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AN INTERVAL OBSERVER FOR NON-MONOTONE SYSTEMS: APPLICATION TO AN INDUSTRIAL ANAEROBIC DIGESTION PROCESS

M. Moisan and O. Bernard

INRIA Comore, BP 93, 06902 Sophia–Antipolis, France
fax: +33 492 387 858, email:
{mmoisan,obernard}@sophia.inria.fr

Abstract: This article proposes a new class of observers in order to estimate unmeasured variables of a biotechnological process. The observer is developed on the basis of interval estimates, which provide guaranteed upper and lower bounds of the unknown variables. The proposed method exploits monotonicity properties of the error dynamics. A bundle of observers is generated by appropriately varying the observer gains. The method is applied to a real industrial anaerobic digestion plant, for the estimation of the key variables on the basis of the available measurements of the methane flow rate.

Keywords: Interval observers, robust observers, anaerobic digestion, monotone systems.

1. INTRODUCTION

Biotechnological processes play an increasing role in many industries, such as food, pharmaceutical or depollution (Stephanopoulos *et al.*, 1998). In contrast with other kinds of processes that are perfectly described by physical laws -like mechanical or electrical systems- biotechnological processes are dealing with living organism. As a consequence, their modelling is uncertain and is known to have a lower aptitude to accurately match experimental results.
On the other hand, online monitoring of biotechnological processes is a very difficult task. The difficulty to measure chemical and biological variables is one of the main limitations in the improvement of monitoring and optimisation of bioreactors. The lack of hardware sensors to perform monitoring tasks has forced the implementation of complicated, and not reliable methods. This problem becomes of great importance in more complex systems like anaerobic wastewater treatment plants, where critical instability of the process must be avoided, making the monitoring of the system variables an important issue (Mailleret *et al.*, 2004).
As an efficient solution for the inherent problem of monitoring biotechnological processes, the internal state reconstruction can be achieved by formulating observers, also called software sensors.
Many types of observers have been proposed and extensively studied, even for nonlinear biological systems (Bastin and Dochain, 1990; Bernard and Gouzé, 2006) and the choice of the design method depends on the kind of available models. Indeed, the quality of the used model is a factor of great importance when choosing an observation strategy. For instance, if a well identified and validated model is available, a high gain observer (Gauthier *et al.*, 1992) may perform good estimations of the internal state. If we have to deal with large uncertainties in model parameters, inputs and measurements, robust state estimation methods,

for example based on interval analysis (Jaulin *et al.*, 2001) and approximation of reachable set using ellipsoids (Kurzhanski and Valyi, 1997), both for discrete time systems, and cooperativity based interval observers (Gouzé *et al.*, 2000; Rapaport and Gouzé, 2003) for the continuous time systems, should provide robust estimates.
Interval observers work under the formulation of two observers: an upper observer, which produces an upper bound of the state vector, and a lower observer producing a lower bound, providing by this way a bounded interval in which the state vector is guaranteed to lie. For the formulation of the interval observer, it is necessary to know bounds of the uncertainties in the model (*i.e.* uncertainties in model parameters, input variables, etc.). These observers are based on hypotheses of monotonicity (Smith, 1995), and the design of such observers for non monotone systems is much less straightforward. We propose here an alternative design strategy to the one proposed in (Moisan and Bernard, 2005) for non monotone systems.
In this paper we propose a guaranteed interval estimation exploiting monotonicity properties of the error dynamics under two approaches. The first correspond to a direct cooperative observer and the second one to dynamics which becomes cooperative after a variable change. Both approaches require a specific choice for the signs of the observer gains. We run then several observers in parallel, obtaining a bundle of observers (Bernard and Gouzé, 2004), and we take the best estimate provided by the bundle.
This paper is organised as follows. In section 2 the considered class system is presented, linking it with example related to a general biotechnological model. Section 3 introduces the observer, considering first a perfect knowledge case and then a general uncertainty framework. Section 4 is devoted to running many observers in parallel to obtain the observer bundle. The application to an anaerobic wastewater treatment process is studied in Section 5.

2. CLASS OF SYSTEMS AND EXAMPLE

We consider a general class of nonlinear systems whose dynamics are expressed as follows:

$$(S): \begin{cases} \dot{\xi} = A\xi + Br(\xi) + d, \quad \xi(0) = \xi_0 \\ y = r(\xi) \end{cases} \qquad (1)$$

where $A \in \mathbb{R}^{n\times n}$ is a diagonal and stable matrix, $B \in \mathbb{R}^n$ and $d \in \mathbb{R}^n$ is a system input. For the sake of simplicity, in this paper we restrict the analysis to the nonlinear term $r(\xi)$ such that $r(\xi) \in \mathcal{C}^1 : \mathbb{R}^n \mapsto \mathbb{R}$.
An example of such a structure can be found in the classical mass balance models for continuously stirred bioprocesses as proposed by (Bastin and Dochain, 1990):

$$\dot{\xi} = -DH\xi + Kr(\xi) + D\xi_{in} - Q(\xi) \qquad (2)$$

In this model, the state vector $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^t$ is the vector of all the process concentrations and biomasses. The matrix K contains the stoichiometric coefficients, also known as yield coefficients of the model. The vector $r(\xi) = (r_1(\xi), r_2(\xi), \ldots, r_k(\xi))^t$ is a vector of reaction rates (or conversion rates) representing the microbial activity (in this paper we only consider the case $r(\xi) \in \mathbb{R}$). The diagonal matrix H stands for the fraction of biomass or substrates in the liquid phase. The influent feeding concentration is represented by the positive vector ξ_{in}. The dilution rate $D > 0$ is the ratio of the influent flow rate and of the volume of the fermenter. Finally $Q(\xi)$ represents the gaseous exchange with the outside of the fermenter.
With such a classical modeling we have *e.g.*:

$$A = -DH, \quad B = K \text{ and } d = D\xi_{in} - Q(\xi)$$

The bacterial kinetics model ($r(\xi)$) is generally a rough approximation and is highly uncertain. Moreover, for wastewater treatment processes, the influent concentrations ξ_{in} are generally not measured.
The objective of this paper is to derive an interval observer for the class of systems (1) considering uncertainties in the measurements, in function $r(\xi)$, as well as uncertainties in the vector d.
Before introducing the observer, let us recall an useful definition.

Definition 1. A square matrix P is said to be cooperative if its offdiagonal terms are nonnegative (Smith, 1995): $p_{ij} \geq 0, \ \forall i \neq j$.

Remark 1. The operator $\leq$ applied between vectors or matrices should be understood as a set of inequalities applied component by component.

The main properties of a cooperative system defined by

$$\dot{X} = PX + b$$

where $X, b \in \mathbb{R}^n$, is that it keeps the (partial) order of the trajectories. If we consider two initial conditions $x^1(0)$ and $x^2(0)$ such that $x^1(0) \geq x^2(0)$, then $x^1(t, x^1(0)) \geq x^2(t, x^2(0)), \forall t \geq 0$.
An interval observer is proposed in (Rapaport and Gouzé, 2003) for system (1) provided that $r(\xi)$ is a monotone function. Here we want to extend these results in the case where $r(\xi)$ is not monotone. For this purpose we will use the following property.

Property 1. The Lipschitz function r can be rewritten as the difference of f and g which are two increasing functions of ξ:

$$r(\xi) = f(\xi) - g(\xi)$$

As a consequence, it follows that, for $\xi^- \leq \xi \leq \xi^+$:

$$\bar{r}(\xi^-, \xi^+) \leq r(\xi) \leq \bar{r}(\xi^+, \xi^-) \qquad (3)$$

where $\bar{r}(\xi^1, \xi^2) = f(\xi^1) - g(\xi^2)$.

Proof. See (Moisan and Bernard, 2005).
Equation (3) implies:

$$\begin{aligned} &\text{(i) } \bar{r}(\xi, \xi) = r(\xi) \\ &\text{(ii) } \left[\frac{\partial \bar{r}}{\partial \xi_1}\right] \geq 0 \quad \text{and} \quad \left[\frac{\partial \bar{r}}{\partial \xi_2}\right] \leq 0 \end{aligned} \qquad (4)$$

Property 2. There exist four positive matrices $N_i(\xi, e^+, e^-) \in \mathbb{R}_+^{1\times n}$, $i = \{1, ..., 4\}$ such that:

$$\begin{aligned} r(\xi^+, \xi^-) - r(\xi, \xi) &= N_1(\xi, e^{+/-})e^+ + N_2(\xi, e^{+/-})e^- \\ r(\xi, \xi) - r(\xi^-, \xi^+) &= N_3(\xi, e^{+/-})e^+ + N_4(\xi, e^{+/-})e^- \end{aligned} \qquad (5)$$

where $e^+ = \xi^+ - \xi$ and $e^- = \xi - \xi^-$.

Proof. Indeed, $\bar{r}(\xi_1, \xi_2)$ corresponds to a function where the increasing and decreasing parts of $r(\xi)$ have been identified. Let us verify equation (5). For the upper bound, one has:

$$\begin{aligned} r(\xi^+, \xi^-) - r(\xi, \xi) &= f(\xi^+) - f(\xi) + g(\xi) - g(\xi^-) \\ &= N_1\bar{e} + N_2\underline{e} \end{aligned} \qquad (6)$$

with $N_1 = \int_0^1 \frac{\partial f}{\partial \xi}(\tau\xi^+ + (1-\tau)\xi)d\tau$ and $N_2 = \int_0^1 \frac{\partial g}{\partial \xi}(\tau\xi + (1-\tau)\xi^-)d\tau$. Matrices N_3 and N_4 are analogously obtained when considering the lower bound.

3. OBSERVER FORMULATION

3.1 Perfect knowledge framework

We consider the following system associated with equation (1).

$$(\mathcal{O}) : \begin{cases} \dot{\bar{\xi}} = A\bar{\xi} + (I - \Gamma_1)By + \Gamma_1 B\bar{r}(\bar{\xi}, \underline{\xi}) + d \\ \dot{\underline{\xi}} = A\underline{\xi} + (I - \Gamma_2)By + \Gamma_2 B\bar{r}(\underline{\xi}, \bar{\xi}) + d \end{cases} \qquad (7)$$

where $\Gamma_1, \Gamma_2 \in \mathbb{R}^{n\times n}$ are the observers gains matrices to be tuned.
Let us write the dynamical system associated with the differential comparison $e = [\bar{\xi} - \xi; \xi - \underline{\xi}]^t$. Considering the presented properties and after some algebraic manipulation we obtain a system of the type $\dot{e} = L(\bar{\xi}, \underline{\xi}, \xi)e$ where matrix $L \in \mathbb{R}^{2n\times 2n}$ is of the form:

$$L(\bar{\xi}, \underline{\xi}, \xi) = \begin{bmatrix} A + \Gamma_1 BN_1 & \Gamma_1 BN_2 \\ \Gamma_2 BN_3 & A + \Gamma_2 BN_4 \end{bmatrix} \qquad (8)$$

Remark 2. Components of matrix L are not exactly known: matrices N_k, $k = \{1, \ldots, 4\}$, depend on the unknown state ξ. However the BN_k have known signs.

3.1.1. Direct cooperative observer. A first interval observer is derived, choosing matrices Γ_k such that matrix L becomes cooperative.

Proposition 1. Let us choose Γ_1 and Γ_2 such that matrix $L(\bar{\xi}, \underline{\xi}, \xi)$ is cooperative. If $\underline{\xi}(0) \leq \xi(0) \leq \bar{\xi}(0)$ then system (7) is an interval observer of system (1): $\forall t \geq 0$ we have $\underline{\xi}(t) \leq \xi(t) \leq \bar{\xi}(t)$.

Proof. Positivity of the error dynamics is deduced from the cooperativity of matrix L (Smith, 1995). Considering that $B = [b_i]$ and $N = [n_i]$. Note that the construction of Γ such that ΓBN_k is positive is straightforward. It suffices to take a matrix Γ whose (fixed) sign for the j^{th} column is the sign of b_j. Then ΓB is a positive vector.

Remark 3. If we choose $\Gamma = 0$ we obtain a (stable) asymptotic interval observer with fixed convergence rate given by matrix A .

3.1.2. Indirect cooperative observer. Now let us focus on the term $F_k = \Gamma_k BN_k$. When these terms are positive to get a cooperative matrix L, they also do affect the diagonal of L. As a consequence the stability of L may be affected. We propose a second design, that guarantees the stability of matrix L.

The idea consists in choosing a matrix Γ_1 (or Γ_2) containing zeros everywhere except on the k^{th} row. This has the **opposite** signs of B, *i.e.* $\gamma_{k,j} b_j \leq 0$. This leads to a matrix L of the form:

$$L = \begin{bmatrix} a_{11} & & & \ldots & 0 \\ & \ddots & & & \vdots \\ (-)_{k1} & \ldots & (-)_{kk} & \ldots & (-)_{kn} \\ \vdots & & & \ddots & \\ 0 & \ldots & & & a_{nn} \end{bmatrix} \qquad (9)$$

this means, the k^{th} line of L is negative and the rest of the matrix has a diagonal form whose entries are the same entries of matrix A.
We can now propose the following interval observer.

Proposition 2. Choosing Γ_1 and Γ_2 such that matrix $L(\bar{\xi}, \underline{\xi}, \xi)$ has the form of matrix (9), and moreover if $\underline{\xi}^i(0) \leq \xi^i(0) \leq \bar{\xi}^i(0)$ for $i \neq k$ and $\bar{\xi}^k(0) \leq \xi^k(0) \leq \underline{\xi}^k(0)$ then system (7) is an interval observer of system (1): $\forall t \geq 0$, $\underline{\xi}^i(t) \leq \xi^i(t) \leq \bar{\xi}^i(t)$ for $i \neq k$ and $\bar{\xi}^k(t) \leq \xi^k(t) \leq \underline{\xi}^k(t)$.

Proof. It consists in considering a change of variable where the k^{th} variable is multiplied by (-1):

$$[\overline{\xi}_1 \dots \overline{\xi}_k \dots \overline{\xi}_{k+n} \underline{\dots \dots} \xi_n]^t$$
$$= [\overline{\zeta}_1 \dots - \overline{\zeta}_k \dots - \overline{\zeta}_{k+n} \dots \overline{\zeta}_n]^t$$

This leads to the error dynamics $\dot{\tilde{e}} = \tilde{L}(\overline{\xi}, \underline{\xi}, \xi)\tilde{e}$, where $\tilde{L}$ is a cooperative matrix (after this variable change, the k^{th} line of matrix L becomes positive, except for the diagonal element $\tilde{l}_{kk}$). The rest of the proof is the same as for Proposition 1.

Remark 4. The observers based on proposition 2 lead to a class of observers for which the matrix $\tilde{L}$ is cooperative and stable. The observers based on proposition 1 may lead to unstable estimates.

3.2 Uncertainty framework

Now we consider the case where the function $r(\cdot)$, the input d and the available measurement y are not perfectly known but upper and lower bounded by two known functions for all time. This is formalized in the following hypothesis:

Hypothesis 1. Function $\bar{r}(\xi^1, \xi^2)$ is assumed to be bounded by two known functions $\bar{r}^+(\xi^1, \xi^2)$ and $\bar{r}^-(\xi^1, \xi^2)$.

Hypothesis 2. A bounded noise δ such that $\delta \leq \Delta < 1$, perturbs the system output. We assume that this noise is of multiplicative nature.

$$\frac{y}{1+\Delta} \leq r(\xi) \leq \frac{y}{1-\Delta} \tag{10}$$

Hypothesis 3. The input vector d is unknown but bounded: $\underline{d} \leq d \leq \overline{d}$.

Now the observer is reformulated considering the bounds on the uncertain terms. Consider the following observer candidate:

$$\begin{cases} \dot{\overline{\xi}} = A\overline{\xi} + \tilde{y}_+(I - \Gamma_1)B + \tilde{r}_+(\overline{\xi}, \underline{\xi})\Gamma_1 B + \tilde{d}_+ \\ \dot{\underline{\xi}} = A\underline{\xi} + \tilde{y}_-(I - \Gamma_2)B + \tilde{r}_-(\underline{\xi}, \overline{\xi})\Gamma_2 B + \tilde{d}_- \end{cases} \tag{11}$$

where $\tilde{y}_+$, $\tilde{y}_-$, $\tilde{r}_+(\overline{\xi}, \underline{\xi})$, $\tilde{r}_-(\underline{\xi}, \overline{\xi})$, $\tilde{d}_+$ and $\tilde{d}_- \in \mathbb{R}^n$ are vectors constructed using the known bounds of the uncertainties and taking into account the signs of Γ_i. For sake of space limitation, the way these bounds are selected is not detailed here, but their choice is straightforward: the objective is to obtain error dynamics of the form $\dot{e} = Le + \phi$, where matrix L is, as in equation (8), either a cooperative matrix (direct cooperativity) or a diagonal matrix plus a k negative row (Equation (9)). Vector ϕ is a residual vector generated be the comparison of system (11) and system (1). It is either a nonnegative vector (direct cooperativity), or nonnegative with the k nonpositive row (indirect cooperativity).

4. BUNDLE OF OBSERVERS AND REINITIALISATION

As we have seen, the cooperativity concept applied to the proposed observer provides us with a guaranteed interval for the state vector. It is worth noting that the gains Γ_1 and Γ_2 introduce some degrees of freedom since the observer convergence can be adjusted using different values for these gains. Taking advantage of this last feature, now we run simultaneously several observers with different values for the gain vectors satisfying always the cooperativity condition (Bernard and Gouzé, 2004). These observers are a mixture of observers based on direct cooperativity (section 3.1.1) or indirect cooperativity (section 3.1.2), associated with a broad range of gains Γ_i. Thus we use both stable (for example with $\Gamma_i = 0$) and unstable observers. In this way, some observers will provide a better estimate during their transitory response and other will have better estimated for the steady state behaviour(Bernard and Gouzé, 2004). Considering different upper [resp. lower] estimations we take the lower [resp. upper] envelope provided by the minimum [resp. maximum] values reached by this bundle of observers (Bernard and Gouzé, 2004).

The bundle can be even improved with a reinitialisation process. Reinitialisation consist in the restarting the whole bundle of observers with the best estimation performed at the time of reinitialisation t_r.

$$[\underline{\xi}_\Gamma(t_r), \overline{\xi}_\Gamma(t_r)] = [\max\{\underline{\xi}_{\Gamma_i}\}(t_r), \min\{\overline{\xi}_{\Gamma_i}\}(t_r)]$$

where $\underline{\xi}_\Gamma$, $\overline{\xi}_\Gamma$ are the lower and upper envelopes, *i.e.* the best estimates at time t_r of $\underline{\xi}$ and $\overline{\xi}$ respectively for the set of considered gain vectors Γ_i. It has already been investigated that reinitialisation of the observers can dramatically improve the interval estimation (Bernard and Gouzé, 2004).

Proposition 3. If, among the set of gains Γ_1 and Γ_2 that produced a bundle of cooperative observers (7), we have a stable observer (for example obtained with $\Gamma_i = 0$), then the envelope of the observer bundle $(\underline{\xi}_\Gamma(t), \overline{\xi}_\Gamma(t))$ is stable.

Proof. The proof is trivial since the $\overline{\xi}_\Gamma(t) \leq \overline{\xi}_{\Gamma_j}(t)$ for any j, and especially for the gain j producing a stable $\xi_{\Gamma_j}(t)$. The same argue holds for the lower bound.

5. APPLICATION TO AN INDUSTRIAL ANAEROBIC DIGESTION PROCESS

5.1 Introduction

Anaerobic digestion is a wastewater treatment process used to remove organic carbon from water using a an ecosystem based on a consortium of anaerobic bacteria. We will here focus on a very simple model where only a single bacterial population is considered. This two dimensional model considers a biomass of bacteria x growing in a bioreactor and consuming the polluting organic substrate s (the Chemical Oxygen Demand (COD)). We also assume that the methane gaseous flow is measured.

The associated model for this ideal CSTR is then the following:

$$\begin{cases} \dot{x} = r(\xi) - Dx \\ \dot{s} = -k_1 r(\xi) + D(s_{in} - s) \\ y = k_2 r(\xi) \end{cases} \tag{12}$$

where s_{in} is the influent substrate, and k_1 and k_2 are yield coefficients.

This model has the form (2) with:

$$H = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \; K = \begin{pmatrix} 1 \\ -k_1 \end{pmatrix}, \quad \xi_{in} = \begin{pmatrix} 0 \\ s_{in} \end{pmatrix}, \quad Q = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{13}$$

As most of the time in biotechnology, the bacterial growth rate $r(\xi)$ is a complex function of the state which is generally badly known. We consider the analytical expression for this term $r(s,x) = \mu(s)x$, where $\mu(s)$ corresponds to the growth rate function. In particular we will use the classical Haldane expression which is a non monotone function of the substrate. This growth rate model is defined by the following equation:

$$\mu_H(s) = \mu_0 \frac{s}{s + k_s + s^2/k_i} \tag{14}$$

for which we consider that the parameter μ_0 is uncertain: $\mu_0^- \leq \mu_0 \leq \mu_0^+$. Moreover, the influent substrate concentration to be processed in the bioreactor is not accurately measured and thus: $s_{in}^- \leq s_{in} \leq s_{in}^+$.

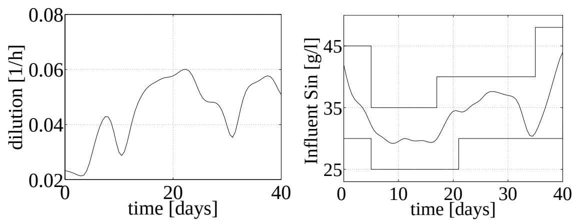

Fig. 1. Dilution rate and influent substrate for the industrial plant.

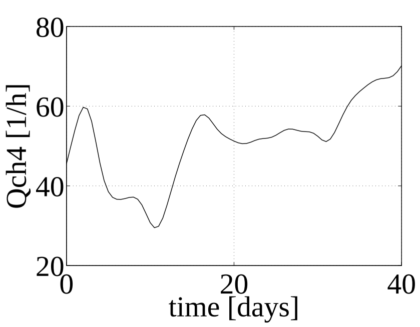

Fig. 2. Measured methane flow rate.

5.2 Observer design

Since the function $\mu(s)$ is non monotone with respect to s we propose to bound it with a function of two variables. We propose an expression as in (Moisan and Bernard, 2005), such that for $\underline{s} \leq s \leq \overline{s}$: $\bar{\mu}(\underline{s}, \overline{s}) \leq \mu(s) \leq \bar{\mu}(\overline{s}, \underline{s})$ where $\bar{\mu}(\underline{s}, \overline{s}) = \mu_0 \frac{\underline{s}}{\underline{s} + k_s + \underline{s}\overline{s}/k_i}$. Now considering the uncertainty on the parameter μ_0, the final bounds on the reaction rate function $r(\xi)$ are:

$$\bar{r}^+(\overline{x}, \overline{s}, \underline{s}) = \overline{x} \frac{\mu_0^+ \overline{s}}{\overline{s} + k_s + \underline{s}\overline{s}/k_i}$$

$$\bar{r}^-(\underline{x}, \underline{s}, \overline{s}) = \underline{x} \frac{\mu_0^- \underline{s}}{\underline{s} + k_s + \underline{s}\overline{s}/k_i}$$

The gains are chosen from the signs of $B = K$ (see Equation (13)): $b_{11} > 0$ and $b_{21} < 0$.

We choose thus $\gamma_{11} > 0$, $\gamma_{22} < 0$, $\gamma_{12} < 0$ and $\gamma_{21} > 0$ that render L cooperative, fulfilling conditions of proposition 1. On the other hand, a matrix of type (9) can be obtained considering $\gamma_{11} < 0$ and $\gamma_{12} > 0$ with $\gamma_{22} = 0$ and $\gamma_{21} = 0$. A second matrix can also be proposed with $\gamma_{21} < 0$ and $\gamma_{22} > 0$ with $\gamma_{11} = 0$ and $\gamma_{12} = 0$. We have run a broad set of observers including $\Gamma = 0$ (asymptotic observer), and 40 direct/indirect cooperative observers with gain values varying in the interval $\gamma_{ij} \in [-24, 24]$.

5.3 System setup

For the application of the method, we have considered a real industrial anaerobic digestion wastewater treatment plant processing raw industrial vinasses of 2000m^3. This plant is owned by the AGRALCO company located in Stella, Spain. The assumed uncertainty on the parameter μ_0 is in a $\pm 15\%$ range with respect to the nominal value. Parameters meaning and values are summarized in Table 1.

Table 1. System parameters.

parameter	meaning	value
μ_0	maximal growth rate	[0.72,1.08]
k_s	saturation constant	40
k_i	inhibition constant	50
$k1$	yield conversion	19.5
$k2$	methane yield conversion	25

The dilution input D and the available online measurements of the methane gaseous flow can be seen in fig. 1 and 2 respectively. A 3% multiplicative noise on the measurement has been assumed to derive the bounds.

Bounds for the unknown influent substrate s_{in} are shown in fig. 1, known to fluctuate around $\pm 30\%$ of the real value. Fig. 3 and 4 show the estimates performed by the proposed set of observers (only the bundle envelope is presented) for the biomass and the substrate in the reactor. It can be seen that the convergence is much more rapid than for the asymptotic observer, especially for the substrate estimation. Note that these observers have been initialized in large intervals.

6. CONCLUSIONS

A new class of interval observers has been proposed, based on an guaranteed interval approach, managing a wide uncertainty framework of a class of nonlinear systems. The presented observers are designed in order to fulfil particular monotone conditions for the error dynamics, considering that the original system dynamics are non monotone. It is worth noting that we have transformed the n dimensional non monotone system into a monotone system in dimension $2n$. Moreover, the combination of the two types of observers, associated with various gain combinations, allowed a strong improvement of the observer performances. The method can be straightforwardly extended to more complex systems with a known vectorial function $r(\xi)$. On the other hand, an optimization criterion as the one introduced in (Moisan *et al.*, 2007) can be applied in order to find the gain values that provide the best estimates.
The application of the method to an industrial plant, where the observer presents good convergence properties illustrates the method efficiency, and its potential of enhancing the convergence rate, especially compared with the classical asymptotic observers.

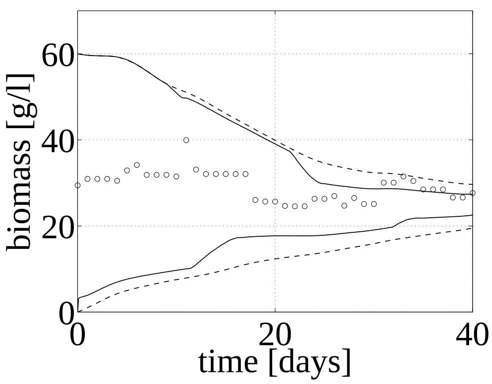

Fig. 3. Biomass interval estimates. —: final observer bundle estimations. _ _: asymptotic observer. ○: data

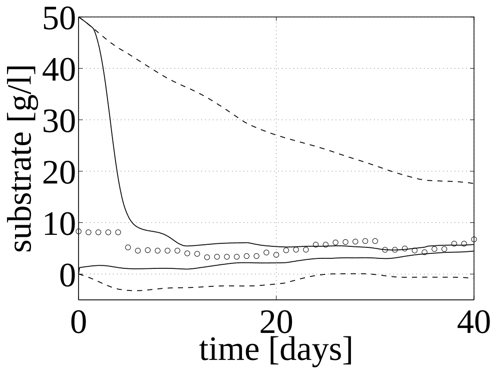

Fig. 4. Substrate interval estimates. —: final observer bundle estimations. _ _: asymptotic observer. ○: data

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