RECURRENT NEURAL CONTROL OF WASTEWATER TREATMENT BIOPROCESS VIA MARQUARDT LEARNING

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Abstract: The aim of this paper is to propose a new Recurrent Neural Network (RTNN) topology and a dynamic recursive Levenberg-Marquardt algorithm of its learning capable to estimate the states and parameters of a highly nonlinear wastewater treatment bioprocess. The proposed RTNN identifier is implemented in a direct adaptive control scheme incorporating feedback/feedforward recurrent neural controllers and a noise rejecting filter. The proposed control scheme is applied for continuous wastewater treatment bioprocess plant model, taken from the literature, where a good convergence, noise filtering and a low Mean Squared Error of reference tracking is achieved.

Keywords: adaptive control, backpropagation algorithms, biotechnology, feedback control, feedforward control, identification, state estimation, learning algorithms, neural network models, recursive algorithms.

1. INTRODUCTION

The rapid growth of available computational resources led to the development of a wide number of Neural Networks (NN) based modelling, identification, prediction and control applications, (Narendra, and Parthasarathy, 1990; Hunt *et al.*, 1992). The main network property namely the ability to approximate complex non-linear relationships without prior knowledge of the model structure makes them a very attractive alternative to the classical modelling and control techniques. Among several possible network architectures the ones most widely used are the Feedforward NN (FFNN) and Recurrent NN (RNN). In a FFNN the signals are transmitted only in one direction, starting from the input layer, subsequently through the hidden layers to the output layer, which requires applying a tap delayed global feedbacks and a tap delayed inputs to achieve a Nonlinear Autoregressive Moving Average (NARMAX) neural dynamic plant model. A RNN has local feedback connections to some of the previous layers. Such a structure is suitable alternative to the FFNN when the task is to model dynamical systems. NN-based techniques were successfully applied in several engineering areas as: prediction of chemical process (Su *et al.*, 1992); modelling and control of wastewater treatment process (Boger, 1992); optimisation of polymerisation process in a twin-screw extruder reactor and acetic anhydride plant (Geeraerd, *et al.*, 1998). In (Boskovic, and Narendra, 1995) a comparative study of linear, nonlinear and neural-network-based adaptive controllers for a class of fed-batch baker's and brewer's yeast fermentation is done. The paper proposed to use the method of neural identification control, given in (Narendra, and Parthasarathy, 1990), and applied FFNNs (multilayer perceptron and radial basis functions NN). The proposed control gives a good approximation of the nonlinear plants dynamics, better with respect to the other methods of control, but the applied static NNs have a great complexity, and the plant order has to be known. The application of RNNs could avoid these problems and could reduce significantly the size of the applied NNs. In some early papers, (see Baruch, *et al.*, 2001, 2002), the state-space approach is applied to design a RNN in an universal way, defining a Jordan canonical two or three layer RNN model, named Recurrent Trainable Neural Network (RTNN) and a Backpropagation (BP) algorithm of its learning. This NN model is a parametric one and it serves as a parameter estimator and a system state predictor, which permits to use the estimated parameters and states directly for process control. In two previous papers (Baruch *et al.,* 2004, 2005) this general RTNN approach is applied in direct and indirect neural control schemes for identification and control of continuous wastewater treatment fermentation bioprocess where unfortunately the plant and measurement noises affected the control precision. In the proposed paper we goes ahead applying the RTNN topology for control of the same wastewater treatment plant, incorporating a filter in a direct RNN control scheme and changing the first order learning with a second order one (the recursive Levenberg-Marquardt (L-M) algorithm of learning).

2. RECURRENT NEURAL NETWORK TOPOLOGY AND LEARNING

The process parameters and states are identified applying a discrete time model of a Jordan canonical RTNN (see Baruch, *et al.*, 2001, 2002), which permits to use the estimated states for direct adaptive neural control systems design. The Fig. 1 shows and example of SISO RTNN topology. The RTNN has the following mathematical description:

$$X(k+1) = AX(k) + BU(k) \qquad (1)$$

$$Z(k) = \varphi[X(k)] \qquad (2)$$

$$Y(k) = \varphi[CZ(k)] \qquad (3)$$

$$A = block-diag(A_{ii}); |A_{ii}| < 1 \qquad (4)$$

Where: X(.) is a N - state vector; U(.) is a M -input vector; Y(.) is a L- output vector, Z(.) is an auxiliar vector variable with dimension N; φ(.) is a vector - valued activation function with appropriate dimension and hyperbolic tangent elements. Equations (1), (2) defined the hidden layer of the RTNN and equation (3) - the feedforward output layer. The matrix A is the feedback weight matrix of the hidden layer, which has a (NxN) diagonal structure, where stability conditions (4) are imposed on its diagonal elements; the matrices B and C are

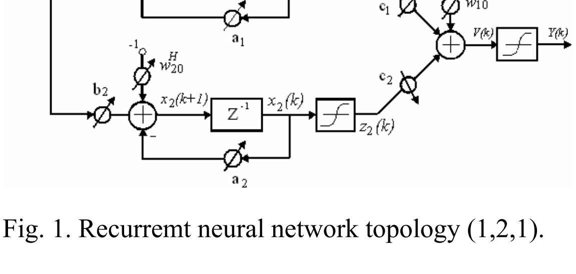

Fig. 1. Recurremt neural network topology (1,2,1).

(NxM) and (LxN) weight input and output matrices, respectively, with structure, corresponding to the structure of A. The main advantage of the proposed two layers Jordan canonical RTNN architecture is that it is an universal hybrid neural model containing one feed-forward output layer, and one recurrent hidden layer with completely decomposed dynamics, as the matrix A is block-diagonal one. Hence, the RTNN has a minimum number of parameters and a completely parallel structure, as the Jordan canonical form is parallel with respect to the autoregressive NARMAX sequential model. The RTNN architecture is described in a state-space form and serves as a one-step ahead state predictor/estimator, therefore it is suitable for identification and control purposes. The tuning of the network weights is based on the recursive Levenberg-Marquardt algorithm, (Ngia and Sjoberg, 2000). Here it is applied for RTNN, which is derived using a sensitivity model. The general updating rule is described by the following equation:

$$W(k+1) = W(k) + P(k)DY[W(k)]E[W(k)] \qquad (5)$$

Where: W(.) is the element update of each weight matrix A, B, C of the RTNN model; P(.) can be interpreted as the covariance matrix of weights estimate W(.); DY[W(.)] is the Jacobian matrix which is defined as the derivative of the RTNN outputs with respect to the weights; finally E[W(.)] is the error of approximation; k is the iteration number. The approximation error is given by:

$$E[W(k)] = Yp(k) - Y(k) \qquad (6)$$

Where: Yp(.), Y(.) are plant and RTNN outputs. The Jacobean matrix includes the corresponding gradient components of the outputs with respect to RTNN weights, derived as:

$$\begin{aligned} &DY[W(k)] = (\partial / \partial W)Y(k) = \\ &\{DY[C_{ij}(k)]; DY[A_{ij}(k)]; DY[B_{ij}(k)]\} \end{aligned} \qquad (7)$$

$$DY[C_{ij}(k)] = D_{1,i}(k) Z_j(k) \qquad (8)$$

$$D_{1,i}(k) = \varphi_i^{'}[Y_i(k)] \qquad (9)$$

$$DY[A_{ij}(k)] = D_{2,i}(k) X_j(k) \qquad (10)$$

$$DY[B_{ij}(k)] = D_{2,i}(k) U_j(k) \qquad (11)$$

$$D_{2,i}(k) = \varphi_i^{'}[Z_i(k)] C_i D_{1,i}(k) \qquad (12)$$

The P(.) matrix is computed recursively by:

$$\begin{aligned} &P(k) = \alpha^{-1}(k)\{P(k-1) - P(k-1). \\ &.\Omega[W(k)] S^{-1}[W(k)] \Omega^T[W(k)] P(k-1)\} \end{aligned} \qquad (13)$$

Where: the S(.), and Ω(.) matrices are given as:

$$S[W(k)] = \alpha(k)\Lambda(k) + \Omega^T[W(k)]P(k-1)\Omega[W(k)] \qquad (14)$$

$$\begin{aligned} &\Omega^T[W(k)] = \begin{bmatrix} & DY^T[W(k)] & \\ 0 \ \cdots & 1 & \cdots \ 0 \end{bmatrix}; \\ &\Lambda(k)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}; 10^{-4} \le \rho \le 10^{-6}; \\ &0.97 \le \alpha(k) \le 1; 10^3 \le P(0) \le 10^6 \end{aligned} \qquad (15)$$

The matrix Ω(.) has dimension (Nwx2), where Nw is the number of weights. The second row of Ω(.) has only one unity element (the others are zero). The position of that element is computed by:

$$i = k \bmod(Nw) + 1; k > Nw \tag{16}$$

Next, this topology and learning is applied for wastewater treatment plant identification and control.

3. DIRECT ADAPTIVE NEURAL CONTROL SYSTEM DESIGN

The block-diagram of the control system is given on Fig. 2. It contains a recurrent neural identifier, two neural controllers (feedback and feedforward), and a low pass noise filter. Let us to write the following *z*-transfer- function representations of the plant, filter, feedback and feedforward controllers:

$$\begin{aligned} &W^P(z) = C^P\left(zI - A_p\right)^{-1} B_p; \\ &W^*(k) = C^*\left(zI - A^*\right)^{-1} B^*; \\ &P^i(z) = \left(zI - A^i\right)^{-1} B^i \end{aligned} \tag{17}$$

$$\begin{aligned} &Q_1(z) = C^c{}_{fb}\left(zI - A^c{}_{fb}\right)^{-1} B_{fb}; \\ &Q_2(z) = C^c{}_{ff}\left(zI - A^c{}_{ff}\right)^{-1} B_{ff} \end{aligned} \tag{18}$$

The control systems *z*-transfer functions (17), (18) are connected by the following equation, given in *z*-operational form:

$$\begin{aligned} &Y^*(z) = W^*(z) W_p(z)[I + Q_1(z) P^i(z)]^{-1}. \\ &.Q_2(z) R(z) + V_3(z) \end{aligned} \tag{19}$$

Where $V_3(.)$ is a generalized noise term, given as:

$$V_3(z) = W^*(z)[W_p(z)V_1(z) + V_2(z)] \tag{20}$$

The RTNN topology is controllable and observable, (see Baruch et al., 2002) and the L-M algorithm of learning is convergent (see Ngia and Sjoberg, 2000), so the identification and control errors tend to zero:

$$E^i(k) = Y^*(k) - Y^i(k) \to 0; \quad k \to \infty \tag{21}$$

$$E^i(k) = Y^*(k) - Y^i(k) \to 0; \quad k \to \infty \tag{22}$$

This means that each transfer functions given by equations (17), (18) is stable with minimum phase.

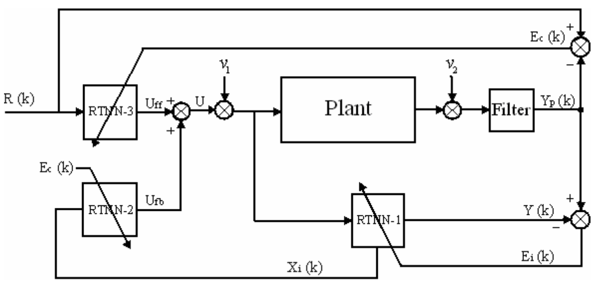

Fig. 2. Block - diagram of the control system containing neural identifier and two adaptive neural controllers.

From (19), it is seen that the dynamics of the stable low pass filter is independent from the dynamics of the plant and it does not affects the stability of the closed-loop system. The closed-loop system is stable and the RTNN-2 feedback controller compensates the combined "plant plus filter" dynamics. The RTNN-3 feedforward controller dynamics is an inverse dynamics of the closed-loop system one, which assure a precise reference tracking in spite of the presence of process and measurement noises.

4. BIOLOGICAL WASTEWATER TREATMENT BIOPROCESS DESCRIPTION

4.1 Analytical derivation of the bioprocess model.

Wastewater treatment is performed in an aeration tank, in which the contaminated water is mixed with biomass in suspension (activated sludge), and the biodegradation process is then triggered in the presence of oxygen. The tank is equipped with a surface aeration turbine, which supplies oxygen to the biomass, and additionally changes its suspension into a homogeneous mass. After some period, the biomass mixture and the remaining substrate go to a separating chamber where the biologic flocks (biologic sludge) are separated from the treated effluent. The treated effluent is then led to a host environment. The maintenance of adequate concentration of active biomass in the aeration tank, which allows the aerobic degradation of the in-coming wastewater, is achieved by the recirculation of the sludge accumulated in the decanter. The aim is good settling of the biomass in the settler and high conversion of the entering organic material in the bioreactor (see Fig. 3). The concentration of the biomass in the recycle stream serves as an indicator of both the sludge activity and the sludge settling characteristics, and is therefore considered as the controlled variable. The main objective of the control system is to keep the recycle biomass concentration close to the reference signal, and this should be achieved in the presence of disturbances and measurement noise acting on the recycle flow rate. The control task is hampered by the strong nonlinearity of the process dynamics, the variations in the reaction kinetics and by unknown and possibly time-varying process parameters. Since the influent flow rate has generally periodic behaviour, the goal is not to keep the recycle biomass concentration constant, but to follow a desired time trajectory, a proportion of the influent flow rate.

Mass balance to the bioreactor. A detailed description of all reactions arising in the bioreactor would lead to a high - order model of differential

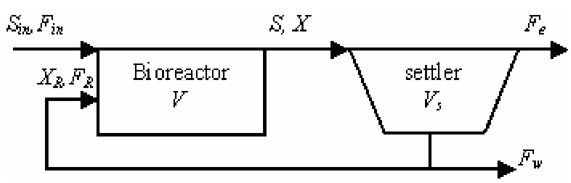

Fig. 3. Biological wastewater treatment with settler.

equations (see Georgieva, and Ilchmann, 2001). For the control strategy developed in this work a simplified reduced order model is sufficient, as far as it preserves the structural properties of the process, (Georgieva, and Ilchmann, 2001). The mass balance to the bioreactor is given by the equations:

$$\dot{X}(t) = \left(\mu(S) - \frac{F_{in}(t) + F_R(t)}{V} - c_d(t) \right) X(t) + + \frac{F_R(t)}{V} X_R(t) \tag{23}$$

$$\dot{S}(t) = -\frac{1}{Y}\mu(S)X(t) + \frac{F_{in}(t)}{V}S_{in} - -\frac{F_{in}(t)+F_R(t)}{V}S(t), \tag{24}$$

Where the state variables are: X(t), biomass concentration, which is considered as the total amount of the sludge present in the mixed liquor and is represented by the Mixed Liquor Suspended Solids; S(t), the substrate measured by the Chemical Oxygen Demand (COD); V is the reactor volume; F_R represents the recycle flow rate (manipulated variable), F_{in} is the influent flow rate; S_{in} is the influent substrate concentration (potential disturbance, also expressed as COD), and Y>0 is the yield coefficient. Here c_d X denotes the decay rate of the biomass concentration (which is added in the model to simulate biomass mortality), with $c_d > 0$, as the decay rate parameter and μ(.) denoted the specific growth rate, given by the Monod-type equation:

$$\mu(S(t)) = \frac{\mu_m(t)S(t)}{K_m(t)+S(t)} \tag{25}$$

Where: μ_m(.) is the maximum growth rate and K_m(.) is the half-saturation constant of biodegradable organic matter which is the substrate concentration of $\mu = \mu_m/2$.

Mass balance to the settler. In the mass balance derivation it is supposed that none of the biomass is left in the effluent F_e of the settler (see Fig. 3), so that the whole biomass in the clarifier is settled. The dynamics of the concentration of the biomass in the settler, X_R (t), can be described by the equation:

$$\dot{X}_R(t) = \left(\frac{F_{in}(t)+F_R(t)}{V_S} \right) X(t) + + \left(\frac{F_W(t)+F_R(t)}{V_S} \right) X_R(t) \tag{26}$$

Where: F_W denotes the waste flow rate and V_s is the volume of the settler. Since the settler has first order dynamics, which is much faster than the bioreactor dynamics, and since we assume that a constant ratio of output to input solids concentration is maintained, we may approximate the settler behaviour by:

$$X_R(t) = q(t)X(t) \tag{27}$$

Where q(t) is considered as continuously differentiable and bounded function with bounded inverse, bounded derivative, and q(t) >1 for all $t \geq 0$. The biomass concentration in the settler is higher than the biomass concentration in the reactor because it accumulates at the bottom of the vessel, and good settling is only possible if the settler is designed such that $X_R(t) > X(t)$.

Process measurements. The sensor dynamics is modelled by:

$$T_m \dot{X}_m(t) = -X_m(t) + X_R(t) + n(t) \tag{28}$$

The bioprocess dynamics is corrupted by some white Gaussian noise n(t). The specific model, we consider for the simulations, is obtained after substitution of $X_R(t)$ from (27) into(23) which yields:

$$\dot{X}_R(t) = \{[\dot{q}(t)/q(t) + \mu[t,S(t)] - -[F_{in}(t)/V] - c_d + [q(t)-1]/V\}X_R(t) \tag{29}$$

$$\dot{S}(t) = -[1/Y(t)]\mu[t,S(t)][1/q(t)]X_R(t) + +[F_{in}(t)/V]S_{in} - \{[F_{in}(t)+F_R(t)]/V\}S(t) \tag{30}$$

Time-varying control reference. The time-varying control reference is based on the basic supposition that the process input has a diurnal periodicity (see Georgieva, and Ilchmann, 2001). Therefore, the control objective is to assure that the biomass concentration in the recycle flow tracks asymptotically a time-varying reference signal, proportional to the influent flow rate which is assumed to be measurable:

$$X_{Rref}(t) = k_{ref}F_{in}(t) \tag{31}$$

The specific model considered for process simulation is the system of nonlinear differential equations (28), (29), (30) and the Monod-type equation (25), with constant parameters: V = 1,5.107 [l], S_{in} = 300 [mg COD /l], T_m = 1/12 [h]. The model uncertainties are taken into account by introducing:

$$\mu_m(t) = 0.2 + 0.1\sin(2\pi t/3 + 4\pi/3) \tag{32}$$

$$K_m(t) = 90 + 30\sin(\pi t/2) \tag{33}$$

$$Y(t) = 0.6 + 0.1\sin(\pi t/3 + \pi/3) \tag{34}$$

$$q(t) = 4 + \sin(\pi t/6) \tag{35}$$

$$c_d(t) = 10^{-4}(25 + 5\sin(\pi t/12)) \tag{36}$$

The control objective is to track the reference signal, given by the equation (31), where the parameters are:

$$k_{ref} = 3.8*10^{-3} \text{ [mgh/l2]}$$

$$F_{in}(t) = 3*10^6(1 + 0.25\sin \pi t/12) \tag{37}$$

The above data coincide with the typical range for domestic wastewater (see Georgieva, and Ilchmann, 2001). We keep these data for all of the following simulations, and the initial conditions are set to:

$$S(0) = 8 \text{ (mgCOD/l)}, \quad X_R(0) = 11.4*10^3 \text{ (mg/l)},$$
$$X_m(0) = 0 \text{ (mg/l)}$$

In order to overcome saturation of the RTNNs, the output and the input of the plant are scaled by:

$$y_p = (X_m - 11400)/5700 \tag{38}$$

$$F_R = \left[\left(\left[\left(U*7.5\times10^5\right)+3\times10^6\right]F_{Conv}\right)-X_m\right]K_{stab} \tag{39}$$

Where the scaling parameters are given equal to: K_{stab}= 3*10^-3 , F_{conv}= 0.0038. These scale factors correspond to the range of the reference signal. Note that the variable U is the bioreactor input control signal, generated by one of the proposed control algorithms, while F_R is the physical process input (the recycle flow rate). Analogous, y_p is the scaled output of the bioreactor, while X_m is the real measured output. Consequently, the reference signal is also normalised following the same procedure:

$$r(k) = \left(X_{Rref}(k) - 11400\right)/\,5700 \tag{40}$$

Substituting (37) into (31), and the obtained result in (40), the reference signal is obtained as:

$$r(k) = 0.5\sin\left(\pi k/12\right) \tag{41}$$

The inverse transformation of (38) leads to:

$$X_m = 5700\,y_p + 11400 \tag{42}$$

Finally, the substitution of the given up scaling parameters and (42) into (39) yields:

$$F_R = \left(0.5U - y_p\right)1.71\times10^8 \tag{43}$$

Note that the recycle flow rate F_R is a function of the control variable U, computed by the feedback control with respect to the estimated state and the feedforward control with respect to the reference r(k).

4.2 Simulation results and discussion.

All simulations are performed using the following set of equations: the process description (28), (29), (30); the Monod-type equation (25); the time variable plant parameters (32) – (36); the plant output scaling equation (38); the scaled reference signal equation (41); the scaled plant input equation (43). In all simulations, a process and measurements noises (see Fig. 2), both with variance 1200, are added. The variance chosen corresponds to 10% noise on the data. The process is simulated over a period of 40 hours, which gives an idea about its periodic behavior (a typical period is about 24 hours) and the period of discretization is set to T_0=0.01h (it is 1 hour of the process time). The learning parameter used in (14) is α=0.95. The results, obtained from the direct adaptive neural control, are given on Fig. 4.

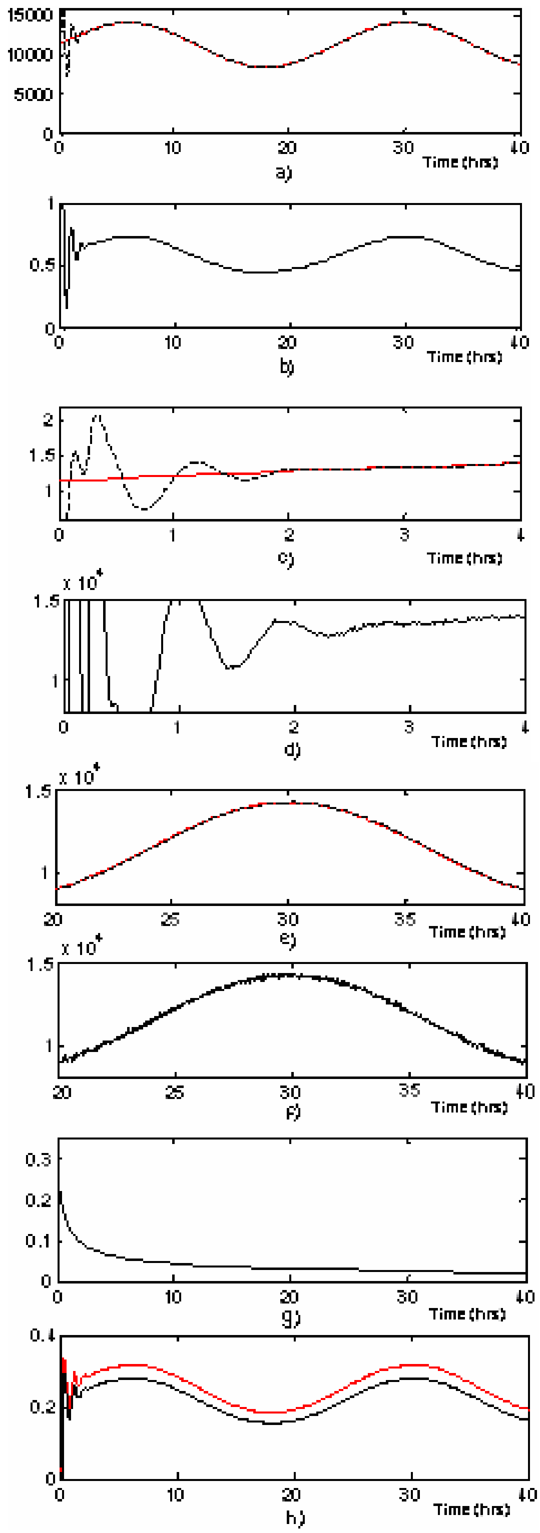

Fig. 4. Graphical results obtained using Direct Adaptive Neural Control. a) Comparison between the filtered plant output and the reference signal; b) The control signal; c) Comparison between the filtered plant output and the reference signal in the first 4 hours; d) The same as c), but with respect to the unfiltered plant output; e) Comparison between the filtered plant output and the reference signal in the last 20 hours; f) The same as e), but with respect to the unfiltered plant output; g) MSE%; h) The two states X of the RTNN, obtained from the identification RTNN and used for feedback control.

The identification RTNN has topology (1, 2, 1). The activation functions of the hidden and output network layers are hyperbolic tangents. The results show a good convergence of the system output to the desired trajectory after approximately 2.1 h and a good filtration of the noise which makes a MSE% reduction up to 1%.

5. CONCLUSIONS

In this paper a Recurrent Trainable Neural Network model and a dynamic Levenberg-Marquardt learning algorithm are proposed to be applied for real time identification and state estimation of a nonlinear bioprocess plants. The proposed RTNN model has a Jordan canonical structure, which permits to use the generated vector of estimated states directly for process control. The obtained states are used to design a feedback direct adaptive control law. The direct neural feedback/feedforward control with noise filter is able to force the system to track a time-varying process-dependent reference signal in noisy plant conditions. It performs very well under restrictive conditions of periodically acting disturbances, parameter uncertainties and inevitable sensor dynamics. The simulation results, obtained with a continuous wastewater treatment bioprocess plant model, confirm the applicability of the proposed identification and control methodology.

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REFERENCES

Baruch I., J.M. Flores, F. Thomas, and R. Garrido (2001). Adaptive neural control of nonlinear systems. In: *Artificial Neural Networks-ICANN 2001, Lecture Notes in Computer Science 2130*, (G. Dorffner, H. Bischof, K. Hornik. (Eds.)), ISBN 3-540-42486-5, 930-936. Springer, Berlin.

Baruch, I. S., J. M. Flores, F. Nava, I.R. Ramirez, and B. Nenkova (2002). An advanced neural network topology and learning, applied for identification and control of a D.C. motor. In: *Proc. of the First Int. IEEE Symposium on Intelligent Systems*, (V. Sgurev, V. Jotsov. (Eds.)), 289-295. IEEE, Varna, Bulgaria.

Baruch, I. S., J. Perez-Medina, and J. Barrera-Cortes (2004). An indirect adaptive neural control of wastewater treatment bioprocess. In: *Proc. of the First International Meeting on Environmental Biotechnology and Engineering, 1IMEBE,* (H.M. Poggy-Varaldo (Ed.)), ISBN 970-94112-0-9, [1MEBE155.pdf]. Sept. 6-8, CINVESTAV, Mexico City, Mexico.

Baruch, I. S., P. Georgieva, J. Barrera-Cortes, and S. Feyo de Azevedo (2005). Adaptive recurrent neural network control of biological wastewater treatment. *International Journal of Intelligent Systems (Special issue on soft computing for modelling, simulation and control of nonlinear dynamical systems), (O.Castillo, and P.Melin (guest eds)).* WILEY Inter-Science, ISSN 0884-8173, **20** (2), 173-194.

Boger, Z. (1992). Application of neural networks to water and wastewater treatment plant operation. *ISA Trans.* **31** (1), 25-33.

Boskovic, J. D., and K.S. Narendra (1995). Comparison of linear, nonlinear and neural-network-based adaptive controllers for a class of fed-batch fermentation processes. *Automatica*, **31**, 817-840.

Geeraerd, A. H., C. H. Herremans, C. Cenens, and J. F. Van Impe (1998). Application of artificial neural networks as a nonlinear modular modelling technique to describe bacterial growth in chilled food products. *Int. J. of Food Microbiology*, **44**, 49-68.

Georgieva, P., and A. Ilchmann (2001). Adaptive λ-tracking control of activated sludge processes. *Int. J. of Control,* **74** (12), 1247-1259.

Hunt, K. J., D. Sbarbaro, R. Zbikowski, and P. J. Gawthrop (1992). Neural network for control systems - a survey. *Automatica*, **28** (6), 1083-1112.

Narendra, K.S., and K. Parthasarathy (1990). Identification and Control of Dynamic Systems Using Neural Networks. *IEEE Transactions on Neural Networks*, **1** (1), 4-27.

Ngia, L. S., and J. Sjöberg (2000). Efficient training of neural nets for nonlinear adaptive filtering using a recursive Levenberg-Marquardt algoritm. *IEEE Trans. on Signal Processing*, 48, 1915-1927.

Su, Hong-Te, Th. J. McAvoy, and P. Werbos (1992). Long-Term Predictions of Chemical Processes Using Recurrent Neural Networks: A Parallel Training Approach. *Ind. Eng. Chem. Res.*, **31**, 1338-1352.