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DESIGN OF A SLIDING-MODE OBSERVER FOR A BIOTECHNOLOGICAL PROCESS

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Abstract: This paper deals with the implementing of a sliding-mode observer for the lipase producing process using *Candida rugosa* yeast. It does consider a simplified model of the process, consisting in three equations, for the two substrates and the biomass. The sliding-mode observer has been designed on the basis of the equivalent control method. For the observer implementation it is necessary a state transformation that has to bring the process to the observable canonic form. The results obtained through numerical simulation show a good behaviour of the sliding-mode observer in the presence of the parameter's uncertainties that interfere with the process.

Keywords**:** Biotechnological Process, Sliding-mode Observer, biomass specific growth rate

1. INTRODUCTION

It is very well known that the biotechnological processes are complex and strong non-linear. In both cases, batch or continuous processes, many difficulties in the modelling and the control of the biotechnological processes occur because these involve the growth of a microorganism's population. They often evolve unforeseeably: for example mutations could appear and in many cases the objectives to be attained can be compromised.

The difficulty to model and to control such processes is bigger in the case of discontinuous processes. They are totally isolated from the outside environment. During the process no additional substrate is fed from the outside, the microorganisms population growing only from the substrate supplied at the beginning of the batch. A number of uncertainties arises right from the beginning of the process with respect to the substrate preparation and nature (the composition of the natural substrates is not precisely known). Generally, these initial batch-preparation operations cannot be mathematically formalized thus generating major problems in the process modelling and synthesis of the control law.

A very big difficulty in the implementation of the biotechnological process control is the lack or the high cost of the instruments for on-line measuring the state variables of the process (biomass, substrates etc.). An alternative solution that is often used in practice consists in the software sensor implementation. These offer estimations of any unmeasured variables on the basis of the other variables that can be directly measured in the process. In this purpose extended state observers are used in the determinist variant, as well as in the stochastic one (Bastin and Dochain, 1990), (Barbu *et al.*, 2004), (Gouze *et al.*, 2004), (Hulboven *et al.*, 2006).

This paper aims to determine a sliding mode observer for an enzyme (lipase) producing process using *Candida rugosa* yeast. It contains 5 sections: section 2 deals with the theoretical background of the sliding mode observer, in section 3 the lipase producing process is presented, section 4 presents the sliding-mode observer implementation and the simulation results and the last section is dedicated to the conclusions.

2. THEORETICAL BACKGROUND ON THE SLIDING-MODE OBSERVERS

An estimation method used for the robustness increase in the presence of the modelling uncertainties is the use of variable structure systems. A first version of the variable structure observer is presented in (Walcott and Zak, 1987), knowing further modifications (Dawson *et al.*, 1992). The use of the sliding-mode control theory in the estimation problem of the process state has known two developing directions: one does consider an extension of the Luenberger observer through the inclusion of a term that allows the appearance of the sliding-mode regime (Slotine *et al.*, 1987) and the second one that is based on the equivalent control method (Drakunov, 1992). In this paper the second method will be used.

Let us consider the general case of a non-linear system:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{pmatrix} = \begin{pmatrix} x_2 + g_1(x_1, u) \\ x_3 + g_2(x_1, x_2, u) \\ \vdots \\ x_n + g_{n-1}(x_1, \ldots, x_{n-1}, u) \\ f_n(x) + g_n(x, u) \end{pmatrix} \tag{1}$$

for which the variable x_1 is measured:

$$y = x_1 \tag{2}$$

and $g_i(\cdot, u = 0) = 0$ for any $i = 1, ..., n$.

For the non-linear system given by equation (1) and on the basis of the results presented in (Drakunov, 1992), (Barbot *et al.*, 2002) proposes the following sliding-mode observer:

$$\begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \vdots \\ \dot{\hat{x}}_{n-1} \\ \dot{\hat{x}}_n \end{pmatrix} = \begin{pmatrix} \hat{x}_2 + g_1(x_1, u) + \lambda_1 \operatorname{sgn}(x_1 - \hat{x}_1) \\ \hat{x}_3 + g_2(x_1, \tilde{x}_2, u) + \lambda_2 \operatorname{sgn}(\tilde{x}_2 - \hat{x}_2) \\ \vdots \\ \hat{x}_n + g_{n-1}(x_1, \tilde{x}_2, \ldots, \tilde{x}_{n-1}, u) + \lambda_{n-1} \operatorname{sgn}(\tilde{x}_{n-1} - \hat{x}_{n-1}) \\ f_n(x_1, \tilde{x}_2, \ldots, \tilde{x}_n) + g_n(x_1, \tilde{x}_2, \ldots, \tilde{x}_n, u) + \lambda_n \operatorname{sgn}(\tilde{x}_n - \hat{x}_n) \end{pmatrix} \tag{3}$$

where:

$$\tilde{x}_2 = \hat{x}_2 + \lambda_1 \operatorname{sgn}(x_1 - \hat{x}_1)$$
$$\tilde{x}_3 = \hat{x}_3 + \lambda_2 \operatorname{sgn}(\tilde{x}_2 - \hat{x}_2)$$
$$\vdots$$
$$\tilde{x}_n = \hat{x}_n + \lambda_n \operatorname{sgn}(\tilde{x}_{n-1} - \hat{x}_{n-1})$$

The *sgn* function is filtered with a low-pass filter, that corresponds to the apply of an equivalent command.

Theorem 1: (Barbot *et al.*, 2002) Considering the BIBS system (Bounded Input Bounded State) in finite time, described by equation (1) and the sliding-mode observer (3), for any initial state and any bounded input, there is a choice of the parameter λ_i so that the state of the observer $\hat{x}$ converges to the real value x in finite time.

The demonstration of the theorem 1 leads to the following condition regarding the observer parameters (Barbu and Caraman, 2006):

$$\lambda_{i-1} > |e_i|_{\max}, \; i = 2, ..., n \tag{4}$$

$$\lambda_n > 0 \tag{5}$$

The design of a sliding-mode observer is done considering the general form of the non-linear systems which is described by the following equations:

$$\dot{x} = f(x) + g(x)u \tag{6}$$

$$y = h(x) \tag{7}$$

The non-linear system (6) can be transformed and brought to the equation (1) through a non-linear transformation of coordinates.

Definition 1: Let us consider a variable change described by the relation (8):

$$z = \Phi(x) \tag{8}$$

where $\Phi(x)$ is a $\mathbb{R}^n$ - function of n variables:

$$\Phi(x) = \begin{bmatrix} \phi_1(x) & \phi_2(x) & \cdots & \phi_n(x) \end{bmatrix}^T \tag{9}$$

that has the following properties:

1) $\Phi(x)$ is convertible, that means the function $\Phi^{-1}(x)$ exists, so that:

$$\Phi^{-1}\left(\Phi(x)\right) = x, \; \forall x \in \mathbb{R}^n \tag{10}$$

2) $\Phi(x)$ and $\Phi^{-1}(x)$ are smooth applications, so that they have continuous partial derivatives of any degree.

In these conditions the equation (8) is a transformation, namely diffeomorfism. If the two properties (1 and 2) are accomplished in the whole space $\mathbb{R}^n$, then the diffeomorfism is global. The system (6) can be brought to the equation (1) considering the following transformation of coordinates:

$$\Phi(x) = \begin{bmatrix} h(x) & L_f h(x) & \cdots & L_f^{n-1} h(x) \end{bmatrix}^T \tag{11}$$

if the transformation $\Phi(x)$ is a global diffeomorfism.

Definition 2: The system given by the equations (6)-(7) is of δ -relative degree in a point x_0 if:

1) $L_g L_f^k h(x)=0, \forall x$ in the neighbourhood of x_0 and $\forall k<\delta-1$
2) $L_g L_f^{\delta-1} h(x_0)\neq 0$

The non-linear system described by the equations (6)-(7), having the relative degree $\delta \le n$, can be brought to the following form, which is identical to the normal equation (1):

$$\begin{aligned} \dot{z}_i &= z_{i+1}\ , i=1,\ldots,\delta-1 \\ \dot{z}_j &= z_{j+1} + L_g L_f^{j-1} h\left(\Phi^{-1}(z)\right)\cdot u\ , j=\delta,\ldots,n-1 \\ \dot{z}_n &= L_f^n h\left(\Phi^{-1}(z)\right) + L_g L_f^{n-1} h\left(\Phi^{-1}(z)\right)\cdot u \end{aligned} \tag{12}$$

3. THE LIPASE PRODUCING PROCESS

Lipase is an enzyme which is produced in batch or fed-batch bioreactors (Selisteanu, 1999). The lipase is obtained using *Candida Rugosa* yeast. The yeast growth takes place in a proper environment where the substrate consumed by the biomass is the oleic acid. The biosynthesis process of the lipase is very complex and strong non-linear. It contains four phases which do appear simultaneously: the liquid phase, the organic phase, the cellular phase and the gaseous phase.

The process is given by the following equations:

$$\frac{dS_1}{dt} = -\eta(S_1)X + F \tag{12}$$

$$\frac{dS_2}{dt} = \eta(S_1) - \mu(S_2)\cdot(Y+S_2) \tag{13}$$

$$\frac{dX}{dt} = \mu(S_2)X \tag{14}$$

$$\frac{dL_{in}}{dt} = \nu_p(S_1, X, \mu) - \nu_{ex}(L_{in}) - \mu(S_2)L_{in} \tag{15}$$

$$\frac{dL_{ex}}{dt} = \nu_{ex}(L_{in})X \tag{16}$$

$$C_{er} = (a\mu(S_2)+b)X \tag{17}$$

where S_1 is the substrate consumed for the biomass growth [g/l], S_2 is the intracellular substrate [g/g], X represents the biomass [g/l], L_{in} is the intracellular enzyme [u/mg] and L_{ex} is the extracellular enzyme [u/ml]. The last equation is algebraic and expresses the specific outflow rate of CO_2, that depends on biomass. The parameter Y is the production coefficient "biomass/substrate". The bioreactor is supplied with a small quantity of substrate through the supply flow F. If F=0, the operating mode of the bioreactor is batch. The reaction rates are given by the equations (18) – (21):

$$\eta = \frac{\eta^* S_1}{K_{M1} + S_1} \tag{18}$$

$$\mu = \frac{\mu^* S_2}{K_{M2} + S_2} \tag{19}$$

$$\nu_{ex} = \frac{\nu_{ex}^* L_{in}}{K_{ex} + L_{in}} \tag{20}$$

$$\nu_p = \frac{\nu_p^*(S_1/X)}{K_p + (S_1/X) + K_i(S_1/X)^2}\cdot\mu(S_2) \tag{21}$$

In the equations (18)-(21) η is the absorption specific rate of the external substrate by the cells, μ - the specific growth rate of the biomass, ν_p - the producing rate of the internal lipase and ν_{ex} - the removal rate of the lipase in the liquid environment.

In industrial conditions the lipase production can be analyzed considering a part of the external lipase (the liquid lipase – L_a).

$$L_a = L_{ex}\left(1 + \frac{K_{a1}\cdot S_1}{K_{a2} + S_1}\right) \tag{22}$$

4. THE SLIDING-MODE OBSERVER IMPLEMENTATION

Further on, only a part of the model - equations (12) – (14) - will be considered. The equations describe the evolution of the substrate S_1, of the substrate S_2 and of the biomass X. The kinetics equations (18) and (19) are also considered. For the simplified model the substrate S_1 is measured and the other two state variables are estimated using a sliding-mode observer.

$$A(\hat{x}) = \left[\frac{\partial f(x)}{\partial x}\right]_{x=\hat{x}} = \begin{bmatrix} -\dfrac{\eta^*\cdot K_{M_1}\cdot\hat{X}}{\left(K_{M_1}+\hat{S}_1\right)^2} & 0 & -\dfrac{\eta^*\cdot\hat{S}_1}{K_{M_1}+\hat{S}_1} \\ \dfrac{\eta^*\cdot K_{M_1}}{\left(K_{M_1}+\hat{S}_1\right)^2} & -\dfrac{\mu^*\cdot\left(K_{M_2}\cdot Y + 2\cdot K_{M_2}\cdot\hat{S}_2 + \hat{S}_2^2\right)}{\left(K_{M_2}+\hat{S}_2\right)^2} & 0 \\ 0 & \dfrac{\mu^*\cdot K_{M_2}\cdot\hat{X}}{\left(K_{M_2}+\hat{S}_2\right)^2} & \dfrac{\mu^*\cdot\hat{S}_2}{K_{M_2}+\hat{S}_2} \end{bmatrix} = \left[a_{ij}(\hat{x})\right]_{i,j=\overline{1,3}} \tag{23}$$

For the observer design and implementation, an analysis regarding the system observability is necessary. Therefore the system is linearized around the point $e = 0$ resulting the matrices given by equations (23)-(24):

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{24}$$

The observability matrix becomes:

$$O = \begin{bmatrix} C \\ C \cdot A(x) \\ C \cdot A^2(x) \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & a_{32}(x) & a_{33}(x) \\ a_{32}(x) \cdot a_{21}(x) & a_{32}(x) \cdot \left(a_{22}(x) + a_{33}(x)\right) & a_{33}^2(x) \end{bmatrix} \tag{25}$$

and it has the determinant
$\det(O) = -a_{32}^2(x) \cdot a_{21}(x) \neq 0$,
because $X > 0$. It results $rang(O) = 3$, therefore the necessary condition of exponential observability is accomplished.

The process model can be written in the form (6)-(7) making the following notations:

$$f(x) = \begin{bmatrix} -\eta(S_1)X \\ \eta(S_1) - \mu(S_2)(Y + S_2) \\ \mu(S_2)X \end{bmatrix} \tag{26}$$

$$g(x) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \tag{27}$$

$$h(x) = S_1 \tag{28}$$

The Lie derivatives of the simplified model, namely $L_f^{i-1}h(x)$, $i = 1,2,3,4$, are given by the equations (29) – (32):

$$L_f^0 h(x) = S_1 \tag{29}$$

$$L_f^1 h(x) = -\eta(S_1)X \tag{30}$$

$$L_f^2 h(x) = -\eta(S_1)\eta'(S_1)X^2 - \eta(S_1)\mu(S_2)X \tag{31}$$

where

$$\eta'(S_1) = \frac{\eta^* K_{M_1}}{(K_{M_1} + S_1)^2}, \tag{32}$$

$$\mu'(S_2) = \frac{\mu^* K_{M_2}}{(K_{M_2} + S_2)^2}, \tag{33}$$

$$\eta''(S_1) = -\frac{2\eta^* K_{M_1}}{(K_{M_1} + S_1)^3}. \tag{34}$$

That leads to the following state transformation:

$$\Phi(x) = \begin{bmatrix} h(x) \\ L_f h(x) \end{bmatrix} = \begin{bmatrix} S_1 \\ -\eta(S_1)X \\ -\eta(S_1)\eta'(S_1)X^2 - \eta(S_1)\mu(S_2)X \end{bmatrix} \tag{35}$$

The Jacobean of the transformation (35) is given by equation (36):

$$d\left(\Phi(x)\right) = \begin{bmatrix} \frac{\partial h(x)}{\partial S_1} & \frac{\partial h(x)}{\partial S_2} & \frac{\partial h(x)}{\partial X} \\ \frac{\partial L_f h(x)}{\partial S_1} & \frac{\partial L_f h(x)}{\partial S_2} & \frac{\partial L_f h(x)}{\partial X} \\ \frac{\partial L_f^1 h(x)}{\partial S_1} & \frac{\partial L_f^1 h(x)}{\partial S_2} & \frac{\partial L_f^1 h(x)}{\partial X} \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -\eta'(S_1)X & 0 & -\eta(S_1) \\ \eta'(S_1)\left(\eta'(S_1)X^2 - \mu(S_2)X\right) + \eta(S_1)\eta''(S_1)X^2 & -\eta(S_1)\mu'(S_2)X & 2\eta(S_1)\eta'(S_1)X - \eta(S_1)\mu(S_2) \end{bmatrix} \tag{36}$$

and it has the determinant
$\det\left(d\left(\Phi(x)\right)\right) = -\eta^2(S_1)\mu'(S_2)X \neq 0$, for almost all situations met in practice. Therefore the Jacobean of the transformation (35) is nonsingular, that leads to the fact that the transformation is a local diffeomorfism in every point.

The Lie derivative of the simplified model, namely $L_g L_f^0 h(x)$, is:

$$L_g L_f^0 h(x) = 1 \tag{37}$$

As $L_g L_f^0 h(x) \neq 0$, it results that the model relative degree is 1 ($\delta = 1$).

The Lie derivatives $L_g L_f^{i-1} h(x)$, $i = 2,3$ are given by the following equations:

$$L_g L_f^1 h(x) = -\eta'(S_1)X \tag{38}$$

$$L_g L_f^2 h(x) = \eta'(S_1)\left(\eta'(S_1)X^2 - \mu(S_2)X\right) + \\ + \eta(S_1)\eta''(S_1)X^2 \quad (39)$$

that leads to the obtaining of the simplified system:

$$\dot{z}_1 = z_2 + F \quad (40)$$

$$\dot{z}_2 = z_3 - \eta'(S_1)X \cdot F \quad (41)$$

$$\dot{z}_3 = -\eta(S_1)X\left(\eta'(S_1)\left(\eta'(S_1)X^2 - \mu(S_2)X\right) + \\ + \eta(S_1)\eta''(S_1)X^2\right) - \eta(S_1)\mu'(S_2)X\left(\eta(S_1) - \\ - \mu(S_2)(Y + S_2)\right) + \mu(S_2)X\left(2\eta(S_1)\eta'(S_1)X - \\ - \eta(S_1)\mu(S_2)\right) + F\left(\eta'(S_1)\left(\eta'(S_1)X^2 - \mu(S_2)X\right) \\ + \eta(S_1)\eta''(S_1)X^2\right) \quad (42)$$

In equations (40) – (42) S_1, S_2 and X are replaced with the relations given by the reverse transformation of the system:

$$x = \Phi^{-1}(z) = \begin{bmatrix} z_1 \\ \dfrac{K_{M_2}\left(\eta(z_1)z_3 - \eta'(z_1)z_2^2\right)}{\eta(z_1)z_2\mu^* - \eta(z_1)z_3 + \eta'(z_1)z_2^2} \\ -\dfrac{z_2}{\eta(z_1)} \end{bmatrix} \quad (43)$$

For the transformed system, described by the equations (41)-(43), the following sliding-mode observer is built:

$$\dot{\hat{z}}_1 = \hat{z}_2 + F + \lambda_1 \tanh\left(z_1 - \hat{z}_1\right) \quad (44)$$

$$\dot{\hat{z}}_2 = z_3 - \eta'(S_1)X \cdot F + \lambda_2 \tanh\left(\tilde{z}_2 - \hat{z}_2\right) \quad (45)$$

$$\dot{\hat{z}}_3 = -\eta(S_1)X\left(\eta'(S_1)\left(\eta'(S_1)X^2 - \mu(S_2)X\right) + \\ + \eta(S_1)\eta''(S_1)X^2\right) - \eta(S_1)\mu'(S_2)X\left(\eta(S_1) - \\ \mu(S_2)(Y + S_2)\right) + \mu(S_2)X\left(2\eta(S_1)\eta'(S_1)X - \\ - \eta(S_1)\mu(S_2)\right) + F\left(\eta'(S_1)\left(\eta'(S_1)X^2 - \mu(S_2)X\right) + \\ + \eta(S_1)\eta''(S_1)X^2\right) + \lambda_3 \operatorname{sgn}\left(\tilde{z}_3 - \hat{z}_3\right) \quad (46)$$

where

$S_1 = z_1$, $S_2 = \dfrac{K_{M_2}\left(\eta(z_1)\tilde{z}_3 - \eta'(z_1)\tilde{z}_2^2\right)}{\eta(z_1)\tilde{z}_2\mu^* - \eta(z_1)\tilde{z}_3 + \eta'(z_1)\tilde{z}_2^2}$

and

$X = -\dfrac{\tilde{z}_2}{\eta(z_1)}$,

with

$\tilde{z}_2 = \hat{z}_2 + \lambda_1 \tanh\left(z_1 - \hat{z}_1\right)$ and

$\tilde{z}_3 = \hat{z}_3 + \lambda_2 \tanh\left(z_1 - \hat{z}_1\right)$.

For the sliding-mode observer the following values of the parameters λ were considered: $\lambda_1 = 5$, $\lambda_2 = 3$ and $\lambda_3 = 3$. Figures 1 - 3 present the results obtained through numerical simulation. The simulation shows very good performances of the sliding-mode observer, therefore a good convergence of the estimated variables to the real ones.

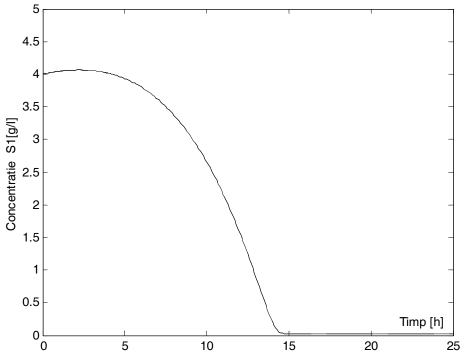

Figure 1: Substrate S_1 evolution (continuous line: process variable, dotted line: estimated variable)

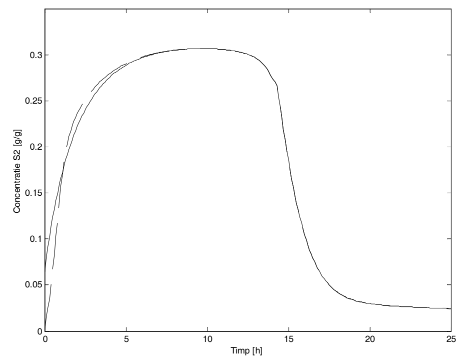

Figure 2: Substrate S_2 estimation (continuous line: process variable, dotted line: estimated variable)

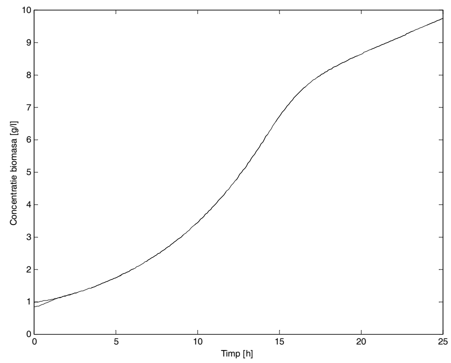

Figure 3: Biomass estimation (continuous line: process variable, dotted line: estimated variable)

The next three figures (4-6) show the sliding-mode observer behaviour in case of parametric uncertainties. In this purpose the values of two important parameters (μ and η) were modified (-5% for each parameter). A very good behaviour of the sliding-mode observer can be noticed, therefore this observer is robust with respect to parameter variations.

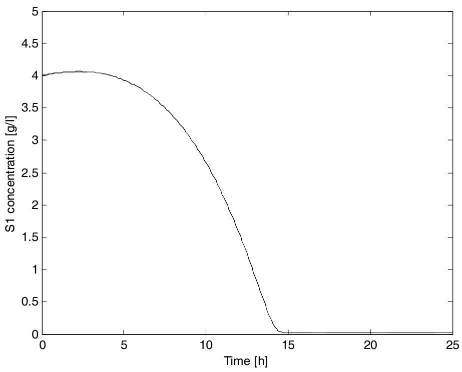

Figure 4: Substrate S_1 evolution

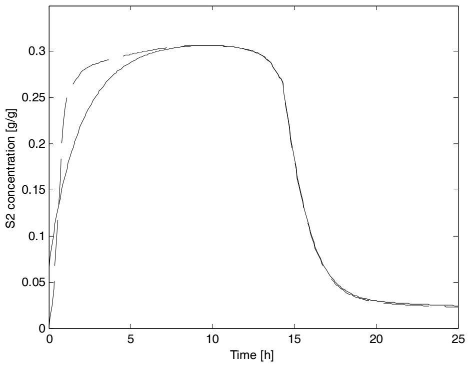

Figure 5: Substrate S_2 estimation (continuous line: process variable, dotted line: estimated variable)

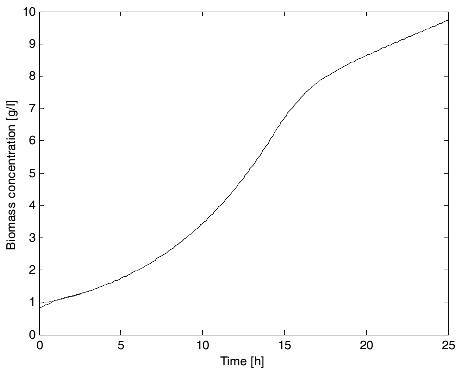

Figure 6: Biomass estimation (continuous line: process variable, dotted line: estimated variable)

5. CONCLUSIONS

An evaluation of the sliding-mode estimator can be done based on the comparison with a classic estimation structure, the extended Kalman filter, implemented on the same process [bar2004].

1. The implementation of the sliding-mode observer, based on the equivalent control method is easier because it needs the tuning of a small number of parameters.
2. The sliding-mode observer provides better results in the presence of the parameter's uncertainties and weak performances in the presence of the measurement noise.
3. The main drawback of the sliding-mode observer design method consists in the difficulty to bring the system to the observable canonic form, due to the complex forms of the parameters used in the biotechnological process models.

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