capital cost (CC), the operating cost (OP) and the variability cost (VC) are functions of the design variables (**d**), the manipulated variables to be used by the control strategy (**u**) and the process variability (ϕ^d), respectively. In addition, the variability cost function is also a function of the controller tuning parameters (λ).

The capital and operating cost of a plant are generally estimated from cost correlations which depend on the design variables of the system. To assess the variability cost, it is necessary first to measure the process variability and assign a cost to it. The process variability function (ϕ^d) is specific for every process and depends on factors such as the goals to be attained by the design, the process inputs and outputs and the nature of the process itself. The allocation of a specific cost to the process variability function also depends on such factors. For example, if the goal is to design a system to keep the property of a product on target, then the cost is related to the deviations in this property with respect to the target. On the other hand, if the sole objective is to design a unit that can reject disturbances in an effective manner, then the variability cost may be associated with the unit's capacity which has a direct impact on the capital cost.

In the current work, to simplify the analysis, the perturbations that may affect the process are classified in two classes: i)-unmeasured disturbances (**υ**) defined as perturbations that vary rapidly in the time domain and ii)-unmeasured perturbations that change very infrequently in time to be referred heretofore as process parameter uncertainty (**ω**). This later type of perturbation is considered to remain at constant values for long periods of time. Thus the transients in **ω** are ignored in the analysis since they are expected to occur very infrequently during the plant's normal operation.

2.2 Process Model

It is assumed in this work that a full nonlinear dynamic process model is available for simulations. Then, a key idea in the proposed approach is to use a robust control model approach to represent the full nonlinear model of the controlled plant. Thus, the plant's closed-loop nonlinear dynamic model is described here as a nominal closed-loop linear state space model supplemented with model uncertainty. The set of uncertain values are defined as model parameter uncertainty (θ). These uncertain values capture the process nonlinearities due to the changes in the disturbance variables υ. This model is given as follows:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}(\theta_A)\mathbf{x} + \mathbf{B}(\theta_B)\boldsymbol{\upsilon} \\ \mathbf{y} &= \mathbf{C}(\theta_C)\mathbf{x} + \mathbf{D}(\theta_D)\boldsymbol{\upsilon} \end{aligned} \quad (2)$$

These models can be obtained from closed loop identification techniques, (Ljung, 1987).The input to this model is the disturbance (**υ**) that affects the process outputs (**y**). The closed loop model (2) describes the process response to changes in the input disturbance (**υ**) in a neighbourhood of a nominal operating condition, specified by the manipulated variables (**u**), the design variables (**d**), the process parameter uncertainty (**ω**) and the controller tuning parameters (λ). Using the uncertain model in (2), robust control tools can be applied to assess robust stability and to calculate bounds on variability. This approach is expected to reduce the computational time since dynamic optimization problems are avoided. Similar state space models can be also developed to represent the closed loop response in **u** with respect to changes in **υ**. Then, the calculated variability in **u** obtained with these models can be used to test constraints on manipulated variables.

2.3 Process Stability

Most of the methodologies proposed in the literature do not include a stability condition within their framework. To assure process asymptotic stability, this approach applies a robust stability criteria based on a quadratic Lyapunov function for linear time-invariant systems (Boyd and Yang, 1989). This condition can be mathematically expressed as:

$$\mathbf{A}(\theta_A)^T\mathbf{P} + \mathbf{P}\mathbf{A}(\theta_A) < 0 \quad (3)$$

Where **P** is a symmetric positive definite matrix and $\mathbf{A}(\theta_A)$ is the **A** state space matrix of the system shown in (2). Inequality (3) represents an infinite number of inequalities corresponding to all the possible values of the uncertain model parameters of the state matrix **A**. However, due to the convexity of the function in the LHS of (3) with respect to θ and **P**, it is possible to evaluate this expression by testing a finite number of Linear Matrix Inequalities (LMI's) estimated at the combinations of the extreme values of θ as follows:

$$\begin{aligned} &\mathbf{A}(w_k)^T\mathbf{P} + \mathbf{P}\mathbf{A}(w_k) < 0 \quad \textit{for} \quad \textit{all } w_k \in \mathbf{W} \\ &\mathbf{W} = \left\{(w_1, w_2, \ldots, w_n) : w_k \in \left[a_{ij} - \theta_{A,ij}, a_{ij} + \theta_{A,ij}\right]\right\} \end{aligned} \quad (4)$$

Where w_k represents a particular combination of the uncertainty values identified for the **A** state space matrix. A sufficient condition for asymptotic stability in a linear time-invariant system is the existence of a matrix **P** such that **P**>0, $\mathbf{P}=\mathbf{P}^T$, which satisfies the finite set of LMI's presented in (4).

2.4 Assessment of the Worst-Case Scenario

A key problem for integrating design and control is the calculation of the critical time dependent disturbance that produces the largest variability in the system. This condition, referred to as the *worst-case* *scenario*, is found by using a Structured Singular Value (SSV) test. The computation of the largest variability at the end of a batch process was calculated by Nagy and Braatz (2003) applying a Mixed Structured Singular Value approach. The current work expands upon that work by proposing a calculation of the variability for a continuous process over a predetermined time horizon. In principle, the calculation for continuous processes would require considering an infinite time horizon, but for practical applications it is sufficient to look at the output's settling time, *N*. To evaluate the worst-case condition around a nominal operating point (**u**, **d** and **λ)** and at a specific value of the process parameter uncertainty (**ω),** the following impulse response model was used:

$$\mathrm{y(j)} = \sum_{q=1}^{m}\sum_{i=0}^{j}\left[h_{iq}\delta\upsilon_q(j-i) + \delta h_{iq}\delta\upsilon_q(j-i)\right];0 \le j \le N \quad (5)$$

Where h_{iq} and δh_{iq} are the nominal and the uncertain values of the impulse response coefficients relating the output **y** to the input disturbance **υ** at time j and **υ** has dimension mx1. The impulse responses of model (2) are simulated for different combinations of the extreme values of the model parameter uncertainty θ's and for each element υ_q of **υ**. From these simulations, upper and lower bounds of **y** are obtained for each time step (j) and for each element of **υ**. Accordingly, the values of h_{iq} and δh_{iq} are calculated as follows:

$$h_{iq} = \frac{h_{iq}^{up} + h_{iq}^{low}}{2}; \delta h_{iq} = \frac{h_{iq}^{up} - h_{iq}^{low}}{2} \quad (6)$$

where h_{iq}^{up} and h_{iq}^{low} represent the upper and lower bounds of the output variable **y** at each time step (j) for each υ_q $\delta\upsilon_q(j-i)$ denotes a change in the disturbance variable υ_q at time (j-i). The disturbance is assumed to be bounded between a priori known bounds.

The goal is to find the critical time-dependent profiles in $\boldsymbol{\delta\upsilon}$ that would drive the output variable to its maximum deviation, i.e.:

$$\max_{\boldsymbol{\delta\upsilon}}|y(j)| \quad (7)$$

In principle, dynamic optimization could be used to find the exact worst perturbation by using the full nonlinear process model. Instead, a bound on the variability can be found by applying a Mixed Structured Singular Value analysis to the uncertain impulse response model given by (5). Following Nagy and Braatz (2003) the output values of **y** for a prediction horizon of N intervals may be bounded as follows:

$$\max_{\boldsymbol{\delta\upsilon}}\left|\sum_{q=1}^{m}\sum_{i=0}^{j}\left[h_{iq}\delta\upsilon_q(j-i) + \delta h_{iq}\delta\upsilon_q(j-i)\right]\right| \ge k \Leftrightarrow \mu_{\boldsymbol{\Delta}}(\mathbf{M}) \ge k \quad (8)$$

Thus, the largest error of **y** at any time interval *j* along a time horizon of N intervals can then be found from the solution of a "skewed' μ problem as follows:

$$\max_{\mu_{\boldsymbol{\Delta}}(\mathbf{M})\ge k} k \quad (9)$$

Where the perturbation block Δ used in the μ calculation has a structured form as follows:

$$\boldsymbol{\Delta} = \mathrm{diag}\left(\boldsymbol{\Delta}_r^1, \boldsymbol{\Delta}_\mathbf{r}^\mathbf{2}, \boldsymbol{\delta}_\mathbf{c}\right) \quad (10)$$

Where $\boldsymbol{\Delta}_\mathbf{r}^\mathbf{1}$ and $\boldsymbol{\Delta}_\mathbf{r}^\mathbf{2}$ are independent real scalar vectors of dimension m x 1 and (N*m) x 1, respectively; and $\boldsymbol{\delta}_\mathbf{c}$ is complex scalar vector of dimension N x 1.The interconnection matrix, **M,** used in (9) has the following structure:

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & k[\mathbf{W_1},\ldots,\mathbf{W_q},\ldots,\mathbf{W_m}]^T \\ \mathbf{S} & 0 & 0 \\ [\mathbf{H_1},\ldots,\mathbf{H_q},\ldots,\mathbf{H_m}] & [\mathbf{R_1},\ldots,\mathbf{R_q},\ldots,\mathbf{R_m}] & 0 \end{bmatrix} \quad (11)$$

Where:

$$\mathbf{W_q} = \begin{bmatrix} & & & & \delta\upsilon_q(0)I_{N+1,N+1} \\ 0 & & & & \delta\upsilon_q(1)I_{N,N} \\ \vdots & \ddots & & & \vdots \\ 0 & \ldots & 0 & & \delta\upsilon_q(j)I_{N-j+1,N-j+1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & \ldots\ 0 & \delta\upsilon_q(N) \end{bmatrix} \quad (12)$$

$$\mathbf{R_q} = \begin{bmatrix} & & & & \delta h_{0,q}I_{N+1,N+1} \\ 0 & & & & \delta h_{1,q}I_{N,N} \\ \vdots & \ddots & & & \vdots \\ 0 & \ldots & 0 & & \delta h_{j,q}I_{N-j+1,N-j+1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & \ldots\ 0 & \delta h_{N,q} \end{bmatrix}^T \quad (13)$$

$$\mathbf{H_q} = \begin{bmatrix} & & & & \mathrm{diag}(h_{0,q}, h_{1,q},\ldots,h_{j,q},\ldots,h_{N,q}) \\ 0 & & & & \mathrm{diag}(h_{0,q},\ldots,h_{N-1,q}) \\ \vdots & \ddots & & & \vdots \\ 0 & \ldots & 0 & & \mathrm{diag}(h_{0,q},\ldots,h_{N-j,q}) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & \ldots\ 0 & h_{0,q} \end{bmatrix}^T \quad (14)$$

$$\mathbf{S} = \mathrm{diag}(\mathbf{T},\ldots,\mathbf{T}),\ \textit{for}\ q = 1,\ldots,m \quad (15)$$

$$\mathbf{T} = \begin{bmatrix} & & & & & \mathbf{B_1} \\ 0 & & & & & \mathbf{B_2} \\ \vdots & \ddots & & & & \vdots \\ \vdots & \ldots & 0 & & & \mathbf{B_j} \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ 0 & \ldots & \ldots & \ldots & 0 & \mathbf{B_{N+1}} \end{bmatrix} \quad (16)$$

$$\mathbf{B_1} = \begin{bmatrix} t_{1,1} & 0 & \ldots & 0 \\ 0 & t_{2,[(N+1)+1]} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \ldots & 0 & t_{N+1,[(N+1)+N+\ldots+(N-(N-2))+1]} \end{bmatrix} \quad (17)$$

$$\mathbf{B_j} = \begin{bmatrix} t_{1,1} & 0 & \ldots & \ldots & \ldots & \ldots & 0 \\ 0 & t_{2,[(N+1)+1]} & 0 & \ldots & \ldots & \ldots & \vdots \\ \vdots & 0 & \ddots & 0 & \ldots & \ldots & \vdots \\ 0 & \ldots & 0 & t_{N-(j+2),[(N+1)+N+\ldots+(N-(N-(j+1)))+1]} & 0 & \ldots & 0 \end{bmatrix} \quad (18)$$

$$\mathbf{B_{N+1}} = [t_{1,1} \quad 0 \quad \ldots \quad 0] \quad (19)$$

In (17), (18) and (19), t_{ij}=k, otherwise 0. Thus, the last N rows of the matrix **M** in (10) corresponds to the different values of y(j) for $0 \le j \le N$. The solution to (9) provides the worst-disturbance vector that produces the largest variability in the output variable **y** as given by the bound *k*.

The previous analysis specifies the worst-case condition in **y** based on the system (2), which in turn was generated around a nominal operating point and at a fixed value of the process parameter uncertainty ω. Then, one must search for the value in ω and the corresponding profile in δυ that produces the largest output error in **y**. The calculation is performed using closed-loop state space models, (2), each identified around a set of operating conditions defined by nominal values of **u**, the design variables **d** and tuning parameters λ. Thus, the calculation of the worst-case scenario is expressed as:

$$\begin{aligned} &\max_{\boldsymbol{\omega}_l \le \boldsymbol{\omega} \le \boldsymbol{\omega}_u} \phi^d(\mathbf{d},\mathbf{u},\boldsymbol{\lambda},k^d) \\ &s.t. \max_{\mu_\Delta(\mathbf{M}_d) \ge k^d} k^d \end{aligned} \tag{20}$$

The optimization problem shown in (20) could be reduced to a single calculation if ω would be considered as a dynamic disturbance within the closed-loop state space model; that is, closed loop identification could be conducted where ω and υ are considered as inputs to system (2). This approach would consider the transients in ω and would also eliminate the need for solving the optimization problem in (20), but it would potentially lead to a system with larger uncertainty and consequently to a more conservative design. Solving (20) requires the identification of a system like (2) for every ω tested. However, the identification process is simple and the identified models are expected to have small model uncertainty since the process is identified in closed loop. Based on the knowledge of the worst-case variability in each of the output variables, a cost can be assigned to quantify the actual economic impact of the variability in (1).

2.5 Process Feasibility

Process feasibility requires that the manipulated variables **u**, must remain within the specified bounds for all possible profiles values υ and ω. The maximum values in **u** can be calculated by using the same methodology applied to calculate the worst output variability by using μ. To evaluate this condition, (9) and (20) can be rearranged as follows:

$$\bar{u} \pm \max_{\boldsymbol{\omega}'_l \le \boldsymbol{\omega}' \le \boldsymbol{\omega}'_u} \max_{\mu_\Delta(\mathbf{M}_u) \ge k^u} k^u \le 0 \tag{21}$$

where $\bar{u}$ represents a steady state value of the manipulated variable, and $\mathbf{M_u}$ is the interconnection matrix with respect to this variable; thus, inequality (21) requires the identification of a robust model as given in (2) from the disturbance υ to the manipulated variable **u**.

2.6 *Optimization Problem*

The formulations in (1), (4), (20) and (21) can be combined into one optimization problem as follows:

$$\begin{aligned} &\min_{\mathbf{d},\boldsymbol{\lambda}} \mathrm{CC}(\mathbf{d},\mathbf{u},\phi^d) + \mathrm{OP}(\mathbf{d},\mathbf{u},\phi^d) + \mathrm{VC}(\mathbf{d},\mathbf{u},\boldsymbol{\lambda},\phi^d) \\ &s.t. \max_{\boldsymbol{\omega}_l \le \boldsymbol{\omega} \le \boldsymbol{\omega}_u} \phi^d(\mathbf{d},\mathbf{u},\boldsymbol{\lambda},k^d) \\ &\quad s.t. \max_{\mu_\Delta(\mathbf{M}_d) \ge k^d} k^d \\ &\quad \bar{u} \pm \max_{\boldsymbol{\omega}'_l \le \boldsymbol{\omega}' \le \boldsymbol{\omega}'_u} \max_{\mu_\Delta(\mathbf{M}_u) \ge k^u} k^u \le 0 \\ &\quad \mathbf{A}(w_k)^T \mathbf{P} + \mathbf{PA}(w_k) < 0 \quad for \quad all\ w_k \in \mathbf{W} \\ &\quad d_l \le d \le d_u \\ &\quad \boldsymbol{\lambda}_l \le \boldsymbol{\lambda} \le \boldsymbol{\lambda}_u \end{aligned} \tag{22}$$

This problem corresponds to a constrained nonlinear optimization problem. It should be noted that the process flow sheet, the control structure and pairing between the manipulated variables and the controlled variables are assumed a priori. Thus, control structure selection and the process synthesis problem are not explicitly considered in this work.

The proposed strategy involves the following steps:

Step 0 (Initialization): Initial values for the process manipulated variables (**u**), the controller tuning parameters (λ), the disturbance variables (υ), and the process parameter uncertainty (ω) are specified.

At iteration k:

Generate a state space model with model uncertainty: Closed loop state space models representing the response of **y** and **u** with respect to changes in υ as given in (2), with a corresponding uncertainty description, are identified. Pseudo random binary signals (PRBS) are designed using the upper and lower bounds specified for the disturbance variables υ. The model is obtained from the simulation of the process closed loop dynamic model around a nominal operating condition defined by the values of λ, **u** and ω. The input/output data collected from simulation of the full nonlinear model of the process is used to identify the desired model. The nominal linear model is determined by least-squares fitting whereas the model parameter uncertainties are calculated from the covariance matrix generated from identification of the nominal linear model (Ljung, 1987).

Step 1 (Worst-case scenario): For the specified conditions, problem (20) is solved to find the critical time profile in ω and υ that produce the maximum variability in the system (function ϕ^d).

Step 2 (Stability Test): Based on the pre-specified conditions and on the critical profiles found in the previous step, the stability test (4) is conducted. If the test fails, the values in **d**, and λ are redefined and the calculations are restarted from step 1. Otherwise proceed to step 3.

Step 3 (Process Constraints): Problem (21) is solved to test whether process constraints are satisfied.
Step 4 (Objective Function Evaluation): The system's cost function (1) is calculated. If the values between the previous and the actual iteration in the objective function are less than a pre-specified tolerance and the process constraints are also within pre-specified tolerances, then an optimal solution has been found; otherwise, k is set to k=k+1, and a new search is performed starting from step 1.

3. CASE STUDY AND RESULTS

The proposed methodology formulated in the previous section is applied to solve for the optimal design of a mixing tank problem, previously studied by Mohideen et al. (1996). Although this process is described by relatively simple equations, the integration of design and control on this problem is a challenging task. The final designed configuration must be able to reject predefined bounded disturbances and process parameter uncertainty changes that may occur during normal operation.

The system consists of two process streams mixed in a stirred tank as shown in Figure 1. The tank's hold up V(t), and the outlet temperature T(t) are state variables that describe the system's behaviour at time t. The hot feed flow rate, $F_h(t)$, is an unmeasured variable that varies very infrequently in time; thus, it is treated in this study as a process parameter uncertainty (ω), whereas the hot stream temperature, $T_h(t)$, is a fast-varying perturbation to be treated consequently as a disturbance variable (υ). The cold feed stream is at a constant temperature, T_c, whereas the corresponding flow, F_c, can be manipulated manually or in closed loop. The outlet flow, F, is assumed to be a nonlinear function of the volume, V(t), and the valve constant, z, which operates within a pre-specified range of values as given in Table 1, and it can also be manipulated manually or in closed loop. The tank is assumed to be well stirred and the density of the substance is assumed constant. Table 1 also includes the process model and numerical values used in the design of the process.

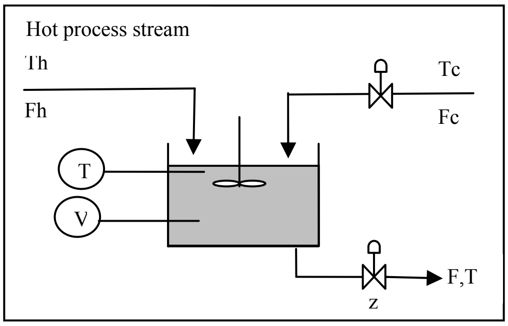

Fig. 1. Case Study: Mixing tank process.

The goal in this case study is to search for the controller tuning parameters (λ) and the nominal values for the design and the manipulated variables (**d** and **u**) that minimize the mixing tank's hold up, $V_d(t)$. The system's final design must remain stable at any time and it has to satisfy process constraints, e.g. outlet temperature should remain between bounds, despite any possible values of the disturbance, T_h, and the process parameter uncertainty, F_h.

Table 1 Process model and data for mixing tank.

Process Model	$\frac{dV}{dt} = F_h + F_c - F(V)$ $F(V) = zV^{1/2}$ $V\frac{dT}{dt} = F_h(T_h - T) + F_c(T_c - T)$
Disturbance Var. (υ)	$350\ K \le T_h \le 390\ K$
Process par. uncert. (**ω**)	$0.05\ m^3/hr \le F_h \le 0.15\ m^3/hr$
Output Variables (**y**)	$V \ge 0.9\ m^3$ $335K \le T \le 370K$
Manipulated var. (**u**)	$0.015\ m^3/hr \le F_c \le 0.2\ m^3/hr$ $0 \le z \le 0.3$
Constant variables	$T_c = 298K$
Design variables (**d**)	T_{sp}, V_{sp}
Controller par. (λ)	$K_{CT}, K_{CV}, \tau_{IT}, \tau_{IV}$

When performing the simultaneous design on this process, the mixing tank's volume was controlled with a proportional-integral (PI) controller using the valve constant (z) as the adjustable variable, and the tank temperature, T, was controlled by another PI controller using the cold feed flow rate, F_c, as the manipulated variable. The cost function in this problem is considered to be the cost due to the tank size; thus, the design mixing tank's hold up is given as follows:

$$V_d(t) = (V_{sp} + k^V) \quad (23)$$

Where V_{sp} represent the capital cost at steady state(CC), and k^V is the cost due to variability (VC) in the volume with respect to steady state. The operating costs (OP) are zero. Using (21), similar expressions to (23), not shown for brevity, are derived to test the constraints on the tank's temperature T and on the manipulated variables F_c and z.

The methodology explained above was used to perform the simultaneous design of this process. The controller's set points (V_{sp} and T_{sp}) and tuning parameters (K_{CT} and K_{CV}) are considered as decision variables within the optimization problem (22). For simplicity, the PI controller's time constants (τ_I) were assumed constants. The resulting optimization problem was implemented in MATLAB®. Table 2 shows the results obtained while applying the present methodology and those obtained by Mohideen et al. (1996). To test the compliance with the constraints, the design obtained by this approach was simulated using the mixing tank's process model (Table 1) and applying different sets of combinations in T_h and F_h. As shown in Figure 2, the actual volume never exceeds the design hold up, $V_d(t)$, for different combinations in T_h and F_h.

A complete comparison with Mohideen et al.'s results is not possible since the strategies applied to perform the simultaneous design were somewhat different. Mohideen et al. (1996) solved this problem applying a dynamic optimization-based approach. Although their design resulted in a smaller tank and only one PI-controller was used to control T by manipulating F_c, z was manipulated based on the nonlinear dynamic optimization to accommodate the changes in F_h for only 30 hours of operation. Hence, online implementation of their methodology will require repeated dynamic optimization based on a priori knowledge of future values of F_h. On the other hand, the present methodology solved this problem based on an off-line constrained nonlinear optimization while explicitly providing an additional PI controller for on-line control of the mixing tank's volume by manipulating the valve constant z. Although the resulting volume is larger than Mohideen et al.'s and the bounds on F_c and T were relaxed, the present approach does not require a priori knowledge of the perturbation variables, T_h and F_h. From the optimization point of view, the application of the criteria developed in this work avoids the task of numerically solving dynamic optimization problems, as used in the most recent methodologies (Sakizlis et al., 2004).

Table 2 Comparison of decision variables values.

Variables	Solution	Mohideen et al. (1996)
V_d (m^3)	1.39	1.0
V_{sp} (m^3)	1.38	--
T_{sp} (K)	337.96	360.0
Kc_T	-3.60	-0.005
Kc_V	-18.23	--
τ_{IT} (hr)	5.0 (fixed)	5.0
τ_{IV} (hr)	5.0 (fixed)	5.0

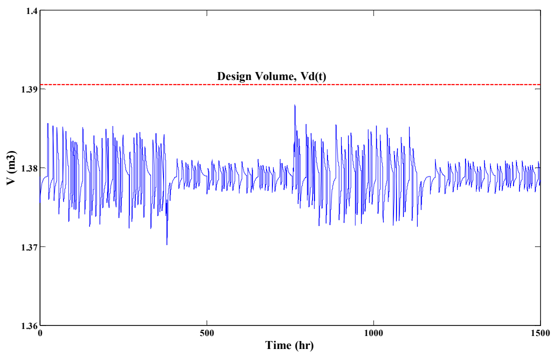

Fig. 2. Mixing Tank's actual volume versus time.

4. CONCLUSIONS

A new approach to integrate process design and control for processes under uncertainty and disturbances has been presented. This methodology uses a robust model, identified from simulations of the full nonlinear model around a steady state condition. Based on the identified robust model, bounds for stability and variability are calculated and are used for optimization. Due to the model parameter uncertainty associated with the robust model, the final design tends to be conservative. However, the current approach does not require dynamic optimizations, which are computationally expensive even in simple cases.

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