USE OF BIFURCATION ANALYSIS FOR MODEL IDENTIFICATION PURPOSES

Vega, M.P., Coimbra, K.B., Araújo, J.M. and Scheid, CM.

Departamento de Engenharia Química - Universidade Federal Rural do Rio de Janeiro, BR 465, km7 – CEP: 23890-000 – Seropédica – RJ – Brasil

Abstract: Nonlinear system identification poses challenging questions because a closed general theory is not available for this field. Particularly, models based on neural networks may present incompatible general process behavior, leading to improper closed loop responses, even when they allow a satisfactory one step ahead prediction of process dynamics, as required by traditional validation methods. It is shown here that performing detailed bifurcation and stability analysis may be very helpful for the adequate analysis of neural models. The study of the many parameters that are defined a priori during the training of the neural network is of paramount importance, as the spurious dynamic behavior is related mostly to the use of incomplete data sets during the learning process. Strategies to improve the quality of the identification procedure are provided and analyzed, using cyclones as a case study.

Keywords: Neural networks, Stability analysis, Bifurcation diagram, Cyclones

1. INTRODUCTION

When all the important characteristics of the process are known, building a phenomenological model is normally an easy task. However, difficulties may emerge during the solution of the resulting system of equations. For this reason, a number of different model reduction techniques have been proposed in the literature, Benallou *et al*. (1986); Pinto *et al*. (1988); Levine *et al*. (1991).

Empirical linear modeling, Luyben (1990), is well studied but, as pointed out by Pearson *et al*. (1997), a well-developed theory for nonlinear system identification is not available. The neural network (NN) approach has proved to be a useful tool and is the most popular framework for empirical model development, although estimating the huge number of parameters frequently present in the model may be regarded as a major problem to be solved, Su *et al*. (1997) and Cybenko (1989).

Nonlinear system identification involves model parameters selection, determination of the forcing function which is introduced into the plant to generate the output response, estimation of model parameters and comparison of plant information and model predictions for data not used in model development. All steps represent very challenging theoretical and practical problems, for a general theory is not available. As a result, further investigation on systematic techniques for nonlinear model validation, characterization of the amount and type of process data required to build nonlinear empirical models with satisfactory predictive capability and the identification of nonlinear model structures which are capable of capturing a wide variety of process behaviors are future research issues that need to be explored.

The main objective of this paper is building a methodology for the analysis of NNs, which allow the development of confident model identification procedure for use in the laboratory and industrial environment. NNs were validated in terms of the traditional methods, Pollard *et al*. (1992) and Sriniwas *et al*. (1995), and in terms of their complex static and dynamic behavior, using bifurcation and stability analysis. As observed through many examples, Vega (2001), the use of traditional validation tests is not enough to guarantee successful use of NNs, as the complex dynamic behavior displayed by the model may be completely different from the one displayed by the plant, resulting in poor identification efficiency. Good identification performance can be detected using bifurcation diagrams, which can be computed with typical numerical packages, such as AUTO, Doedel (1986). It is proposed here that standard bifurcation analysis be used as an additional validation procedure for implementation of NN models. In order to illustrate this point, big and small cyclones, separating solid particles from gases, are taken as a case study. Bifurcation techniques are used to allow the development of confident NNs, based on experimental data. Experimental data are obtained elsewhere, Halasz *et al*. (2000). It is shown that the bifurcation analysis of NNs may be very helpful for the appropriate development and implementation of model identification. Therefore, bifurcation analysis should be included as an innovative validation criterion in the nonlinear system identification methodology.

2. THE PROCESS ANALISED

A cyclone is a particle removal device without moving parts which spins a gas stream to collect entrained particles by centrifugal force. Figure 1 shows a typical cone-under-cylinder cyclone design. In this design, particle-laden gas enters the cyclone at the top of the cylinder and makes several revolutions due to the shape of the entry forming a vortex with a high tangential velocity which accelerates particles outward to the wall for collection. Below the bottom of the gas exit tube, the spinning gas gradually migrates inward, to a central core axially along the cylinder centerline, and from there up, finally out to the exit tube. Cyclones are used in the field of air pollution control with small cyclones for ambient and source sampling and large ones for industrial particulate control. Their simple design, low capital and maintenance costs, and adaptability to a wide range of operating conditions have made cyclones one of the most widely used industrial dust collectors.

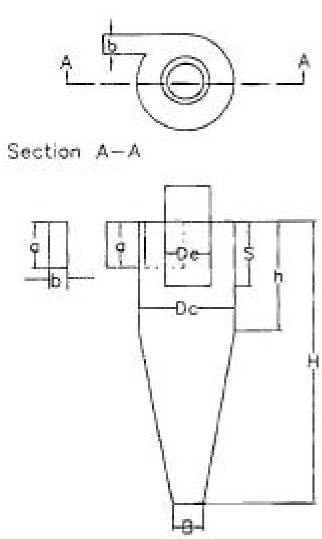

Figure 1 – Typical cyclone design

Due to the complex three-dimensional fluid flow in cyclones the exact mechanisms of removing particulate are still not fully understood. In addition, different operating conditions such as temperature, pressure and flow rate add even more difficulties to the already complicated problem. Therefore, most cyclone theories are based on a simplified model or depend upon empirical correlation equations. Although these theories are valid for certain cyclone operating conditions, none of them has been satisfactorily validated. Therefore, study on cyclones is still largely based on experiment methods and design of cyclones relies upon experience, trial and design guides.

Halasz *et al*., 2000 employed a FLN (Functional Link Network) neural network for modeling the particle collection efficiency, according to Equations 1 and 2, using literature data for big and small diameter cyclones, respectively. The NN models were a function of operational conditions [flow (Q), diameter of the particle to be separated (dp) and ratio between fluid viscosity and particle density $P=(\mu/\rho_S)$], the equipment characteristic relations [(a/Dc), (b/Dc), (De/Dc), (H/Dc), (h/Dc), (B/Dc), (S/Dc)] and cylindrical section diameter, Dc. The lower and upper limit values for the parameters are reported in Table 1.

$$\begin{aligned}
\eta(dp) &= 0.7152 + 20.048 D_c - 164.69 \frac{P}{D_c} \\
&- 0.0036 \frac{1}{Q^2} - 25.40 \left(D_c^{0.5}\right)^3 - 0.4738 \left(\frac{D_c}{Q}\right) \\
&+ 1.4098 \left(\frac{D_c^{0.5} P}{dp}\right) + 0.013 \frac{D_c^{0.5}}{Q^2} \\
&- 4.9078 D_c^{0.5} \left(\frac{b}{D_c}\right)\left(\frac{D_e}{D_c}\right)^2 \\
&+ 2.4907\times 10^{8} \left(\frac{D_e}{D_c}\right)^2 \left(\frac{b}{D_c}\right) P + 1.984\times 10^{-7} \times \\
&\frac{\left(b/D_c\right)}{Q dp} - 12.0252 \left(\frac{b}{D_c}\right)\left(\frac{a}{D_c}\right)\left(\frac{D_e}{D_c}\right)^2 \\
&+ 2.2\times 10^{-18} \frac{1}{dp^3} - 3.4044 \left(\frac{b}{D_c}\right)^2 \left(\frac{S}{D_c}\right) \\
&+ 0.0025 \left(\frac{B}{D_c}\right)^2 \left(\frac{h}{D_c}\right)^4
\end{aligned} \tag{1}$$

3. STABILITY ANALYSIS

Bifurcation theory provides tools for a system stability analysis under its parametric changes. As the parameters undergo changes, the existence of multiple steady states, sustained oscillations and traveling waves might occur for highly nonlinear processes, Ray *et al*. (2000).

The quality of the different NNs was evaluated by analyzing their dynamic structure (attractors and respective stability characteristics). In order to do that, bifurcation and stability analyses were carried out to unveil the attractors of the NNs, employing well-known continuation methods. The computations presented in this paper were carried out with routines provided by AUTO, Doedel (1986). Steady states are stable if all eigenvalues of the Jacobian matrix are inside the unity circle. If any of the eigenvalues is outside the unity circle, the steady-state solution is unstable. At a Limit Point, an eigenvalue becomes identically equal to +1. At this point, multiple steady state solutions usually appear and a change in stability occurs. At a Hopf (Thorus) Bifurcation Point, a pair of complex eigenvalues crosses the unit circle with non-zero imaginary component and a branch of oscillatory solutions may appear. At a Period Doubling Bifurcation Point an eigenvalue becomes equal to -1 and branches of periodic solutions usually develops. It is assumed here that a good NN model should exhibit a steady-state bifurcation diagram that resembles the bifurcation diagram of the original process. This means that NNs should present the same bifurcation structure and bifurcation points of the original process. Therefore, in our particular case, a good NN should not display multiple steady-state solutions and steady-state unstable operation conditions.

$$\begin{aligned}\eta(dp) = {} & 1.26 - 2.49\times10^{-14}\left(\frac{D_e{}^2 S}{Q dp^2}\right) - 0.312h \\ & - 6.64\times10^{-19}\left(\frac{S}{Q^2 dp^2}\right) - 0.039\left(D_c{}^{0.5} H h^2\right) + \\ & 2.21\times10^{-48}\left(\frac{1}{P^4 Q dp^2}\right) - 7.75\left(D_c a h\right) - 2.24 \\ & \times10^{-14}\left(\frac{B^4}{Q dp^2}\right) - 3.18\times10^{-32}\left(\frac{D_e{}^2}{P^2 Q dp^2}\right) \\ & + 2.75\times10^{-26}\left(\frac{B^2}{Q dp^4}\right) - 19.1\left(D_c{}^{3/2}\right) \\ & - 6.97\times10^{-14}\left(\frac{D_c}{Q dp^2}\right) + 3.05\times10^{-15}\left(\frac{Sh}{Q dp^2}\right) \\ & + 1.45\times10^{-4}\left(\frac{D_e{}^2 S^2}{Q}\right) + 3.12\times10^{-18}\left(\frac{D_e{}^2}{Q^2 dp^2}\right) \\ & - 1.09\times10^{-7}\left(\frac{D_e{}^4}{Q^2}\right) + 2.31\left(D_e{}^8\right) + 5.4\left(D_c{}^{0.5} b^{0.5} h\right) \\ & - 7.86\times10^{-44}\left(\frac{1}{P^2 Q dp^4}\right) - 2.14\times10^{-17}\left(\frac{b^{0.5} D_e{}^2 S}{P^2}\right)\end{aligned} \qquad (2)$$

AUTO automatically detects bifurcation points and provides routines for computation of the multiple steady state solutions, oscillatory and periodic solutions that arise at these special points. Unstable behavior usually occurs in the vicinities of these bifurcation points, as at least one of the eigenvalues crosses the unity circle. Therefore, identification of the bifurcation points may allow the understanding of how and why the NNs models fail at certain process operation conditions.

Table 1 – Parameter range

	big diameter cyclone		small diameter cyclone	
parameter	lower limit	upper limit	lower limit	upper limit
D_c, m	0.18	0.4	0.01	0.05
$P\times10^9$, m^2/s	4	80	4	80
$Q\times10^3$, m^3/s	43	240	0.15	4.8
a/D_c	0.25	0.7	0.21	0.59
b/D_c	0.15	0.3	0.15	0.32
D_e/D_c	0.3	0.58	0.2	0.8
S/D_c	0.35	3.5	0.4	1.64
H/D_c	3	6	2.3	4.3
h/D_c	0.5	3.5	1.1	2.1
B/D_c	0.38	1	0.3	0.69
dp , m	1×10^{-6}	1×10^{-4}	1×10^{-6}	1×10^{-4}

4. RESULTS AND DISCUSSION

Literature information provides that as the parameter Dc improves, the collection efficiency decreases. The increase of P produces a decrease in the collection efficiency. As Q, S, h, B and dp increase the collection efficiency increases. In addition, as b and De increases, the collection efficiency diminishes. Finally, it is well-known that the collection efficiency varies from zero to unity.

The bifurcation diagrams presented in Figures 2 and 3 for big and small cyclones, respectively, were built using central parameter values (see Table 1). All NN models present incompatible behavior for the collection efficiency. Besides, as b increases the small cyclone NN model predicts a wrong increase in the collection efficiency. The small cyclone NN model also indicates that the collection efficiency is insensitive to parameters B, De, S and P. The big cyclone NN model displays an inverse behavior for the continuation parameter De. The big cyclone NN model presents an incompatible collection efficiency behavior as the continuation parameter dp varies. The small cyclone NN model presents opposite collection efficiency behavior as the continuation parameter h varies. Finally, the big and small cyclone NN models present spurious collection efficiency behavior as the continuation parameters P and S varies.

5. CONCLUSIONS

It was observed that NNs built to represent cyclones presented incompatible open loop steady state stable behavior. Also, the NNs generalization capacity was very poor in the central data region, producing spurious solutions, incompatible with the real system performance. Bifurcation diagrams built in the extreme parameter limits, not shown in the text, presented the same incompatible collection efficiency values. The collection efficiency is supposed to be severely affected by Q, dp, De, B and Dc. However, none of the bifurcation diagrams presented such behavior.

It may me concluded that although NNs may be used successfully for identification purposes, care must be taken regarding the number and range of data used, as the simple manipulation of the neuron activation functions, NN architecture and initial guesses used for NN training are not enough to guarantee the building of proper models. It is also shown that, detailed bifurcation analysis may be very useful for the proper design and identification. Bifurcation diagrams indicate whether spurious model responses are predicted by the model and therefore indicate whether additional effort is needed for proper model development. The result obtained with the bifurcation analysis validation criterion motivates further analysis for adequate NN cyclone model building. The authors are now working on the analysis of the training data set (500 points), used by Halasz *et al*. (2000), in order to investigate the reasons for the weak NN performance. Bifurcation analysis was used as an efficient tool for validating models, which were built in a supervisory fashion, using available experimental data. Unknown systems can be unveiled if the convergence of the bifurcation diagram to a final structure is used as a quality index in an iterative procedure. This sophisticated validation procedure is indicated for complex units operating in a large range of operating conditions and using nonlinear model based controllers.

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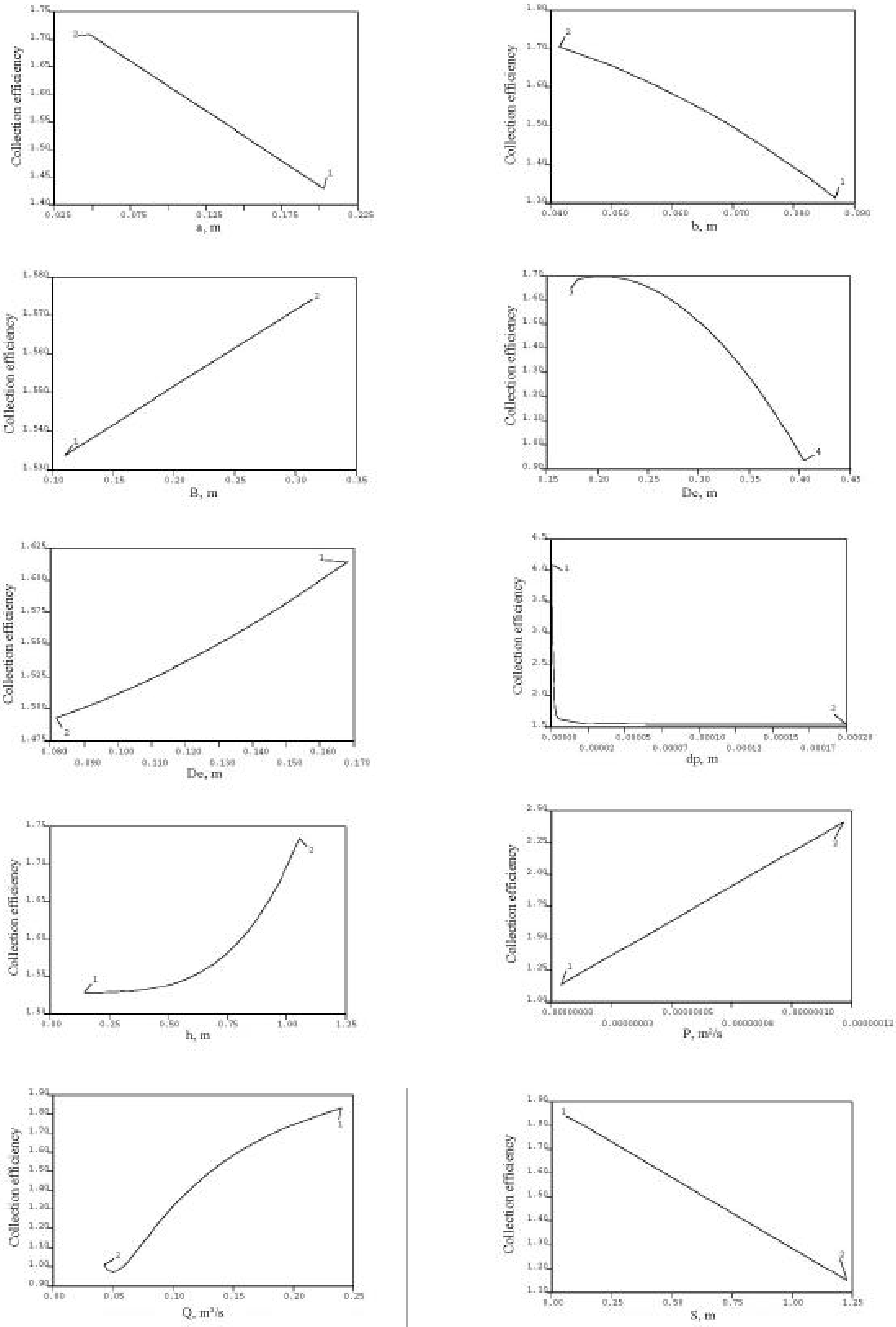

Figure 2 – Bifurcation diagrams for big diameter cyclones

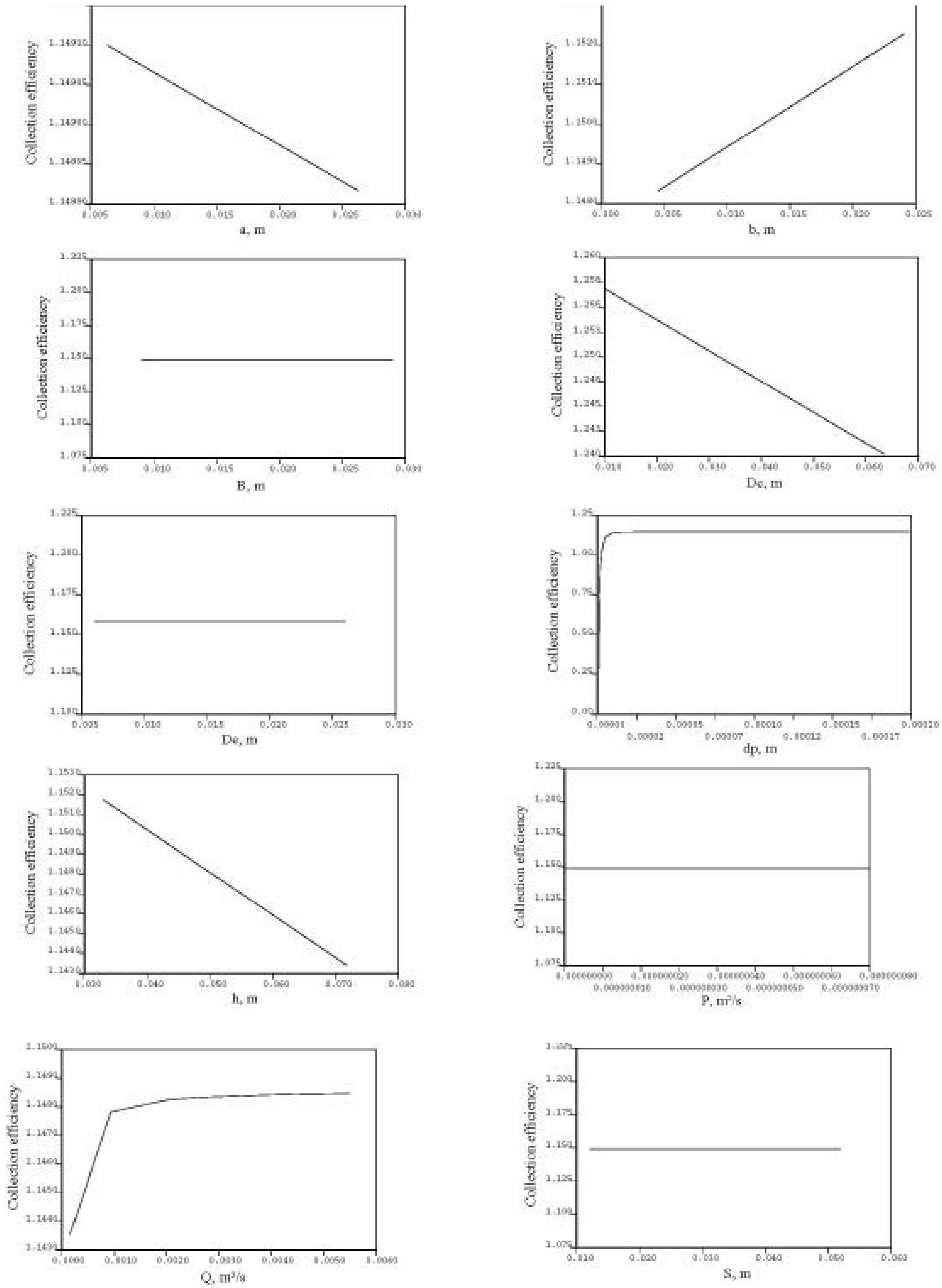

Figure 3 – Bifurcation diagrams for small diameter cyclones