A FILTER BASED APPROACH FOR ESTIMATION OF PI ACHIEVABLE PERFORMANCE

Mranal Jain and S. Lakshminarayanan *

Department of Chemical and Biomolecular Engineering
National University of Singapore, Singapore 117576
**Corresponding Author. Telephone: +65-68748484 Email*: chels@nus.edu.sg

Abstract: We present a new filter based method for the calculation of PI achievable performance for simple feedback control systems. We also extend this new approach to multiloop systems and address some key questions such as (a) what would be the performance with alternate loop pairing? (b) what performance improvement will accrue through the use of decouplers in multiloop control systems?

Keywords: Control loop performance, PI achievable performance, Multiloop systems, Pairing, Decouplers

1. INTRODUCTION

In a typical process plant, there are thousands of control loops designed and implemented in order to achieve specific objectives. Continuous good performance of existing controllers is indispensable to meet the demands of the consumer in a cost effective way and to generate sustained benefits. This indicates the need for control loop performance benchmarks and measures. Astrom (1970), Harris (1989) and Stanfelj *et al.* (1993) used minimum variance control as the benchmark to assess control loop performance. Since then, there has been an interest in the academia and the industry for developing measures and benchmarks for control loop performance. In addition to the minimum variance benchmark, several alternate benchmarks also exist (Tyler and Morari, 1996; Kendra and Cinar, 1997; Swanda and Seborg, 1999). Applications of performance assessment techniques can be found in Thornhill *et al*. (1999). Huang *et al*. (1996) and Harris *et al*. (1996) extended the performance assessment concepts to MIMO feedback controllers. These works employ the multivariable minimum variance controller as the benchmark.

In the works described above, no restriction is placed on the form that the feedback controller Q can possibly take. If the structure of controller Q is complicated, it is not amenable for implementation on standard industrial DCS. Desborough and Miller (2001) surveyed the status of controllers employed in the chemical industry and concluded that a typical chemical plant has 98% PID type controllers and a vast majority of these controllers are PI controllers. Hence, it is essential to know to what extent can the given process be effectively regulated (for stochastic disturbances) with PI controllers. Therefore, the PI achievable performance is more realistic benchmark from the perspective of the chemical industry. The Approximate Stochastic Disturbance Realization (ASDR) technique developed by Ko and Edgar (1998) for estimating the PI achievable performance assumes that the process model is known. The ASDR method approximates the noise model from routine operating data. Recently, Agrawal and Lakshminarayanan (2003) described a method to determine the control loop performance achievable with PI type controllers, the optimal control settings that will yield the best performance and the expected robustness margins using closed loop transfer functions identified from closed loop experimental data.

The objective of this article is to describe a method to calculate the PI achievable performance of feedback control systems using the knowledge of process model alone. The key idea is the derivation of a filter that can provide routine closed loop operating data for any controller Q^* using information of the process model and original controller Q alone.

The paper is organized as follows. Section 2 provides a brief overview of existing methods that are available for the calculation of the PI achievable performance. In section 3, we propose a new method for the calculation of the PI achievable performance in single input single output systems. The proposed method is extended to multiloop systems in section 4. In section 4, we also outline a method to obtain PI achievable performance for alternate control loop pairing and for multiloop control systems with decouplers. Examples to demonstrate the proposed method are shown in section 5, followed by conclusions.

2. COMPUTATION OF PI ACHIEVABLE PERFORMANCE

We assume the reader to be familiar with the basics of the control loop performance assessment theory that employs minimum variance controller as the benchmark. The interested reader is referred to the exceptional coverage provided by Qin (1998), Huang and Shah (1999) and Harris *et al*. (1999).

The maximum control loop performance that can be attained by restricting the controller structure to PI type is called PI achievable performance. Assuming that the open loop model (including time delay) is known, Ko and Edgar (1998) used routine closed loop operating data (no set point change is made to excite the process) to estimate the PI achievable performance. Using an ARIMA (p,1,1) model with $2 \leq p \leq 5$, they approximate the disturbance (noise) transfer function N by matching the first few coefficients of the estimated closed loop disturbance impulse response model. Once, the process and noise models are known, Ko and Edgar (1998) employ a numerical optimization procedure to estimate the highest control performance index reachable by restricting the feedback controller Q to a PI or PID form.

Agrawal and Lakshminarayanan (2003) proposed an alternate way of determining the PI achievable performance from closed loop experimental data (set point excited data). Their method uses identified closed loop process as well as disturbance models. The relationship between the controlled variable and the set point under closed loop is:

$$Y = \frac{QT}{1+QT} Y_{sp} + \frac{N}{1+QT} a = G Y_{sp} + Ha \qquad (1)$$

In equation (1), Y represents the controlled variable, Y_{sp} its set point; T is the open loop process model; Q is the feedback controller that is probably of the PID type; N is the open loop disturbance model; G is the closed loop servo response model and H is the closed loop disturbance model.

From equation (1), we can write

$$T = \frac{G}{(1-G)\,Q} \qquad (2)$$

and

$$N = \frac{H}{(1-G)} \qquad (3)$$

Assuming time invariant process (T) and noise dynamics (N), for a new controller Q^* the closed loop impulse response H^* is given by

$$H^* = \frac{N}{1+Q^* T} = \frac{\dfrac{H}{(1-G)}}{1+Q^* \dfrac{G}{(1-G)Q}} = \frac{H}{1+G\left(\dfrac{Q^*}{Q}-1\right)} \qquad (4)$$

We know that the control loop performance index could be obtained from the estimated closed loop impulse response H and the process delay d. Equation (4) implies that with the knowledge of the current closed loop impulse response H, closed loop servo transfer function G and the current controller Q, it is possible to estimate the closed loop impulse response H^* for any given controller Q^*. Given that the process delay d remains constant, it is possible to determine the optimal PI type controller Q^* that maximizes the performance. Hence, the PI achievable control loop performance can be computed from the knowledge of the current controller and current closed loop servo and disturbance transfer functions. Agrawal and Lakshminarayanan (2003) demonstrated the workability of the above scheme using several examples. They also ensured that deterministic control loop performance measures like the normalized integral absolute error, gain and phase margins are also within acceptable limits.

3. THE PROPOSED METHOD FOR PI ACHIEVABLE PERFORMANCE

Now, consider the process driven by white noise sequence a_t. The expression for the controlled variable Y_t is given by

$$Y_t = \left\{ \frac{N}{1+T\,Q} \right\} a_t \qquad (5)$$

If the current controller Q is replaced by the new controller Q^*, then new output data series Y_t^* is given by

$$Y_t^* = \left\{ \frac{N}{1+T\,Q^*} \right\} a_t \qquad (6)$$

Again, assuming a time invariant process T and noise dynamics N, equations (5) & (6) can be used to derive the following expression

$$\frac{Y_t^*}{Y_t} = \frac{1+TQ}{1+TQ^*} \qquad (7)$$

The right hand side of equation (7) represents the filter which gives the new "routine closed loop data series" Y_t^* when the current output data series Y_t^* passes through it. For any new controller Q^*, the filter is specified (if the open loop process model T and the original controller Q are known) and the original routine data Y_t can be used to "generate" the routine closed loop data Y_t^* that would be obtained with the controller Q^*. Using this Y_t^*, one can calculate the control loop performance index corresponding to the new controller Q^*. Incorporating this methodology in conjunction with an optimization routine, it is possible to determine the controller Q_{opt} that provides the maximum possible control loop performance. In this work, we are interested in having the optimal controller Q^* to be of PID type (more specifically PI type).

Also, substituting the expression for T from equation (2) into equation (7) will result in

$$\frac{Y_t^*}{Y_t} = \frac{1 + \frac{G}{(1-G)}}{1 + \frac{GQ^*}{(1-G)Q}} = \frac{Q}{(1-G)Q + GQ^*} \quad (8)$$

Equations (7) and (8) demonstrate that it is possible to obtain controllers that maximize the performance index with only the knowledge of the original controller and the open loop model T (equation 7) or the closed loop servo model G (equation 8). Notice that, in contrast to the earlier methods, we do not explicitly use the closed loop noise model H to determine the "best" controller. The noise component is implicit in the original closed loop data Y_t and will reflect in the generated Y_t^*. Y_t^* is then modelled using an ARMA structure to get the performance index for controller Q^*. Obviously, this method (like other methods which try to compute the PI achievable performance) requires fairly accurate open loop process model T or closed loop servo model G. This would inevitably require some experimentation on the process. The proposed method is therefore most useful when a first level control audit indicates poor performance of this loop and one desires to improve its performance.

4. EXTENSION TO MULTILOOP SYSTEMS

The above-described method can be easily extended to multiloop systems. Again, the central idea is to estimate the output data series for the new controller using known process model (open loop process model or closed loop servo model) and the present controller.

For the multiloop case, Equation (7) becomes

$$\frac{Y_t^*}{Y_t} = \frac{I + TQ}{I + TQ^*} \quad (9)$$

Equation (8) becomes

$$\frac{Y_t^*}{Y_t} = \frac{Q}{(I-G)Q + GQ^*} \quad (10)$$

where I is the identity matrix of appropriate dimension.

Consider the simplest multiloop example, i.e a 2 x 2 system. In this case, G, T, Q, Q^*, Y_t and Y_t^* take the following matrix form:

$$T = \begin{bmatrix} T_1 & T_3 \\ T_2 & T_4 \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & G_3 \\ G_2 & G_4 \end{bmatrix},$$

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}, \quad Q^* = \begin{bmatrix} Q_1^* & 0 \\ 0 & Q_2^* \end{bmatrix},$$

$$Y_t = \begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} \quad \text{and} \quad Y_t^* = \begin{bmatrix} Y_{1t}^* \\ Y_{2t}^* \end{bmatrix}$$

Again using equation (9) or equation (10), the output data series for any new controller Q^* can be predicted. This predicted series can be used to estimate the control loop performance index if the controller Q^* were in place. Using these equations within an optimization routine enables the calculation of the PI achievable control loop performance for the multiloop case.

The key element again is the knowledge of the process model T or the closed loop servo transfer function model G. Estimating either of these will involve some experimentation on the process. Some experiment is indispensable because of the need to compute the interactor matrix (a generalization of the time delay for multivariate systems) for performance assessment of multivariate systems (Huang *et al.*, 1997).

PI achievable with alternate pairing: In multiloop systems, the choice of input-output pairing is an important issue. Even after employing a certain pairing, one needs to ascertain if with alternate pairing the control loop performance can be improved. This question can be answered using the method proposed in this paper.

Continuing with the 2 x 2 example, the alternate pairing is chosen by changing the Q^* structure, i.e.

$$Q^* = \begin{bmatrix} 0 & Q_1^* \\ Q_2^* & 0 \end{bmatrix} \quad (11)$$

Equations (8) and (9) can still predict the data corresponding to any new controller with alternate pairing. Again, the use of an optimization procedure, will give us the optimal controller Q^* (corresponding to alternate loop pairing) that maximizes the CLPI for the process.

Optimal control loop performance with decouplers: Another well recognized problem in multiloop systems is that of interaction. Severe interaction among the loops deteriorates the performance. Decouplers can improve the performance in the interacting system by diagonalizing the plant effectively removing the interactions. The question of the extent of performance improvement possible with the use of decouplers is explored next.

Assuming that decoupled process is T^* is equivalent to the diagonalized T,i.e.

$$T = \begin{bmatrix} T_1 & T_3 \\ T_2 & T_4 \end{bmatrix} \quad \text{and} \quad T^* = \begin{bmatrix} T_1 & 0 \\ 0 & T_4 \end{bmatrix}$$

Now, for the process driven by white noise sequence, expression for Y_t is given by

$$Y_t = \left\{ \frac{N}{1 + T\,Q} \right\} a_t \tag{12}$$

For the decoupled process driven by white noise sequence with new controller Q^*, Y_t^* can be written as

$$Y_t^* = \left\{ \frac{N}{1 + T^*\,Q^*} \right\} a_t \tag{13}$$

Using equations (12) and (13) gives

$$\frac{Y_t^*}{Y_t} = \frac{1 + TQ}{1 + T^* Q^*} \tag{14}$$

Utilizing the filter obtained in equation (14), the PI achievable performance with decoupler and the corresponding controller parameters for Q^* can be found out by using an optimizer.

5. CASE STUDIES

The theory outlined thus far is validated with many examples in this section. For the SISO case, the process model is identified from the experimental data, obtained by set point changes in the plant. For the multiloop case, perfectly known process models are used just to demonstrate the efficacy of the developed theory. The developed method will be of greater practical relevance if identified models can be used. This will also demonstrate the robustness of the proposed method. Such investigations are currently in progress. The "fminsearch" routine available in the Optimization Toolbox (Matlab Version 6.5 Release 13) is employed in all the following examples.

<u>Example of a simulated SISO system</u>: This is a simulation of the closed loop system for a first order plus time delay process regulated by a PI controller. In particular, the process, noise and controller transfer functions are given by

$T(z^{-1}) = \dfrac{0.2\,z^{-3}}{1 - 0.8\,z^{-1}}$, $N(z^{-1}) = \dfrac{1}{1 - 0.95\,z^{-1}}$ and

$Q(z^{-1}) = \dfrac{0.14 - 0.12\,z^{-1}}{1 - z^{-1}}$ respectively. With the present controller CLPI is calculated to be 0.3922.

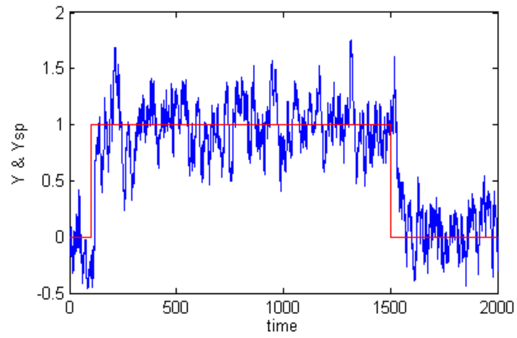

Fig. 1: Closed loop experimental data for example 1

With the experimental closed loop data shown in Figure 1, the closed loop servo transfer function G is identified. Standard tools from MATLAB's System Identification Toolbox are employed for this purpose. This identified model is used in Equation (8) to determine the optimal PI controller Q^* that maximizes the control loop performance. Using this approach, we predict the best possible performance for a PI controller (PI achievable performance) to be 0.78 and the optimal controller to be $Q^*(z^{-1}) = \dfrac{1.568 - 1.543\,z^{-1}}{1 - z^{-1}}$. With complete knowledge of the open loop process model, we get the PI achievable performance to be 0.83 and the optimal PI controller is $Q^*(z^{-1}) = \dfrac{1.696 - 1.632\,z^{-1}}{1 - z^{-1}}$. We therefore note that our method provides a very good estimate of what is theoretically achievable for this process with a PI controller.

<u>A Multiloop example</u>: Consider a 2 x 2 example with the following models for the process and noise. Note that we have integrated white noise affecting the process.

$$T = z^{-2} \begin{bmatrix} \left(\dfrac{-0.19}{1 - 0.905 z^{-1}} \right) & \left(\dfrac{0.948}{1 - 0.368 z^{-1}} \right) \\ \left(\dfrac{0.948}{1 - 0.368 z^{-1}} \right) & \left(\dfrac{0.19}{1 - 0.905 z^{-1}} \right) \end{bmatrix}$$

$$N = \begin{bmatrix} \left(\dfrac{1}{1 - z^{-1}} \right) & \left(\dfrac{-0.6}{1 - z^{-1}} \right) \\ \left(\dfrac{0.5}{1 - z^{-1}} \right) & \left(\dfrac{1}{1 - z^{-1}} \right) \end{bmatrix}$$

Initially, the process is controlled using diagonal pairing (1-1/2-2). The controller matrix Q is as follows

$$Q(1,1)=\frac{-0.01+0.005z^{-1}}{1-z^{-1}} \quad ,$$

$$Q(2,2)=\frac{0.01-0.005z^{-1}}{1-z^{-1}} \text{ and}$$

$$Q(2,1)=Q(1,2)=0$$

With the present PI controllers the obtained performance index is 0.0562.

Using the theory described in section 4 and taking the present controllers as the initial guess, the PI achievable performance for multiloop system is estimated to be 0.3453. The optimal PI controllers obtained in this case are:

$$Q^{*}(1,1)=\frac{-0.4761+0.4106z^{-1}}{1-z^{-1}}$$

and

$$Q^{*}(2,2)=\frac{0.4437-0.3920z^{-1}}{1-z^{-1}} .$$

Further, for finding out the PI achievable performance with the alternate (1-2/2-1) pairing, a stable initial guess for controllers Q(1,2) and Q(2,1) are provided based on the knowledge of process model. In this example, the initial guess used is

$$Q(1,2)=\frac{0.0654-0.0327z^{-1}}{1-z^{-1}}$$

and

$$Q(2,1)=\frac{0.0654-0.0327z^{-1}}{1-z^{-1}} .$$

With this initial guess, the performance index is 0.1021. Again, proceeding with the developed equations, the PI achievable with alternate pairing is predicted to be 0.7217. The optimal controllers with off-diagonal pairing are:

$$Q^{*}(1,2)=\frac{0.7038-0.4137z^{-1}}{1-z^{-1}} \text{ and}$$

$$Q^{*}(2,1)=\frac{0.6859-0.4435z^{-1}}{1-z^{-1}} .$$

It is quite evident from the results obtained that the off-diagonal pairing gives a much better regulatory performance for this process than does the diagonal pairing.

Finally, to estimate the effect of decouplers in improving the control performance of the process, the theory developed in section 4 is followed. Starting with the present controllers as the initial guess, the PI achievable performance obtained with decouplers is found to be 0.8472. The optimal controllers for this decoupled process comes out as

$$Q^{*}(1,1)=\frac{-2.3662+2.190z^{-1}}{1-z^{-1}} \text{ and}$$

$$Q^{*}(2,2)=\frac{2.4389-2.2505z^{-1}}{1-z^{-1}} .$$

We therefore conclude that, for this example, the off-diagonal control pairing can raise the PI achievable performance by more than 100%. Alternately, if the diagonal pairing is employed, the performance of the control system (for stochastic disturbances affecting this process) can be improved significantly (from 0.35 to 0.85) if perfect decouplers are employed. All of these predictions are being made with only the knowledge of the process model T and current controller Q. It is possible to estimate the enhancement in the controller performance without actually implementing the decouplers.

<u>Another Multiloop example</u>: We next consider the well-known Wood & Berry model of an experimental distillation column. The process and noise models are as follows

$$T=\begin{bmatrix}\left(\frac{12.8*e^{-s}}{16.7s+1}\right) & \left(\frac{-18.9*e^{-3s}}{21s+1}\right)\\ \left(\frac{6.6*e^{-7s}}{10.9s+1}\right) & \left(\frac{-19.4*e^{-3s}}{14.4s+1}\right)\end{bmatrix}$$

$$N=\begin{bmatrix}\left(\frac{3.8*e^{-8.1s}}{14.9s+1}\right) & \left(\frac{0.22*e^{-7.7s}}{22.8s+1}\right)\\ \left(\frac{4.9*e^{-3.4s}}{13.2s+1}\right) & \left(\frac{0.14*e^{-9.2s}}{12.1s+1}\right)\end{bmatrix}$$

The Relative Gain Array (RGA) (Bristol, 1966) for this process is given by

$$\Lambda=\begin{bmatrix}2.0095 & -1.0095\\ -1.0095 & 2.0095\end{bmatrix}$$

The negative $(1,2)$ element of the RGA matrix suggests that no PI controller can achieve stability when off-diagonal pairing is used. Also, the Niederlinski index (Niederlinski, 1971) for the off-diagonal pairing turns out to be -0.9906, which confirms the instability.

The initial controller for the process is given by

$$Q(1,1) = \frac{0.5625 - 0.4946z^{-1}}{1 - z^{-1}}$$

$$Q(2,2) = \frac{-0.1125 + 0.1077z^{-1}}{1 - z^{-1}}$$

This controller gives a performance index of 0.5210. Further, using these present controllers as initial guess, the maximum performance index equal to 0.54. Hence, we can say that present controller (initial controller) is quite good and controller tuning is not contributing to the poor loop performance. Also, the alternate pairing need not be considered as it points to potential instability.

Interaction may be the factor for this low performance (0.54). Further analysis gives the PI achievable performance of 0.6627 with perfect decouplers in place. From the results obtained, it is quite evident that eliminating interaction does not significantly improve the loop performance. Further increase in performance may be possible with more complex controllers or by improved process design.

6. CONCLUSIONS

A novel method is developed to compute the PI achievable performance for SISO as well as multiloop systems. The effect of loop pairing and the implementation of decouplers are examined and the PI achievable performance is determined for these scenario *in silico*. The proposed method can therefore quantify the potential benefits in the performance that will result if alternate pairing or decouplers are employed.

REFERENCES

Agrawal P. and S. Lakshminarayanan, (2003). Tuning PID controllers using Achievable Performance Indices, *Industrial and Engineering Chemistry Research,* **42**(22), 5576-5582.

Åström, K.J. (1970). *Introduction to stochastic control theory*, Academic press, New York.

Bristol, E. H. (1966) On a new measure of interaction for multivariable process control, *IEEE Transactions on automatic control*, **AC-4**, 133-134.

Desborough, L. D.; Miller, R. M. (2001). Increasing Customer Value of Industrial Control Performance Monitoring – Honeywell's Experience, *Proceedings of CPC VI, Tucson*, USA.

Harris, T. J. (1989). Assessment of control loop performance. *Canadian Journal of Chemical Engineering*, **67**, 85.

Harris, T. J., F. Boudreau, and J. MacGregor. (1996). Performance assessment of multivariable feedback controllers *Automatica*, **32**(11), 1505-1518.

Harris, T.J., C. Seppala, and L. Desborough (1999). A review of performance monitoring and assessment techniques for univariate and multivariate control systems. *Journal of Process Control*, **9**, 1-17.

Huang, B., S. L. Shah, and K. Kwok (1996). How good is your controller? Application of control loop performance assessment techniques to MIMO processes. *Proceedings of 13th IFAC World Congress,* **Volume M**, San Franciso.

Huang, B., S. L. Shah, and H. Fujii (1997). The unitary interactor matrix and its estimation using closed-loop data. *Journal of Process Control*, **7**(3), 195-207.

Huang, B. and Shah, S.L. (1999). *Performance Assessment of Control Loops: Theory and Applications*, Springer-Verlag, London.

Ko, B.-S., & Edgar, T.F. (1998). Assessment of achievable PI control performance for linear processes with dead time. *Proceedings of the American Control Conference*, Philadelphia, USA, 1548-1552.

Kendra, S. and A. Cinar (1997).Controller performance assessment by frequency domain techniques. *Journal of Process Control*,**7**(3), 181-194.

Niederlinski, A. (1971).A heuristic approach to the design of linear multivariable interacting control systems. *Automatica*,**7**,691-701.

Optimization Toolbox, Version 2.2, *MATLAB* (Release 13), The MathWorks, Inc. USA.

Qin, S. J. (1998).Control performance monitoring – a review and assessment. *Computers and Chemical Engineering*, **23**, 173.

Stanfelj, N.; Marlin, T. E., MacGregor, J. F(1993). Monitoring and diagnosing control loop performance: The single loop case. *Industrial and Engineering Chemistry Research*, **32**, 301.

Swanda, A.P., & Seborg, D.E. (1999).Controller performance assessment based on setpoint response data. *Proceedings of the American Control Conference*, San Diego, USA, 3863-3867.

System Identification Toolbox, Version 2.2, *MATLAB* (Release 13), The MathWorks, Inc. USA.

Thornhill, N. F., Oettinger, M.; Fedenczuk, P. (1999).Refinery-wide control loop performance assessment. *Journal of Process Control*, **9**, 109.

Tyler, M. and M. Morari (1996). Performance monitoring of control systems using likelihood methods. *Automatica*, **32**, 1145- 1162.