SENSITIVITY ANALYSIS OF A SIMPLIFIED CHEESE RIPENING MASS LOSS MODEL

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Abstract: This paper is focused on the mass loss rate during the ripening of surface-mould cheese. From mechanistic laws, the idea is to carry out simplifications without loss of accuracy and to perform sensitivity analysis for model inputs. The high significance of accurate relative humidity and gas composition measurements and the low influence of the atmospheric temperature are pointed out. The model reliability according to three key parameters (emissivity, surface water activity and average heat transfer coefficient) is also described.

Keywords: modelling, model reduction, sensitivity analysis, error estimation, food processing, cheese ripening.

1. INTRODUCTION

Ripening is a key step in the manufacture of surface-mould cheese. A microbial consortium activity is mainly responsible for the organoleptic characteristics of these cheeses. It is influenced by the ripening chamber atmosphere, which is characterized by temperature, relative humidity and gas composition.

In parallel of these transformations, gases exchanges due to evaporation and carbon dioxide emission are noted. Ramet (Ramet, 2000) has estimated a mass loss of 3 to 8% during the ripening depending on the cheese-type. As a general rule, cheese mass loss during ripening acts on process productivity. For protected designation of origin (PDO) cheese, the weight is a conformity criterion (e.g., Camembert-Normandie PDO requires a final weight of 0.25 kg).

Heat and mass transfer are commonly studied in cooking and drying processes but little data have been published about the cheese ripening. Two specificities of the cheese ripening differ from the cooking and drying processes: On one hand, the temperature variations are lower and the atmosphere relative humidity is close to saturation. On the other hand, biological transformations are more important than physical and chemical changes. Cheese ripening can be considered as solid substrate fermentation (SSF). Raimbault (Raimbault, 1998) summarized several aspects of SSF:

- Biomasses measurements are very difficult and direct evaluations are mostly impossible. In some cases, microbial growth is estimated with respiratory metabolism (oxygen consumption and carbon dioxide release measurements).

- Heat transfer limitation is probably the most crucial factor in large-scale SSF processes. Respiration is highly exothermic and evaporation is the major element for temperature regulation.

In industrial context, measurements bias (*e.g.* wrong calibrations or inadequate sensor positions) cannot be neglected in regard to the process monitoring. The aim of a sensitivity analysis is to assess the relative importance of input factors in the presence of finite ranges of uncertainty (Saltelli *et al.*, 2006). Several methods exist (Saltelli *et al.*, 2006; Rabitz, 1989; van Keulen *et al.*, 2005) and variance based methods are mostly used. "One factor at a time" approaches are dedicated to linear systems; for non-linear systems, second and higher order effects must be taken into account, *e.g.* the (Sobol', 1993) or the (Morris, 1991) methods. In fact sensitivity analysis must be regarded as a part of the modelling process to allow extrapolation and validation.

In this paper, we give an overview of a cheese mass loss dynamic modelling. A simple model is built with the smallest number of parameters values as possible determined by model fitting. The constants are then determined from the literature or, in a few cases, from experimental measurements. In this model, the atmospheric variations are not represented and are considered as inputs.

The sensitivity analysis of this study has two objectives. First, the idea is to represent the consequences of online acquisition sensors errors on the mass loss dynamic. For this, deviations are determined according to input errors with unbiased parameter values. Secondly, in order to check the model reliability according to estimated constants, sensibility analysis for the parameters is carried out with input ranges corresponding to ripening conditions.

2. MATERIALS AND METHODS

Soft cheeses (Camembert type) were manufactured in a sterile environment as previously described (Leclercq-Perlat *et al.*, 2004). After drainage, 45 cheeses were aseptically transferred to a sterile ripening chamber. The average weigh of cheese was 0.333 kg with a standard deviation of 0.023 kg.

The ripening chamber ($0.91 m^3$) was placed into a refrigerated room to allow the temperature regulation. The defrosting cycle had an 8 hours period, which induced temperature changing: an increase of 1°C during the first hour of the cycle.

A cheese was continuously weighted with an electronic balance. A combined sensor measured atmospheric temperature and relative humidity 6 cm above the weighted cheese. Atmospheric changes were also characterized with CO_2 and O_2 sensors (Picque *et al.*, 2006). When the ripening chamber was used without input airflow, variations of these gas concentrations were depending only of cheese respiratory activity (CO_2 production and O_2 consumption). The ripening was performed with a periodically renewed atmosphere: if necessary, the CO_2 concentration was decreased to 2% by daily air injection with 6 m^3/h flow rate. In practical, the atmosphere was not renewed except 30 min per day.

The ripening duration was 15 days, cheese were turned over on day 5. All online data were carried out with a 6 min acquisition period.

3. MODELLING OVERVIEW

3.1 Cheese mass loss

Biological activities induce a matter flux between the cheeses and atmosphere of the ripening chamber: oxygen consumption and carbon dioxide release. Aerobic metabolism is the major pathway when the respiration quotient (RQ) is close to one, as observed for Camembert-type cheeses, (Picque *et al.*, 2006). r_{o_2}, the O_2 consumption, and r_{co_2}, the CO_2 production rates ($mol.m^{-2}.s^{-1}$) are obtained by deriving CO_2 and O_2 atmospheric concentrations. The respiration matter flux ($kg.m^{-2}.s^{-1}$), is obtained by the difference between these two rates balanced by the molar masses

$$\phi_r = w_{o_2} r_{o_2} - w_{co_2} r_{co_2} \quad (1)$$

with w_{o_2} and w_{co_2} the respective molar masses ($kg.mol^{-1}$). Because the O_2 consumption and CO_2 production rates have the same dynamic, the following simplification is used when the RQ is closed to one:

$$\phi_r \simeq (w_{o_2} - w_{co_2})\, r = w_c r \quad (2)$$

with

$$r = \left(\frac{r_{o_2} + r_{co_2}}{2} \right) \quad (3)$$

The two rates are merged in r, corresponding to the respiratory activity. This simplification can be easily done because the carbon loss represents only 3% of the total mass loss.

The difference between water vapor pressure in the atmosphere and at the cheese surface causes an evaporative flux ϕ_w classically represented as following:

$$\phi_w = k \left(a_{ws} P_{sv}(T_s) - rh P_{sv}(T_\infty) \right) \quad (4)$$

with a_{ws} the cheese surface water activity, T_s and T_∞ the average surface and atmospheric temperatures respectively (K), rh the relative humidity (expressed between 0 and 1), $P_{sv}(T_\star)$ (Pa) the saturation vapor pressure at the temperature $T_\star$, and k the average water transfer coefficient ($kg.m^{-2}.Pa^{-1}.s^{-1}$). Atmospheric variables, T_∞ and rh, are online measured. The saturation vapor pressures are classically calculated with empirical relations as the Goff-Gratch equation (WMO, 2000). However, the ripening temperature is usually between 12 and 14°C. For this low range of temperature, an approximation can be done for saturation vapor pressure values, using a linear regression on the Goff-Gratch equation. The following relation is used:

$$P_{sv}(T_\star) = \beta_1 T_\star + \beta_2 \tag{5}$$

where $\beta_1 = 102$ Pa.K^{-1} and $\beta_2 = -27643$ Pa. The relative error (residual standard deviation over value range) is equal to 0.48%.

Surface water activity (a_{ws}) is a key parameter for this relation. The link between water activity and water content is classically represented by a sorption isotherm. However, the values of soft cheese water activity are high, more than 0.95 (Hardy, 2000); for this part of the sorption curve, an important variation of water content only implies a low water activity variation. Consequently, modelling of water content dynamic is not necessary and a_{ws} is assumed to be a constant.

3.2 Cheese surface temperature

Direct heat exchange between the cheese and the atmosphere result from convective and radiative fluxes

$$\psi_{cr} = h\,(T_s - T_\infty) + \epsilon\sigma\left(T_s^4 - T_\infty^4\right) \tag{6}$$

with h the average convective heat transfer coefficient ($W.m^{-2}.K^{-1}$), ϵ the product emissivity (dimensionless) and σ the Stefan-Boltzmann constant ($W.m^{-2}.K^{-4}$). The radiative heat flux relation causes a strong nonlinearity; it can be approximated as following:

$$\epsilon\sigma\left(T_s^4 - T_\infty^4\right) \simeq 4\epsilon\sigma\overline{T}_\infty^3\,(T_s - T_\infty) \tag{7}$$

where $\overline{T}_\infty$ is the atmospheric temperature mean value. It is then possible to define a global heat transfer coefficient $h^\star = h + 4\epsilon\sigma\overline{T}_\infty^3$.

With $h^\star$ and taking cheese heat conductivity in an 0.3 to 0.4 $W.m^{-1}.K^{-1}$ interval (Hardy and Scher, 2000), its Biot number is comprised between 0.24 and 0.32. This result (close to 0.1) shows that the heat conduction inside the product is faster than at its surface. Thus, we allow to neglect temperature gradient inside the cheeses and to take T_s as representative of the product temperature.

In addition, the moisture loss induces an heat consumption flux $\psi_w = \lambda\phi_w$ for the evaporation, with λ the water latent vaporization heat ($J.kg^{-1}$).

High biological activity is observed during the ripening for Camembert-type cheeses with an important mycelial development on the rind. This phenomenon induces a respirative heat production. The generic glucose aerobic respiration equation is

$$\begin{aligned} C_6H_{12}O_6 + 6O_2 \longrightarrow\ & 6H_2O + 6CO_2 \\ & +2.816 \times 10^6 J.mol^{-1} \end{aligned} \tag{8}$$

This equation gives a respiration quotient (RQ) equal to one because of the equimolarity between O_2 and CO_2. During ripening, many substrates are oxidized (lactose, lactate, lipids and proteins), which can induce RQ variation close to one. The variability of RQ is then represented by the average of the gases rates r.

The cheese temperature dynamical model is

$$\frac{dT_s}{dt} = \frac{s}{mC}(-\psi_{cr} - \lambda\phi_w + \alpha\ r) \tag{9}$$

with m the mass of a cheese, s (m^2) the surface exchange of the cheese, C the specific heat ($J.kg^{-1}.K^{-1}$) and α the respiration heat ($J.mol^{-1}$) determined according to (8).

The mass loss dynamic is very slow compared to temperature dynamic, therefore the mass can be consider as constant for the temperature equation. In this case, the time constant of the temperature linear differential equation can be calculated and the response time after a perturbation is approximatively equal to 90 min. This short period of time compared with the ripening duration allow to take T_s at the steady-state

$$\overline{T}_s = \frac{h^\star T_\infty - \lambda k\,(a_{ws}\beta_2 - rh\,(\beta_1 T_\infty + \beta_2)) + \alpha r}{h^\star + \lambda k a_{ws}\beta_1} \tag{10}$$

and the mass loss rate q_m is defined by

$$\begin{aligned} q_m = \ & \gamma h^\star(a_{ws} - rh)(\beta_1 T_\infty + \beta_2) \\ & + (\gamma a_{ws}\beta_1\alpha + w_c)\,r \end{aligned} \tag{11}$$

with

$$\gamma = \frac{k}{h^\star + \lambda k a_{ws}\beta_1}$$

3.3 Values of the parameters

Parameter values used to the mass loss estimation are shown in table 1. a_{ws} and C are the averages of measurements during the ripening. Plaster value is used for the cheese emissivity (Mirade *et al.*, 2004). Except the transfer coefficients, the others parameters are easily determined from the literature. Parameters h and k are mainly determined by the product shape and the airflow properties (Kondjoyan and Daudin, 1997). From (Mirade *et al.*, 2004), we define

$$k = 0.66 \times 10^{-8} h^{1.09}$$

Symbol	Unit	Value (SEM or SE)[1]
a_{ws}	dimensionless	0.976 (0.001)
C	J.kg^{-1}.K^{-1}	2.194×10^3 (59)
h	W.m^{-2}.K^{-1}	2.97 (3×10^{-4})
k	kg.m^{-2}.Pa^{-1}.s^{-1}	2.16×10^{-8} (1×10^{-12})
s	m^2	2.25×10^{-2}
w_c	kg.mol^{-1}	1.2×10^{-2}
α	J.mol^{-1}	4.693×10^5
λ	J.kg^{-1}	2.47×10^6
ϵ	dimensionless	0.91
σ	W.m^{-2}.K^{-4}	5.67×10^{-8}

[1]SEM: Standard error of the mean, used for experimental acquisitions; SE: Standard error, used for optimization result.

Table 1. Parameters values.

an empirical relation between h and k for product with cheese shape. Only h is obtained by non-linear least square regression, between the estimated and the measured cheese mass.

Figure 1 illustrates a comparison between the mass loss rate obtained from online acquisition and model estimation. The variations due to defrosting cycles or air injections are shown by the right left zoom insertions respectively. These two experimental phenomena could correspond to industrial faults such as dysfunctions of temperature or relative humidity controls. The representation of these abrupt variations is better with the eq. (9) than with eq. (11) but the model (11) succeed in representing the global dynamic. The increase of the mass loss rate which occurs on days 4 and 5 is due to the mycelial growth (respiratory activity) and shows that the biological activity cannot be neglected (Picque *et al.*, 2006).

4. SENSITIVITY ANALYSIS

4.1 Inputs bias consequences

Let e_{rh}, e_{T_∞} and e_r the respective errors on inputs acquisitions, *e.g.* $e_{T_\infty} = \hat{T}_\infty - T_\infty$ with $\hat{T}_\infty$ the biased atmospheric temperature and T_∞ the real atmospheric temperature. From the equation (11), e_{qm} the mass loss rate error can be easily represented as following.

$$\begin{aligned} e_{qm} = & -\gamma h^\star \beta_1 e_{T_\infty} e_{rh} + \gamma h^\star \beta_1 (a_{ws} - rh) e_{T_\infty} \\ & -\gamma k(\beta_1 T_\infty + \beta_2) e_{rh} + (\gamma a_{ws} \beta_1 \alpha + w_c) e_r \end{aligned}$$

In addition to the three first order errors, the second order error $e_{T_\infty} e_{rh}$ must be considered. Note that e_{qm} according to e_r or $e_{rh} e_{T_\infty}$ does not depend on input values.

To overview ripening conditions, the relative humidity is comprised between 0.9 and 1, the atmospheric temperature between 11 and 15°C and the respiration rate between 0 and 1.5 mol.m^{-2}.d^{-1}. The consequences of different biases with taking errors between ±25% of input ranges are shown in figure 2. The intervals result from the combination of minimal and maximal T_∞ and rh values.

The errors on relative humidity and on respiration activity have the most important consequences. For the extreme values, e_{qm} is equal to ±0.05 and ±0.035 kg.m^{-2}.d^{-1} respectively. First order errors on atmospheric temperature and second order error $e_{T_\infty} e_{rh}$ have a low impact.

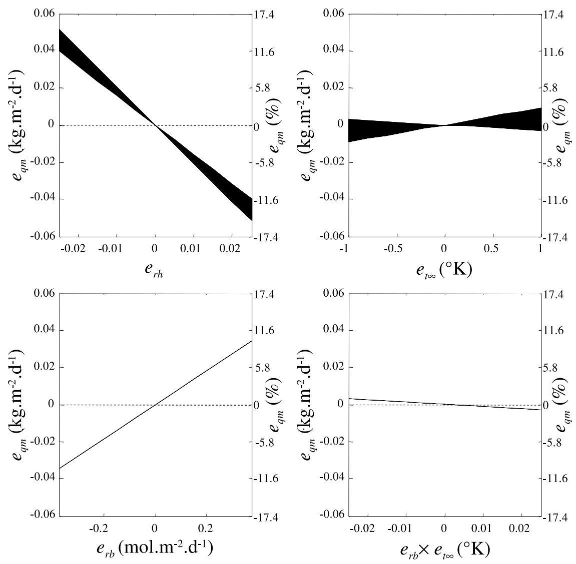

Figure 2. Mass loss rate error (kg.m^{-2}.d^{-1}, left axis) and relative mass loss rate error (%, right axis) as a function of error on input.

4.2 Sensitivity analysis for the parameters

4.2.1. Parameters studied Several parameters are not considered in this study:

- Two parameters of (9) are not used in the model (11), (i) *the cheese surface*, indeed the sensitivity analysis is carried out for a mass loss rate in kg.m^{-2}.d^{-1} and not for an individual cheese, and (ii) *the specific heat of cheese*, this parameter only influences the temperature dynamic and not the steady-state.
- As $\gamma a_{ws} \beta_1 \alpha$ is 8 times higher than w_c, errors on α are similar to errors on r, thus this parameter is not included in the sensitivity analysis.
- Physical constants are assumed to be unbiased and consequently they are not used in the sensitivity analysis.

Thus, three key parameters are studied:

- the average heat transfer coefficient, resulting from an optimization,
- the emissivity, for which plaster value is used,
- the surface water activity, which is obtained by experimental measurements.

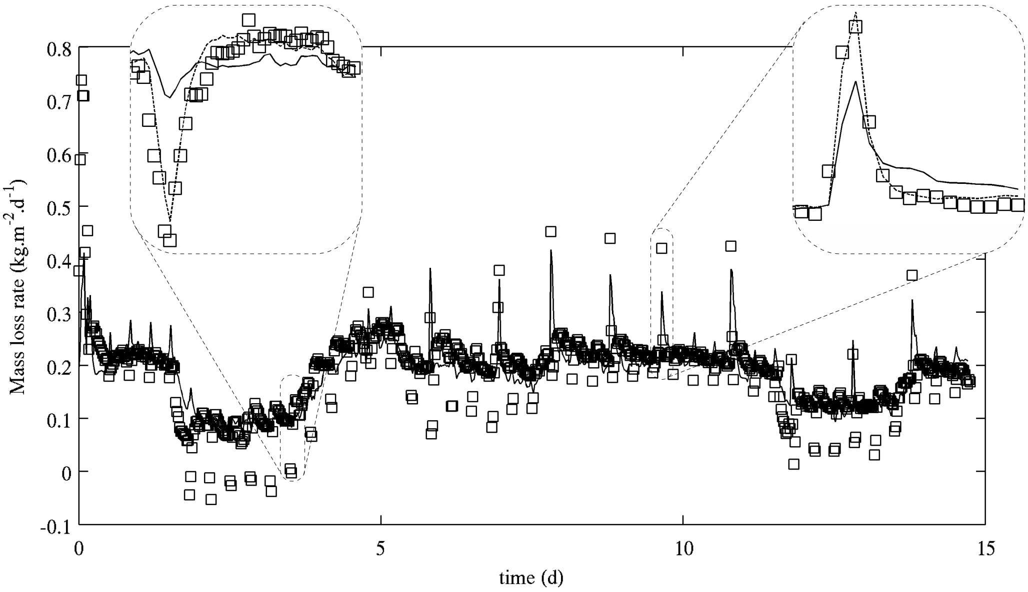

Figure 1. Measured (□) and estimated with surface temperature at the steady-state (−) mass loss rate of cheese vs. ripening time. 30 min mean values are represented for clarity reason. For the zoom insertions, the estimation with the dynamical temperature equation is also represented (- -) and the acquisition step is 6 min.

4.2.2. First and high order effects The high order effects are more complex for parameters than for inputs in the sensitivity analysis. Let

$$e_x = f(U, P, \{e_1, \ldots, e_i, \ldots, e_n\}) \qquad (12)$$

the error on a variable x with U the inputs set, P the parameters set and $E = \{e_1, \ldots, e_i, \ldots, e_n\}$, the parameters errors set. To study e_i, (12) can be decomposed as follow:

$$\begin{aligned} e_x = & \; f(U, P, \{0, \ldots, 0, e_i, 0, \ldots, 0\}) \\ & + f(U, P\{e_1, \ldots, e_{i-1}, 0, e_{i+1} \ldots, e_n\}) \\ & + g(U, P, \{e_1, \ldots, e_i, \ldots, e_n\}) \end{aligned}$$

where

- $f(U, P, \{0, \ldots, 0, e_i, 0, \ldots, 0\})$ is the first order error according to e_i,
- $f(U, P, \{e_1, \ldots, e_{i-1}, 0, e_{i+1} \ldots, e_n\})$ is the first and high orders errors according to $E \backslash \{e_i\}$,
- $g(U, P, \{e_1, \ldots, e_i, \ldots, e_n\})$ is the high order errors due to e_i.

f can be calculated for the different conditions and thus g is determined.

4.2.3. Sensitivity analysis results for parameters Figure 3 represents the first and high orders effects of these three parameters considering the input ranges. Intervals of ± 0.09 for emissivity error, ± 1.2 W.m^{-2}.K^{-1} for average heat transfer coefficient error and ± 0.02 for water activity error are used. These sets are considered as representative of cheese characteristics.

The sensitivity of the model for emissivity is very low, less than 0.01, more than -0.01 kg.m^{-1}.d^{-1} for the extreme error values. Consequently, it is not necessary to define precisely this parameter and the plaster value used in this study does not imply large errors.

Imprecision on the average heat transfer coefficient has important consequences on the mass loss rate estimation. An error of 1.2 W.m^{-2}.K^{-1} induces a mass loss rate error comprised between -0.026 and 0.068 kg.m^{-1}.d^{-1}. Note that this error is strongly influenced by the input: the error increases when the gap between rh and a_{ws} increases.

The result is also sensitive to water activity error but input values have less impact on e_{qm} with $e_{a_{ws}}$ than with e_{rh}. For this parameter, high order effects are not negligible, until 68% of the error according to $e_{a_{ws}}$.

5. CONCLUSION

During cheese ripening, an important mass loss occurs, resulting from heat and mass transfers from cheese to atmosphere. This phenomenon is based on physical laws and biological activity. A simple but efficient mass loss model for cheese ripening has been established and a sensitivity analysis has been carried out.

For model inputs, mass loss rate errors are mainly due to relative humidity and microbial respirative dynamic errors, a wrong measurement of atmospheric temperature have a low impact. We also tested the consequences of some parameter value variations. The product emissivity value can be

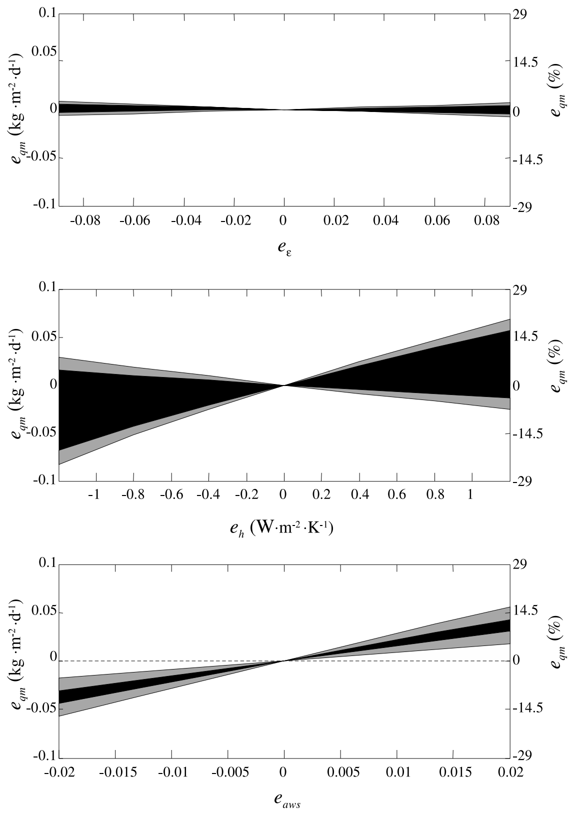

Figure 3. Mass loss rate error (kg.m^{-2}.d^{-1}, left axis) and relative mass loss rate error (%, right axis) as a function of error on parameters considering the complete input ranges; first (black) and high (grey) order effects are represented.

very roughly approximate without important result error. At the opposite, heat transfer coefficient and surface water activity must be precisely determined for each application.

Considering the robustness of this model, this mass loss description could be an interesting way to (i) predict final mass of cheeses and (ii) to estimate microbial activity during cheese ripening with software sensor approach, and thus to improve cheese ripening monitoring.

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