CONTROL OF A BIOREACTOR WITH SAMPLED DELAYED MEASUREMENT

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Abstract: A robust regulator for input delayed systems is applied to stabilize a class of bioreactors with a sampled delayed measurement, exhibiting partially known nonlinear dynamic behavior. An uncertain environment with the presence of unknown inputs is considered. This approach is applied on an anaerobic digestion model for the treatment of wastewater where the objective is the regulation of the chemical oxygen demand (COD) by using the dilution rate as the manipulated variable. Despite large disturbances on the input COD and state and parametric uncertainties, this regulation law perform quite well, leading the output COD towards its set-point.

Keywords: Robust control, bioreactor, input delayed system, waste treatment.

1. INTRODUCTION

The last two decades have seen an increasing interest in the application of advanced control techniques to the wastewater treatment field since the involved processes require careful monitoring in order to fulfill the requirements related to water quality and ecological norms (Dochain and Vanrolleghem, 2001). However, the optimal control of wastewater treatment processes faces important uncertainties arising from the intrinsic complexity of the plant design. Moreover, due to the lack of reliable on-line sensors and to the complex nature of the process inputs, the proper operation and control of these processes is a very tedious task (Van Impe et al., 1998). In order to control such processes, reliable information about the key variables has to be available. This information is usually obtained from fast and simple measurements (e.g., biogas flowrate, biogas composition, pH when dealing with anaerobic digestion), however key variables like chemical oxygen demand (COD) should be monitored to guarantee the waste removal. In one hand, the COD measurements on-line are expensive and usually rely on the use of soft-sensors based on other measurements; in the other hand COD measurements with traditional method like laboratory analysis implies the acquisition of sampled information usually with large time-delay (Gordon, 2005).

The control with discrete measurement for nonlinear systems has been addressed mostly with continuous measurement designs followed by ad hoc discrete-time implementations where the choices of the sampling time and gains are resolved with trial and error procedures assisted by the knowledge on the plant dynamics (Hernández and Alvarez, 2003). In dynamical systems such as biological systems (Cushing 1977), time delay arises frequently and can severely degrade closed-loop system performance and in some cases drive the system to instability. Since controllers designed with the assumption of instantaneous information may fail to stabilize dynamic systems with time delay (Frankl'm *et. al.*, 1994) it is of paramount importance that delay system dynamics be accounted for in the control-system design process. In addition, it is well known that a system with delayed measurements can be represented by a system with input delay.
Recently, has been proposed a linear controller (García-Sandoval, 2006) which solves the regulation problem for systems with input delay, this class of robust regulators is an error feedback controller which relies on the existence of an internal model, obtained by finding if possible, an immersion of the exosystem dynamics into an observable one, which allows to generate all the possible steady state inputs for the admissible values of the system parameters (Castillo-Toledo and Di Gennaro 2002). Thus, this

control law is capable of coping with both the lack of reliable information and uncertainty. In this paper, we propose the application of this approach to the single-input-single-output (SISO) regulation of the chemical oxygen demand in the output of an anaerobic digestion (AD) process using sampled delayed measurements where a larger number of uncertainties are introduced in the nonlinear functions that describe the kinetics of the process and the usefulness of the approach is demonstrated under the maximum uncertainty conditions that may characterrize AD processes. This paper is organized as follows, in section 2 we present the robust regulator, in section 3 we describe the plant model, and develop the particular controller which is tested in section 4 before some conclusions are drawn.

2. BASIC FACTS ON THE REGULATION PROBLEM FOR A CLASS OF NONLINEAR SYSTEMS WITH IMPUT DELAY

Consider a nonlinear system described by the differential equations

$$\dot{x}(t) = f\left(x(t), u(t-\tau), \omega(t), \mu\right) \tag{1}$$

$$\dot{\omega}(t) = s\left(\omega(t)\right) \tag{2}$$

$$e(t) = h\left(x(t), \omega(t), \mu\right) \tag{3}$$

with the initial conditions

$$u(t_0 - \sigma) = \phi(\sigma), \forall \sigma \in [-\tau, 0]$$

where $x(t) \in$ Ñ^n and $u(t) \in$ Ñ are respectively, the state and input variables of the plant, subject to disturbances and/or references signals $\omega(t)$. Equation (2) describes an autonomous exosystem, defined in a neighborhood of the origin of Ñ^s, while $e(t) \in$ Ñ represents an output tracking error between the system output and the reference signal and $\mu \in$ Ñ^q is a parameters vector which may take values in a neighborhood P ⊂ Ñ^q. We assume that the mappings f, s and h are smooth in their arguments and that $f(0,0,0,0) = 0$, $s(0) = 0$ and $h(0,0,0) = 0$.

The *nonlinear Input Delayed Discrete Robust Regulation Problem* (IDDRRP) considering that there exists an integer m, and a scalar δ such that $\tau = m\delta$, consists in finding, if possible, a feedback dynamic discrete controller with sampling time δ, such that, for all admissible parameter values μ, the following conditions are satisfied

(DS) *Stability.* The solution of the close-loop system, without disturbances but with parametric variations, at the sampling instants goes asymptotically to zero.

(DR) *Regulation.* For each initial condition ($x(0)$, $z(0)$, $\omega(0)$) in a neighborhood of the origin, the solution of the closed-loop system, with disturbances and parametric variations, guarantees that $\lim_{t\to\infty} e(t) = 0$.

Remark 1: Because m is an integer, the input delay is τ and δ is the sampling time, it means that we have a delay equal to m times the sampling period.

Several results on the stability for the system (1) can be found in the literature (see. e.g. Lee *et. al.* 1994a, b) for continuous measurement, but in this work, we assume a different scheme to accomplish the stabilization requirements for sampled measurement. The following assumption is instrumental in the solution of the previous problem:

Assumption A1: To impose that the inputs are persistent in time for the tracking problem, the equilibrium point $\omega = 0$ is stable in the sense of Lyapunov, and the eigenvalues of $S = \frac{\partial s}{\partial \omega}\Big|_{=0}$ are on the imaginary axis.

A sufficient condition for the solution of the IDDRRP is presented in the next theorem.

Theorem 2: (Isidori, 1995) Assume that *A1* holds. Consider that the pairs (A_0, B_0) and (A_0, C_0) are stabilizable and detectable, respectively, where

$$A_0 = \frac{\partial f}{\partial x}\bigg|_{\substack{x=0\\ \omega=0\\ \mu=0}}, \quad B_0 = \frac{\partial f}{\partial u}\bigg|_{\substack{x=0\\ \omega=0\\ \mu=0}}, \quad C_0 = \frac{\partial h}{\partial x}\bigg|_{\substack{x=0\\ \omega=0\\ \mu=0}},$$

are the linearized matrices of system (1)-(3). The IDDRRP is solvable if there exist C^k ($k \geq 2$), mapping $x_{ss}(t) = \pi(\omega(t),\mu)$, and $u_{ss}(t-\tau) = \gamma(\omega(t),\mu)$, with $\pi(0,0) = 0$, and $\gamma(0,0) = 0$, both defined in a neighborhood of $(\omega,\mu) = (0,0)$, that satisfies the equations

$$\frac{\partial \pi(\omega,\mu)}{\partial x} s\left(\omega(t)\right) = f\left(\pi(\omega,\mu), \gamma(\omega,\mu), \omega, \mu\right)$$
$$0 = h\left(\pi(\omega,\mu), \omega, \mu\right) \tag{4}$$

In (Isidori and Byrnes, 1990) it has been shown that the existence of a solution might be obtained if for some set of real numbers, $a_0, a_1, \ldots, a_{r-1}$, it holds that

$$L_s^r \gamma(\omega,\mu) = \sum_{k=0}^{r-1} a_k L_s^k \gamma(\omega,\mu)$$

where $L_s^k \gamma(\omega,\mu) = \left[\frac{\partial L_s^{k-1}\gamma(\omega,\mu)}{\partial \omega}\right] s(\omega)$ $k > 1$ with $L_s^0 \gamma(\omega,\mu) = \gamma(\omega,\mu)$, for all (ω,μ), and

$$\begin{pmatrix} A_0 - \lambda I & B_0 \\ C_0 & 0 \end{pmatrix}$$

is nonsingular for every λ which is a root of the polynomial

$$p(\lambda) = a_0 + a_1\lambda + \ldots + a_{r-1}\lambda^{r-1} - \lambda^r$$

having non-negative real parts, then the IDDRRP is solvable by a linear controller. Where the exosystem is immersed into a system

$$\dot{\varsigma}(t) = \Phi\varsigma(t)$$
$$\gamma\left(\omega(t),\mu\right) = H\varsigma(t) \tag{5}$$

where

$$\Phi = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_0 & a_1 & a_2 & \cdots & a_{r-1} \end{pmatrix}, H = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}_{1\times r}$$

and

$$\varsigma(t) = \begin{pmatrix} \gamma(\omega,\mu) & L_s\gamma(\omega,\mu) & \cdots & L_s^{r-1}\gamma(\omega,\mu) \end{pmatrix}^T. \blacksquare$$

For the case of not input delay ($\tau = 0$) the discrete linear controller, with sampling time δ, presented by Castillo and Di Genaro (2002) which solves the Robust Regulation Problem has the form

$$\xi_1(j+1) = (A_{d0} + B_{d0}K_0 - G_1C_0)\xi_1(j) + G_1e_d(j) \quad (6a)$$

$$\xi_2(j+1) = -G_2C_0\xi_1(j) + \Phi_d\xi_2(j) + G_2e_d(j) \quad (6b)$$

$$u(j) = K_0\xi_1(j) + He^{\Phi\theta}\xi_2(j), \quad (6c)$$

$$\theta = t - j\delta \in [0,\delta)$$

where $A_0 = e^{A_0\delta}$, $B_{d0} = \int_0^{\delta} e^{A_0\lambda}B_0 d\lambda$ and $\Phi_d = e^{\Phi\delta}$ are the discretized version of matrices A_0, B_0 and Φ.

Remark 3: Controller (6) is compose by an error observer (6a), an immersion estimator (6b) and the input (6c) can be seen as an hybrid input, because contains a discrete error feedback and a continuous signal $He^{\Phi\theta}\xi_2(j)$ which incorporates an exponential holder $He^{\Phi\theta}$. Notice that the signal θ in the exponential holder is a periodic sawtooth signal. A method of constructing such an exponential holder is described in (Castillo and Obregon, 2003).

Remark 4: In order to construct the controller (6), it is not necessary to know neither the continuous steady state x_{ss} nor the discrete one x_{dss}, but only the continuous steady state input u_{ss}. However, for nonlinear systems, in the particular case of polynomials describing function $\gamma(\omega,\mu)$, (such as triangular systems describing by polynomial terms), Φ can be determined without knowing exactly $\gamma(\omega,\mu)$, but only the maximum degree of the polynomial. Another remarkable feature of controller (6) is that it is based on the discretized linear part of the system description.

Now the robust controller which solves the IDDRRP is presented in the next theorem.

Theorem 5: (García-Sandoval, 2006) Assume condition A1 holds and there exists a solution $\pi(\omega,\mu)$ and $\gamma(\omega,\mu)$ for equation (4), where $\gamma(\omega,\mu)$ can be generated by a linear immersion of the form (5). Additionally, assume that the pairs

$$(A_{d0}, B_{d0}) \text{ and } \left[\begin{pmatrix} A_{d0} & -M_{d0} \\ 0 & \Phi_d \end{pmatrix}, \begin{pmatrix} C_0 & 0 \end{pmatrix}\right]$$

with

$$M_{d0} = \int_0^{\delta} e^{A_0\lambda}B_0He^{\Phi(\delta-\lambda)}d\lambda \quad , \quad \Phi_d = e^{\Phi\delta},$$

are controllable and observable, respectively. Assume that for a given integer m the matrix $K_d = (K_{d1}\, K_{d2})$, calculated as

$$\begin{aligned} K_{d1} &= K_e A_{d0}^m \\ K_{d2} &= K_e\begin{pmatrix} B_{d0} & A_{d0}B_{d0} & \cdots & A_{d0}^{m-1}B_{d0} \end{pmatrix} \end{aligned}, \quad (7)$$

where $(A_{d0} + B_{d0}K_e)$ is Hurwitz, is such that condition DS holds and the matrix $G_d = \begin{pmatrix} G_{d1}^T & G_{d2}^T \end{pmatrix}^T$ makes stable the matrix

$$\begin{pmatrix} A_{d0} & -M_{d0} \\ 0 & \Phi_d \end{pmatrix} - \begin{pmatrix} G_{d1} \\ G_{d2} \end{pmatrix}\begin{pmatrix} C_0 & 0 \end{pmatrix}$$

Then, if G_d is calculated assuming that there exists matrices $Q > 0$, $Z > 0$, R, X, Y, U and V such that the LMI's

$$\begin{pmatrix} (1,1) & (2,1)^T & A^TR - C^TU \\ (2,1) & -Q & A_1^TR \\ RA - UC & RA_1 & -R \end{pmatrix} \le 0$$

$$\begin{pmatrix} X & Y \\ Y^T & Z \end{pmatrix} \le 0$$

$$R - mZ > 0$$

$$\begin{pmatrix} R - mZ & (1,2) \\ (1,2)^T & R - mZ \end{pmatrix} \ge 0$$

with

$$\begin{aligned} (1,1) &\triangleq -R + mX + Y + Y^T + Q + 2mZ \\ &\quad - m(A^TZ + ZA) + m(C^TV^T + VC), \\ (1,2) &\triangleq A^T(R - mZ) - C^T(M - mN)^T, \\ (2,1) &\triangleq -Y^T - mA_1^TZ, \end{aligned}$$

are satisfied for

$$A = \begin{pmatrix} A_{d0} & b_zB_{d0} & 0 & 0 \\ B_zK_{d1} & M + B_zK_{d2} & B_zK_{d1} & 0 \\ 0 & 0 & A_{d0} & -M_{d0} \\ 0 & 0 & 0 & \Phi_0 \end{pmatrix},$$

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & M_{d0}e^{\Phi\tau} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, G_d = \overline{P^i}(U + mV),$$

with $\overline{P^i}$ as the pseudoinverse of the last $n + r$ columns of $P = R + mZ$ and

$$M = \begin{pmatrix} 0 & \cdots & 0 & 0 \\ 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{pmatrix}_{m\times m}, B_z = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{m\times 1},$$

$$b_z = \begin{pmatrix} 0 & \cdots & 0 & 1 \end{pmatrix}_{1\times m}.$$

Then the IDDRRP for system (1)-(3) is solvable by the controller

$$\begin{aligned} \xi(j+1) &= A_{d0}\xi(j) - M_{d0}\varsigma(j) + M_{d0}e^{\Phi\tau}\varsigma(j-m) \\ &\quad + B_{d0}b_zz(j) - G_{d1}\left[C_0\xi(j) - e(j)\right] \end{aligned} \quad (8a)$$

$$\varsigma(j+1) = \Phi_d\varsigma(j) - G_{d2}\left[C_0\xi(j) - e(j)\right] \quad (8b)$$

$$\bar{u}(j) = K_{d1}\xi(j) + K_{d2}z(j) \quad (8c)$$

$$u(t) = He^{\Phi(\theta+\tau)}\varsigma(j) + \bar{u}(j), \theta = t - j\delta \in [0,\delta) \quad (8d)$$

where

$$z(j\delta) = \begin{pmatrix} \bar{u}(j-1) & \bar{u}(j-2) & \cdots & \bar{u}(j-m) \end{pmatrix}^T. \blacksquare$$

Remark 6: Controller (8) stabilizes system (1)-(3) using a sampled measurement described by $e(j)$ with a sampling period equal to δ, while the delay in the input is equal to τ. If $m > 1$ it means that the delay is bigger than the sampling time and because m, which defines the dimension of matrices M, B_z and b_z as well as the dimension of the vector $z(j\delta)$, must be an integer, we must choice the sampling period in order to accomplish the equation $\tau = m\delta$.

Despite system (1)-(3) is an input delayed system, it is well known that defining the variable

$x(t) = x_\tau(t-\tau)$ and $\omega(t) = \omega_\tau(t-\tau)$ this system can be seen as an output delayed system,

$$\dot{x}_\tau(t) = f\left(x_\tau(t), u(t), \omega_\tau(t), \mu\right) \quad (9)$$

$$\dot{\omega}_\tau(t) = s\left(\omega_\tau(t)\right) \quad (10)$$

$$e(t) = h\left(x_\tau(t-\tau), \omega_\tau(t), \mu\right) \quad (11)$$

hence we can design a controller for system (9)-(11) using the procedure describe in theorem 5 developed for system (1)-(3). In this case we obtain a discrete controller which regulates system (9)-(11) using a sampled delayed output, where delay can be a multiple of the sampling delay.

3. COD REGULATION

In this section we analyze the system to control and we develop a controller for the COD in a wastewater AD using the procedure presented in theorem 5, then we test this controller via numerical simulation.

3.1 The process model.

AD is a multistep biological process in which complex organic matter is degraded into a gas mixture of CH_4 and CO_2. It reduces the inlet organic matter by using acidogenic bacteria and methanogenic archae to produce valuable energy (*i.e.*, CH_4) (Henze and Harremoës, 1995). When AD is performed in continuous biofilm reactors, the acidogenic phase can be described by the following two ordinary differential equations (Bernard *et al.*, 2001):

$$\dot{X}(t) = \left(\mu(t) - \alpha(t)D(t)\right)X(t)$$
$$\dot{S}(t) = -\frac{\mu(t)X(t)}{Y} + D(t)\left(S_i(t) - S(t)\right) \quad (12)$$

where X, S and S_i are respectively the concentrations of acidogenic bacteria, COD, and inlet COD. The parameter α ($0 \le \alpha \le 1$) denotes the biomass fraction that is retained by the reactor bed, *i.e.*, $\alpha = 0$ for the ideal fixed-bed reactor and $\alpha = 1$ for the ideal continuous stirred tank reactor. Y is the biomass yield coefficient for COD degradation. The variable $D = D(t) \ge 0$ denotes the dilution rate. The specific growth rate is given by the highly nonlinear Monod equation in which most parameters are badly or inadequately known (Van Impe *et al.*, 1998; Dochain and Vanrolleghem, 2001):

$$\mu(t) = \frac{\mu_{\max}(t)S(t)}{K_S(t) + S(t)} \quad (13)$$

where μ_{max} and K_S are the maximum specific growth rate and the half saturation parameter associated with S, respectively. Assumptions A2 describe the available on-line information while the maximum uncertain scenario depicted in the introduction is formally stated by hypotheses H1.

Assumptions A2:

a) $D = D(t) \ge 0$, is measured on-line and is supposed to be a persisting input, *i.e.*, $\int_0^\infty D(\tau)d\tau > 0$. In addition, $D(t)$ is bounded that is, there exist D^- and D^+ such as $D^- \le D(t) \le D^+$.

b) The COD concentration, $S(t)$, the variable to be regulated, is measured in a sampled delayed manner, with a sampling time δ and a delay $\tau = m\delta$, *i.e.*,

$$S_m(j\delta) = S((j-m)\delta), \quad (14)$$

where S_m is the measurement.

c) The biomass concentration, X cannot be measured.

Hypotheses H1: Typical uncertainties in anaerobic digestion processes:

a) $\mu_{\max}$, K_S, Y, α and S_i are unknown and possibly time varying around their nominal value; *i.e.*, $\hat{\mu}_{\max}$, $\hat{K}_S$, $\hat{Y}$, $\hat{\alpha}$ and $\hat{S}_i$, respectively.

b) Initial conditions on X and S are unknown.

c) The reference to track is piecewise continue and equal to $S_r(t)$.

3.2 Controller design.

System (12) together with the sampled delayed measurement (14) is similar to system (9)-(11), subsequently it can be rewritten as an input delayed system with sampled measurement

$$\dot{x}_1(t) = -\mu\left(x_1(t)\right)x_2(t) + \left(\omega - x_1(t)\right)u(t-\tau)$$
$$\dot{x}_2(t) = \left(\mu\left(x_1(t)\right) - \alpha u(t-\tau)\right)x_2(t) \quad (14)$$
$$x_m(j\delta) = x_1(j\delta)$$

where

$x_1(t) = S(t-\tau)$, $x_2(t) = X(t-\tau)/Y$,
$u(t) = D(t)$, $\omega = S_i(t-\tau)$,
$x_{1r}(t) = S_r(t)$ and $x_m(j\delta) = S_m(j\delta)$.

If we want to track a constant reference, x_{1r}, for the COD concentration, then solving the Francis-Isidori-Byrnes equations (4), the mappings $x_{ss}(t) = \pi(\omega(t),\mu)$ and $u_{ss}(t-\tau) = \gamma(\omega(t),\mu)$ for system (14) are

$$x_{1ss} = x_{1r} =: \pi_1(\omega,\mu)$$

$$x_{2ss} = \frac{(\omega - x_{1r})x_{20}}{\alpha x_{20} - \left((\omega - x_{1r}) - \alpha x_{20}\right)e^{-\mu(x_{1r})t}} =: \pi_2(\omega,\mu)$$

$$u_{ss} = \frac{\mu(x_{1r})}{(\omega - x_{1r})}\pi_2(\omega,\mu) =: \gamma(\omega,\mu)$$

where x_{20} is the initial conditions for x_2. Because x_2 is time varying, u_{ss} so does and it can be generated by a nonlinear immersion free of uncertain parameters with the form

$$\left.\begin{array}{lcl} \dot{\varsigma}_1 & = & \varsigma_1\varsigma_2, \\ \dot{\varsigma}_2 & = & \varsigma_2\varsigma_3, \\ \dot{\varsigma}_3 & = & \varsigma_2\varsigma_3, \end{array}\right\} \quad , \quad \gamma(\omega,\mu) = \varsigma_1.$$

However, for large enough time, both mappings, $\pi_2(\omega,\mu)$ and $\gamma(\omega,\mu)$ get a constant steady state equal to $\pi_2(\omega,\mu) = (\omega - x_{1r})/\alpha$ and $\gamma(\omega,\mu) = \mu(x_{1r})/\alpha$. At this point, the immersion can be described by a linear system like equation (5) of dimension one ($r = 1$) with $\Phi = 0$, *i.e.*,

$$\dot{\varsigma}_1 = 0 \quad , \quad \gamma(\omega,\mu) = \varsigma_1. \quad (15)$$

Additionally, the linear matrices around the nominal values for system (14) are

$$A_0 = \hat{\mu}(x_{1r})\begin{pmatrix} -\frac{1}{\hat{\alpha}}(\Theta+1) & -1 \\ \frac{1}{\hat{\alpha}}\Theta & 0 \end{pmatrix} \quad , \quad C_0 = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

Table 1: Nominal values and percentage of variations for model parameters in the simulation of figure 2.

Time of Change	Parameter				
	S_i	μ_{max}	K_S	$1/Y$	α
Nominal	10	1.25	4.95	6.6	0.5
$0 \le t < 20$	40%	30%	-20%	-30%	25%
$20 \le t < 40$	20%	30%	-20%	-30%	25%
$t > 40$	20%	-10%	-10%	30%	25%

$$B_0 = (\omega - x_{1r})\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad , \quad \Theta = (\omega - x_{1r})\frac{\hat{K}_S}{\hat{\mu}_{max}}\frac{\hat{\mu}(x_{1r})}{x_{1r}^2}.$$

With theses matrices and the immersion (15) we are able to design a linear discrete controller of dimension three, as described in theorem 5, in order to control the COD concentration. In the next subsection we present the application of such controller.

3.3 Numerical simulations.

In this section we illustrate the features of the robust SISO regulation law controlling the COD concentration of an anaerobic reactor. We consider that we obtain measurements of the COD three times at day and each measurement has a delay equal to one day. In this case m = 3 and τ = 1 d. Additionally, the nominal parameters used are taken from (Alcaraz-González *et. al.*, 2000) and reported in Table 1. For this particular case, using the structure of controller (8), the discrete controller has the form

$$\begin{aligned} &\xi(j+1) = \Psi_0\xi(j) + \Psi_1\xi(j-m) + \hat{B}\hat{u}(j) + G_d e(j) \\ &u(j) = sat\left[\xi_3(j) + \bar{u}(j)\right] \\ &\hat{u}(j) = u(j) - \xi_3(j) \\ &\bar{u}(j) = K_{d1}\left(\xi_1(j) \quad \xi_2(j)\right)^T + K_{d2}z(j) \\ &z(j) = \left(\bar{u}(j-1) \quad \cdots \quad \bar{u}(j-m)\right)^T \end{aligned} \qquad (16)$$

Here, because of the bounds of the dilution rate (0.1 ≤ $D(t)$ ≤ 1.5), we have introduced the saturation function as an anti-window prevention. The numerical values of the matrices are

$$\Psi_0 = \begin{pmatrix} -0.7519 & -0.0772 & -1.8373 \\ 1.5879 & 0.9694 & 1.9568 \\ 0.4250 & 0 & 1 \end{pmatrix},$$

$$\Psi_1 = \begin{pmatrix} 0 & 0 & 1.8373 \\ 0 & 0 & -1.9568 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{B} = \begin{pmatrix} 1.8373 \\ -1.9568 \\ 0 \end{pmatrix}$$

Additionally, the feedback matrix K_e is calculated in order to matrix (A_{d0} + K_eB_{d0}) be Schur, with eigenvalues (0.77, 0.1), while matrices K_{d1} and K_{d2} are calculated as described in equation (7), obtaining

$$K_{d1} = \left(0.3122 \quad 0.3904\right),$$

$$K_{d2} = -\left(0.4724 \quad 0.3140 \quad 0.2344\right).$$

In the other hand, matrix G_d is calculated using the LMI equations presented in theorem 5 obtaining

$$G_d = \left(1.1267 \quad -1.1478 \quad -0.4350\right)^T.$$

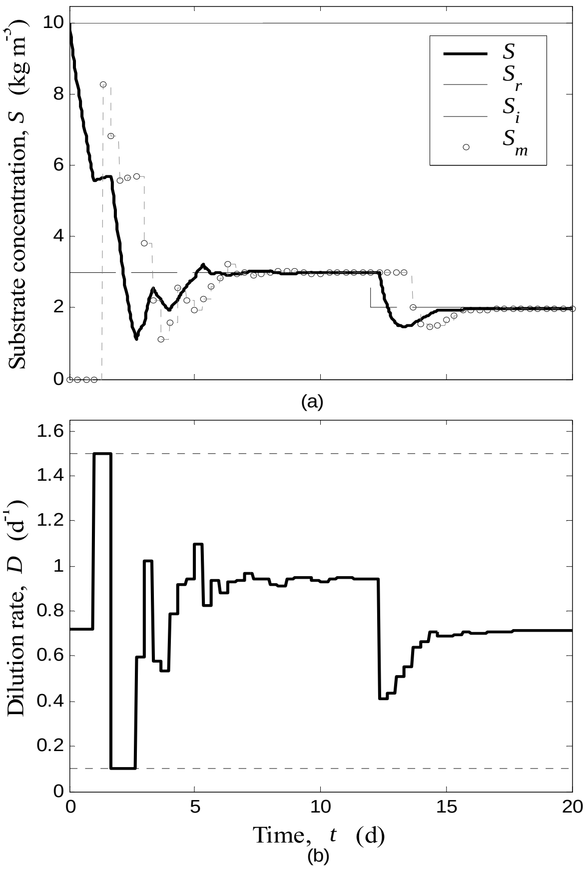

Fig. 1. Simulation of close-loop system with sampled delayed measurements and nominal values. (a) COD concentrations (actual ━, reference −−, input — and measured ○). (b) Dilution rate.

Results and discussion. To verify the behavior of the close-loop system we test the controller without parametric variations and disturbances; the results are presented in figure 1. As can be seen in the figure 1a, the controller shows an excellent performance even though the resident time for the COD is approximately two times the measurement delay. After 12 days we impose a reference change and control begins to act immediately and approaches to the new reference before the first delayed measurement show any change. In the other hand, in figure 2 we present a comparison of the purposed controller and a PI controller with Smith predictor tuned with traditional methods (Smith and Corripio, 1997). We test both controllers in the face of parametric variations, input COD disturbances and reference changes reported in table 1. At the beginning of the simulation all the parameters are different from their nominal values, at time equal to 20 d we introduce a change in the COD input concentration and at t = 30 d we impose a reference change from 3 to 2 kg m^{-3}; during these changes both controllers stabilize the system almost with the same response, been the PI a little bit slower. However, at t = 40 d we induce a change in the kinetic parameters (μ_{max} and K_S) and the PI controller can not handle this parametric variation. As can be seen, with the purpose controller, even tough parametric variations up to 40% and delay in the measurement, the error approaches to zero. Notice in both figures 1b and 2b, that at the beginning of the simulation, because of the initial error, the dilution rate gets saturation, but controller is able track the system towards the reference. The controller (16) can be easily implemented and when there is not saturation, is completely linear.

4. CONCLUSIONS

The control of AD is a huge challenge, because of the uncertain environment of the process. In addition, if we want to control those systems using sampled delayed measurements the problem is not trivial. In this work, we present a linear discrete controller which can be easily implemented. This controller shows robustness in face of uncertainties and parametric variations.

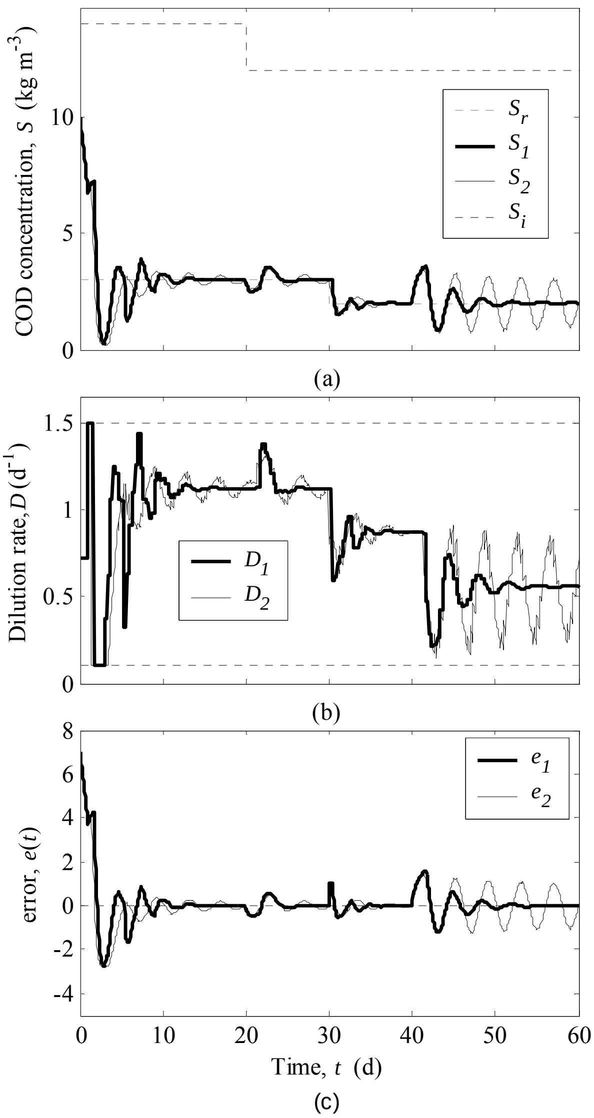

Fig. 2. Simulation of close-loop system with sampled delayed measurements, parametric variations and input COD disturbances. (a) COD concentrations (Reference --, Input --, Purposed controller — and PI control —). (b) Dilution rate (Purposed controller — and PI controller —). (c) Error (Purposed controller — and PI controller —).

REFERENCES

Alcaraz-González V., Harmand J., Steyer J. P., Rapaport A., González-Alvarez. V. and Pelayo-Ortiz C. (2000). Robust interval-based SISO regulation of an anaerobic reactor. *3rd IFAC Symposium on Robust Control Design, ROCOND2000*, Prague, Czech Republic, 6 pp. on CD-ROM.

Bernard O., Hadj-Sadok Z., Dochain D., Genovesi A., Steyer J.P. (2001) Dynamical model development and parameter identification for anaerobic wastewater treatment process, *Biotechnology & Bioengineering*, **75**(4), 424-439.

Castillo-Toledo, B., y Di Gennaro, S. (2002), On the nonlinear ripple free sampled-data robust regulator. *Europ. J. of Contr.*, **8**, 44- 55.

Castillo-Toledo, B. and G. Obregon-Pulido, (2003) Guaranteeing asymptotic zero intersampling tracking error via a discretized regulator and exponential holder for nonlinear systems. *J. App. Reserch and Tech.*, 203-214.

Cushing, J. M. (1977), Integrodifferential equations and delay models in population dynamics, *in Lecture Notes in Bio-Mathematics*, **20**, Springer-Verlag, Berlin.

Dochain D. and Vanrolleghem P. (2001). *Dynamical Modelling and Estimation in Wastewater Treatment Processes*. IWA Publishing.

Frankl'm, G.F., J.D. Powell, and A. Emami-Naeini, (1994) *Feedback Control of Dynamic Systems*, Addison Wesley, New York.

García-Sandoval, J. P. (2006) *The Robust Regulation Problem using Immersions: Reactors Applications*, PHD Thesis, CINVESTAV, México.

Gordon L., (2005) Basic Regulatory Control, *Control Engineering*, January.

Henze M., Harremoës P., LaCour Jansen J. and Arvin E. (1995). *Wastewater Treatment: Biological and Chemical Processes*. Springer, Heidelberg.

Henández, H. and J. Alvarez (2003), Robust estimation of cont. nonlinear plants with discrete measurements, *J. of Proc. Contr.* **13**, 69-89.

Isidori, A. and C. I. Byrnes, (1990). Controller for Linear Systems with Time Delays in both State and Control Variables. *J. of Dyn. Syst. Means. and Control*, **115**, 179-183.

Isidori, A. (1995) *Nonlinear Control Systems*, (3rd edition), Springer, London.

Lee, J. H., Y. I. Lee, and W. H. Kwon, (1994). H_∞ controllers for input delayed systems. *Proc. of the 33rd. Conference on Decision and Control*, 870-871, USA.

Lee J. H., S. W. Kim and W: H. Kwon, (1994). Memoryless H_∞ controllers for state delayed systems. *IEEE Trans. Aut. Cont.*, **39**(1), 159-162.

Smith, C. A. and A. B. Corripio (1997), *Principles and Practices of Automatic Process Control*, 2nd Ed., John Wiley & Sons, USA.

Van Impe J.F.M., Vanrolleghem P.A. and Iserentant D.M. (1998) *Advanced Instrumentation, Data Interpretation, and Control of Biotechnological Processes*. Kluwer Academic Publishers.